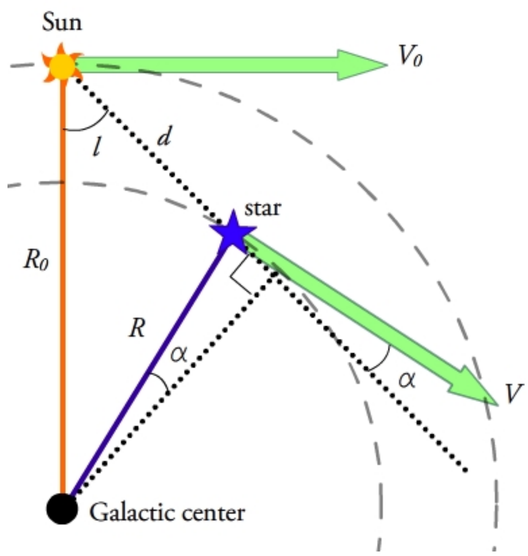
PHYS2160 Part I – 2015

Tutorial Problems

Working through these problems will help you grasp the material presented in lectures. I will set aside some time in lectures to go through the problematic ones in class. But you will need to put your hand up and say "Can I get assistance with this …"

1) If two stars have magnitudes $m_1 = 1$ and $m_2 = 6.5$, what is the ratio of their apparent brightness? Which is the brighter one?
2) What is the absolute magnitude of M of a star with apparent magnitude 6.5 at a distance of 15pc from the Earth?
3) HII regions
   a) Briefly explain what a HII region is.
   b) Calculate the radius of the Strömgren sphere for a rate of ionizing photons emitted into a cloud of $5 \times 10^{45} s^{-1}$, where the cloud has an electron density of $n_e=100 cm^{-3}$, a recombination coefficient $\alpha=2 \times 10^{-3}\ cm^3 s^{-1}$.
4) Oort's constants
   a) Calculate Oort's constants A and B for the case $R_0=7.1\pm1.2$kpc. Include uncertainty estimates for A and B in your answer.
   b) Now briefly explain in words what Oort's A and B constants tell us about the rotation of our Galaxy.
   c) Work through the Oort constant derivation presented in class **<u>yourself</u>**, working through the steps elided over. (The geometry is shown in the figure shown below)
      i) Then show how for a galaxy with a uniform mass density the tangential velocity (V) of material should be proportional to the galactocentric radius (R). i.e. $V \propto R$
      ii) While for stars in orbit about a mass enclosed within the radius R show $V \propto R^{-1/2}$



   d) Show that along a line of sight with longitude l, the maximum value of radial velocity ($v_{r,max}$) obeys

$$V(R_{min}) = v_{r,.max} + V_0 \sin l$$

Assume l<90 or > 270 (i.e. the line of sight is interior to the solar position) so you can Taylor expand $V(R_{min})$ (noting that $d<<R_0$, so that $\Delta R=-R_0(1-\sin l)$) to derive

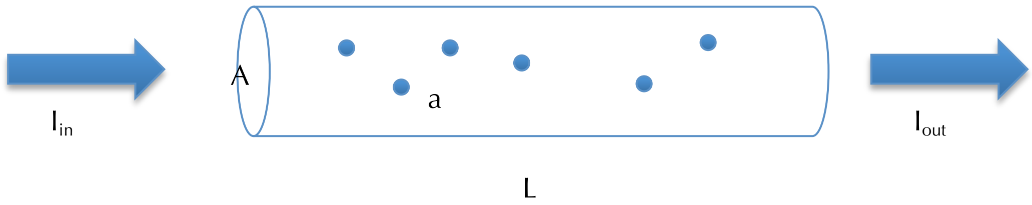
$$V(R_{min}) = V_0 - dV/dR|_{R0}\, R_0\, (1-\sin l)$$

then substitute and so that

$$v_{r,.max} = 2AR_0 (\sin l)(1-\sin l)$$

Explain all the symbols you use, and be sure you understand each step!

5) Radiative Transfer / Extinction – Work through the derivation of the 'optical depth' parameterisation for extinction along a line of sight. Consider the transfer of radiation through a cylinder of space with cross section A and length L, containing n absorbers each with radius a.



We can define a cross section for the absorbers $\sigma = \pi a^2$, and a volume for the cylinder of LA, so the total number of absorbers is nLA, and (as long as the absorbers are sufficiently sparse that they don't block each other) the fractional area blocked is

$$\sigma_{total} = \sigma\, n\, L\, A$$

and the fraction of light blocked is $F_{abs} = \sigma_{total}/A = n\sigma L$. Lets call this the optical depth τ.

Consider the cylinder as made up of a series of thin slices with length dL. The change in flux intensity produced by each slice is then the product of the fraction of light blocked times the light incident on that slice

$$dI = -\, I\, (n\sigma\, dL) = -I\, d\tau$$

or

$$dI/I = -d\tau$$

Integrate both sides over their full range to derive

$$I_{out} = I_{in}\, e^{-\tau}$$

Note that in class we defined a slightly different optical depth by adding an absorption efficiency coefficient to the $n\sigma L$ definition of optical depth.

$$\tau = \pi a^2\, Q_\lambda\, n\, L$$

6) Jean's Criteria for Cloud collapse (adapted from Maoz 2007)

a) Derivation

Let us assume, for simplicity, a spherical gas cloud of constant density ρ and temperature T, composed of particles with mean mass $\bar{m}$. The gas is ideal, classical, and nonrelativistic. The cloud's mass is M, its radius is r, and its gravitational energy is

$$|E_{\text{gr}}| \approx \frac{GM^2}{r}. \tag{5.1}$$

If the cloud undergoes a radial compression dr, the gravitational energy will change (become more negative) by

$$|dE_{\text{gr}}| = \frac{GM^2}{r^2}dr. \tag{5.2}$$

The volume will decrease by

$$dV = 4\pi r^2 dr, \tag{5.3}$$

and the thermal energy will therefore grow by

$$dE_{\text{th}} = PdV = nkT4\pi r^2 dr = \frac{M}{\bar{m}\frac{4}{3}\pi r^3}kT4\pi r^2 dr = 3\frac{M}{\bar{m}}kT\frac{dr}{r}. \tag{5.4}$$

The cloud will be unstable to gravitational collapse if the change in gravitational energy is greater than the rise in thermal energy (and the pressure support it provides),

$$|dE_{\text{gr}}| > dE_{\text{th}}. \tag{5.5}$$

Now show the substitutions to drive the Jeans criteria for the Jeans mass M_J (as a function of temperature, total mass and mean molecular weight)

$$\boxed{M_J = \frac{3kT}{G\bar{m}}r. \qquad (5.6)}$$

as well as equivalent expressions for the Jeans radius and the Jeans density.

b) Evaluate the Jeans density and radius for a $3M_{\odot}$ H_2 cloud with T=15K.

7) Planet transits

   a) How much light is blocked when a Uranus-sized planet crosses the disk of a Sun-like star? (Find and provide an estimate for Uranus' radius to do this calculation).

   b) How much light is blocked by a Mars-sized planet?

   c) What about for a $0.3M_{\odot}$ star crossing the disk of a Sun-like star (recall radius scaling relations from lectures).

8) Explain in a few lines the difference between synchrotron radiation and thermal bremsstrahlung radiation.

9) Briefly discuss one piece of evidence for the existence of significant amounts of *dark matter*.

10) Briefly explain why there are two possible ground states of atomic hydrogen, and so how the 21cm atomic line is formed.