Last time

- flux, luminosity and distance
- apparent magnitude (or more usually just "magnitude"), absolute magnitude & distance modulus
- flux/luminosity in a given bandpass vs bolometric (or "total integrated") flux/luminosity
- trigonometric parallax and definition of the parsec as a fundamental distance unit
- star counts as a way to probe the properties of a population of stars
- basic picture of the Milky Way





Disk – stars (young and middleaged stars like Sun), gas, dust

Central Bulge/Spheroid – stars (but largely hidden from view by the disk of the Milky Way).

Halo – globular clusters and old stars (which are asymmetric relative to the Sun)

The Sun orbits at 8kpc with velocity 220 km s⁻¹.

Galactic 'year' ~230 Myr. Mass of Galaxy ~ $6x10^{11}M_{\odot}$.

Co-ordinates

The Cartesian (x,y,z) three dimensional co-ordinate system (that we all know and love) is not very useful in astronomy. We observe the Universe projected onto a sphere at enormously large distances, so a spherical-polar co-ordinate system makes much more sense.

We can easily measure the pair of angles – (α, δ) in the example below – to very similar levels of precision, and be left with the separate problem of how to measure the distance (r)

We most commonly use an Earth-based co-ordinate "equatorial" system where the place that defines the coordinate α – known as "right ascension" – is based on the Earth's equator, and the axis that defines the δ co-ordinate – known as "declination" – is based on the Earth's axis of rotation.

It's basically a celestial projection of the longitude/latitude system used to navigate on the surface of the Earth.



Co-ordinates

In this co-ordinate system, the stars have (more-or-less) fixed co-ordinates, and the whole system *appears* to rotate above us.

As noted before measuring (α, δ) to the same levels of precision is straightforward. Typically getting precisions of 0.1" (1/1,296,000th of a full circle) is straightforward, and to 0.001" (1 milliarcsecond or mas) is doable with effort.

Measuring distances to objects in the Universe to better than 10% is typically quite hard to do.

 (α, δ) can be reported in units of decimal degrees or in radians, but are more commonly reported in sexagisamal notation – Hours:Minutes:Seconds for α and Degrees:Minutes:seconds for δ .



Velocities

Just as we can define a velocity vector for any particle in a three dimensional space ...

So we can define one in our equatorial co-ordinate system (α , δ , r). The relevant velocities are μ_{α} , μ_{δ} , v_r

The first two are angular "proper motions" across the sky (i.e. velocities in the plane tangent to the celestial sphere at that point, and aligned with the right ascension and declination directions at that location on the sky) and a "radial velocity", which is the velocity towards or away from us along the line of sight.

 $\mu_{\alpha,}\mu_{\delta}$ are straight forward to measure, by taking pairs of observations separated in time ...



Proper motions

For example - at right are three images of the very cool "brown dwarf" UGPS0722 from 1998 to 2010. Three circles highlight the object's at each epoch. Additional observations (below) allow one to perform a solution for both proper motion *and* parallax.

Proper motions are typically expressed in arcsec/yr or sometimes as millarcsec/yr = mas/yr

The orthogonal proper motion components in the right ascension and declination directions can be combined in quadrature to give the total amplitude of the proper motion. So in the case of UGPS0722

$$\mu^{2} = \mu_{\alpha}^{2} + \mu_{\delta}^{2}$$

$$\mu = \sqrt{(906.9^{2} + 351.0^{2})} = 972.4 \text{ mas/yr}$$

The total proper motion is 0.972"/yr. (These data are how the parallax mentioned last time are measured. In this case the object lies just 4.2pc away.

The fastest known object (measured by apparent proper motion) is Barnard's Star at 10.37"/yr at a distance of 1.834pc.





Lucas et al. 2010 MNRAS, 408, 56. http://arxiv.org/abs/1004.0317v2

Tangential Velocity

If the proper motion *and* distance is known, then one can determine the physical velocity of the object (in the tangential plane of the sky) using

$$v_T = 4.74 \ \mu \ / \pi$$

for v_T in units of km/s, if μ has units of arcsec/yr and π has units of arcsec.

Radial Velocity

Radial velocities are determined using the Doppler shift to provide the velocity along the line of sight.

