EXERCISE 1
THE MONOCHORD: PYTHAGORAS, HARMONIA AND COSMOS

EXPERIMENTAL APPARATUS

This exercise uses the monochord: a device which was commonly used in teaching the theory of harmony from the time of the Greeks to the Renaissance. It is one of the few experimental tools actually used by the Greek philosophers who usually preferred to seek the nature of the universe by contemplation rather than by observation. We shall use a **monochord** a little like that used by Pythagoras in the sixth century BC and we shall measure lengths with a **metre rule**. If you cannot accurately divide one number by another (shame!), you will need a calculator, but using such a device is rather contrary to the spirit of the exercise. For an added touch of authenticity, please feel free to wear a toga.

INTRODUCTION: Some background on intervals and vibrating strings.

Pythagoras of Samos and the philosophers, mathematicians and lyre players of the Pythagorean school investigated the ratios of the lengths of pairs of identical strings which produced "harmonious" intervals in the scales with which they were familiar. An important example is the pentatonic scale (in our notation and the key of C, this scale is C-D-E-G-A-C). The Pythagoreans discovered that the length ratios were the ratios of whole numbers, and because they were very interested in mathematical order, they cited this arithmetic order in music as an example of "harmonia". For the Greeks, "harmonia" had an even larger figurative sense than "harmony" does for us today: Homer used it to mean either geometrical "fitting together" (as in say carpentry) or mental and spiritual agreement.

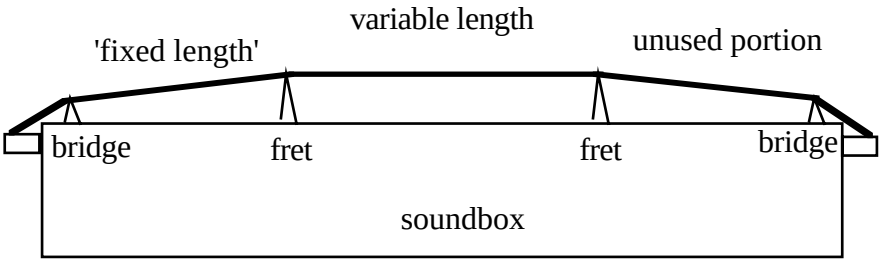
Harmony in music and mathematics was the paradigm of the order (*cosmos*) that Pythagoras and subsequent philosophers tried to impose on what turned out to be a less than compliant universe. They ascribed almost mystical importance to the numbers 1 to 4 and to their sum (10), and they sought arithmetic harmony in geometry, chemistry, astronomy, metaphysics and even moral philosophy. Plato, for example, described (perhaps allegorically) the universe as comprising wheels bearing the planets and stars. To each was attributed a pitch and, under the direction of the Sirens, Muses and Fates, they together produced the "music of the spheres".

Pitch and vibrating strings

A little background from modern knowledge. The musical parameter **pitch** is closely related to what physicists call **frequency**. If a particular sound has a pattern of sound pressure which is repeated 440 times per second, for example, it has a frequency of 440 cycles per second and in most cases it would be recognized as the note A in the treble clef. In western music at least, the size of a musical interval is (almost exactly) determined by the ratio of the frequencies. We shall deduce in this course that the frequency of vibration of an ideal string is inversely proportional to its length, so if two identical strings are used to sound two notes, the ratio of the lengths gives the reciprocal of the ratio of the frequencies. Although Archytas associated "high" pitch with speed and power, the relation between pitch and frequency was unknown to the Greeks. We shall largely ignore it for the purposes of this practical session, in which we shall investigate the relationship between between "harmonious" musical intervals and the lengths of pairs of identical strings used to sound them.

OBSERVATION

The monochord has a (fixed) bridge at either end and frets (or moveable bridges) that can be positioned under the string at any positions along the soundboard. Check that you have two frets of the same height, and position them under one string. You can choose whether to tension the string using a tuning peg or by placing masses on the hook attached at the end. **Do not over tighten the strings.** The tension should be such that a length of about 300 mm is comfortably pitched for you to "sing along" with the experiment.



First, position one fret at 400 mm from the bridge. Measure the length very carefully. This will be the fixed length. You will vary the position of the second fret to produce the variable length. Initially, set the variable length to 400 mm. Sound the two lengths by striking them lightly with a pen or pencil a few cm away from the fret, and adjust the position of the second fret until the notes are the same. Measure the two lengths of string. This interval (a consonance) is not very interesting: we shall now look for the length ratios that produce other intervals.

1. **The octave** (or diapason or do-do').

   Move the fret, strike the longer length, let it sound, and then stop the vibration with your hand. Then strike the shorter, but DO NOT LET THEM SOUND TOGETHER. Strike the pair in sequence, always taking care not to let them sound together, and, by repeating this procedure, adjust the second so that it sounds the musical interval of one octave higher than the first. (The first and last notes of an ordinary eight note scale are one octave apart.) Repeat the striking and adjust carefully to make the octave sound "in tune" to you. **Your ears are very sensitive to frequency, so you will probably notice the difference made by a change of less than 1 mm.** Measure the lengths carefully and record them and their ratio below. **Aim for an accuracy of better than 1 mm**. *In this measurement, and in all of science, it is very important to write down exactly what you do observe rather than to write down what you think your are supposed to observe.* If you and your lab. partner disagree on the tuning of an octave, note down both and keep the results for later discussion.

   a. The octave (notes sounded sequentially):

   long string; short string; ratio (long/short).

   Now strike the two strings and allow them to sound together. If there is any change required to improve the harmony, record the new ratio.

   b. The octave (notes sounded simultaneously):

   long string; short string; ratio (long/short).

2. For the rest of the intervals, use only the method of simultaneous sounding. We shall consider in order the intervals of the **perfect fifth** (the diapente), the **fourth** (the diatessaron), the **major sixth**, the **major third** and the **major second**. These intervals are respectively those between first and fifth; first and fourth etc. notes of an ascending major scale. You will probably may find it easiest to judge them by singing (fairly quietly) up the scale. Sing the first note a few times to establish it in your mind. You may find it easiest to sing a scale in numbers ("one-two-three" etc or in solfa names "do-re-mi" etc). Alternatively, you may find it helpful to look at this list of tunes; the first interval of which are given:

Octave (do-do'): "Somewhere over the rainbow", "I dig rock and roll music".

Fifth (do-so): "Scarborough fair", "It ain't necessarily so".

Fourth (do-fa): "Advance Australia fair", "The internationale".

Major Sixth (do-la): "The lord is my shepherd", "Jingle bells" (verse).

Major Third (do-mi): "When the saints go marching in", "Morning has broken".

Major Second (do re): "Sweet Georgia Brown", "The man I love".

Check the length of your "fixed length" from time to time, and remember to measure lengths accurately.

a. The fifth (do-so): long string; short string; ratio (long/short).

b. The fourth (do-fa): long string; short string; ratio (long/short).

c. The major sixth (do-la): long string; short string; ratio (long/short).

d. The major third (do-mi): long string; short string; ratio (long/short).

e. The major second* (do-re): long string; short string; ratio (long/short).

* The major second is the interval over which there is often the most variation in tuning. You and your lab partner may have quite different ideas on what is the best tuning. Record both opinions if they are different.

Calculation:

Complete the following table with the decimal approximations (to three places) of the ratios of the natural numbers up to twelve: divide the number at the head of each column by the numbers at the left of the rows. You need only fill in the ratios which fall between one and two.

	1	2	3	4	5	6	7	8	9	10	11	12
1	1.000	2.000	*	*	*	*	*	*	*	*	*	*
2	*	*	1.500	*	*	*	*	*	*	*	*	*
3	*	*	*			*	*	*	*	*	*	*
4	*	*	*	*				*	*	*	*	*
5	*	*	*	*	*					*	*	*
6	*	*	*	*	*	*						*
7	*	*	*	*	*	*	*					
8	*	*	*	*	*	*	*	*				
9	*	*	*	*	*	*	*	*	*			
10	*	*	*	*	*	*	*	*	*	*		
11	*	*	*	*	*	*	*	*	*	*	*	
12	*	*	*	*	*	*	*	*	*	*	*	*

Now write down the whole number ratio that most nearly approximates the length ratios you measured for the pentatonic intervals. (Remove common factors from the ratios where they occur—in other words, reduce 6:3 to 2:1.)

Major second,	Major third,	Fourth,	Fifth,	Major sixth,	Octave
:	:	:	:	:	:

CONCLUSION

1. This is a very interesting stage in science: you now have some data and it is time to look for an hypothesis or theory to explain them, and to predict the results of new experiments.

   Is there any simple order (any arithmetic harmonia) that you can see in musical intervals? Is there any obvious common property of the arithmetic ratios that you suspect might be important? Is there anything about the ratios that you have found that seems to suggest which will be most harmonious and which will be least harmonious? (It may help to rank them in order of decreasing harmony.) If you find that there is, express the general rule as briefly and as clearly as you can.

2. Now try to make some predictions from your general rule. For instance, how harmonious would you expect the following intervals to be: 3:1? 5:3? 17:8? Try them out and fit them into your ranking of "more or less harmonious". If necessary, try to find a new law.

3. Consider the following series of twelve ascending fifths: C G D A E B $F^{\#}$ $C^{\#}$ $G^{\#}$ $D^{\#}$ $A^{\#}$ $E^{\#}$ $B^{\#}$. Using your ratio for the fifth, what would be the ratio of the lengths of two identical strings producing the first and last note in this series of twelve fifths?

   Now on a piano, $B^{\#}$ is the same note as C, so the series of twelve fifths separates two C's which are seven octaves apart. Using your ratio for the octave, what is the length ratio of strings producing notes seven octaves apart?

   What is the difference between these two values? How big is this interval in comparison with a major second? (Express both as a percentage difference: $\frac{x_2 - x_1}{x_1}$ 100 %.)

   Does this pose any problems for the rule you have cited above? Comment.

CONTEMPLATION

The Pythagoreans did not confine their arithmetical analysis to (audible) music: every topic in the universe was open for discussion. In that spirit, the following questions are posed for contemplation and discussion. If you have any thoughts about them, note them down to share with your colleagues.

1. Here is a list of the orbital periods (in earth years) of the planets. Can you see any similar arithmetic harmonia relating their orbits?

Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
0.241	0.615	1.00	1.88	11.9	29.5	84.0	165	248

1. Can you see any *a priori* reason why there should be a harmonic relation in either or both of music and astronomy?
2. Can you write down notes that might form the scale for Plato's music of the spheres?
3. To continue the metaphor of Plato's cosmogony, what musical roles would you give to Kepler and Newton? (Einstein's corrections would be quite inaudible).
4. Is there any arithmetic harmony in the "music of the spheres" of Gustav Holst?