If you know the rest wavelength of a spectral line as obtained in the laboratory here on Earth (λ_{rest}), and then observe the same spectral line in an astronomical object at a different wavelength (λ_{obs}) then the difference between those observations is the Doppler shift and provides the relative line-of-sight velocity

For $\Delta \lambda = \lambda_{rest} - \lambda_{obs}$ then $v_r = c \Delta \lambda / \lambda_{obs}$

Where *c* is the speed of light. It is always quoted in the sense that motions toward you (called "blue shifted" because λ_{obs} is smaller [and so bluer] than λ_{rest}) are +ve, and motions away ("redshifted") are -ve. Total space velocity *V* is then just the sum of all three components of the space velocity

 $V^{2} = v_{r}^{2} + v_{T}^{2} = v_{r}^{2} + v_{\alpha}^{2} + v_{\delta}^{2}$

The measurement of positions, velocities and brightnesses form the core of much of modern astronomy.

Galactic Co-ordinates

Just as one can define an *equatorial* spherical-polar co-ordinate system based on the orientation of the Earth (because it's useful), so one can define a *Galactic* spherical-polar co-ordinate system based on the orientation of the Galaxy.

Galactic co-ordinates are centred on the Sun, and defined by two angles

I is the Galactic longitude defined to be zero in the direction the Galactic centre

b is the Galactic latitude defined to be zero in the galactic plane

The Galactic centre direction lies in the constellation of Sagittarius

at α = 17h45m40.04s, δ = -29° 00' 28.1"



Galactic co-ordinates viewed from outside Milky Way



Galactic & equatorial co-ordinates viewed from the Earth

The Disk (1)

Geometry: Flattened structure of material (stars, gas, dust) that orbits the centre of the Milky Way on predominantly circular orbits. Its density is reasonably well parameterised as an exponential disk in both the radial and vertical directions

 $\rho(r,z) = \rho_0 \exp(-r/r_d) \exp(-z/h_d)$

The radial scale length is r_d = 3.5±0.5kpc, so at the Sun's 8kpc radius its density is only ~10% of that at the Galactic centre, placing us well into the outer regions of our Galaxy. The characteristic scale height h_d is around 330pc for older stars like the Sun. The Sun lies within about 30pc of the mid-plane of the disk.

Gas and Stars : As well as a dense population of stars, the Disk also contains a significant reservoir of gas. It is this gas that is responsible for on-going star formation in the Milky Way.

The gas and dust disk has a significantly lower scale height (around 160pc) than the disk of older stars (330pc), as do the very young stars currently forming from this gas and dust.

It is also this gas that is slightly concentrated by travelling spiral density waves to produce the spiral arms, that are the dominant visual feature of most galaxy disks. These density waves are believed to trigger gravitational instabilities in dense clouds of gas, which in turn initiate the formation of young stars. It is these bright, hot young stars which produce the clearly visible spiral arms.





Young hot stars trace out the spiral arms in M51 (HST)

Galaxies are almost entirely "collisionless" systems as far as stars are concerned

The mass of stars in the Milky Way disk within the inner 3.5kpc (one scale radius) is about $10^{10}M_{\odot}$. If we assume the average mass of a star is about $0.5M_{\odot}$, than that implies some 2×10¹⁰ stars. To get a number density lets assume they occupy a volume given a cylindrical disk of radius 3.5kpc and thickness 2×330pc ... so

 $n_* \sim 2 \times 10^{10} / (\pi (3500 pc)^2 \times 2 \times 330 pc) \sim 1 pc^{-3}$.

Or a mean distance between stars of $d=1/\sqrt[3]{n_*} \sim 1pc$.

Ignoring gravity for the time being, the mean free path for a star to make a direct collision with another star will be

l ~ 1/(n₊ σ)

Where σ is the geometric cross-section for collision. For a solar radius star $\sigma = \pi (2r_{\odot})^2$, which means $l \sim 4.8 \times 10^{30}$ m = 1.5×10^{14} pc. That's a very large number compared to the size of the galaxy!

Or put another way, given the average random velocities of stars relative to each other of about 20km/s, this corresponds to a mean time between direction collisions of 2.4×10^{26} s or 7.6×10^{18} yr – almost a billion times longer than the age of the Universe.

In practice, gravitational focussing (i.e. nearby stars attracting each other) increases this cross section by a factor of ~1000. But this is not enough to change the fact that collisions in a disk are incredibly rare.

The Bulge/Spheroid and the Stellar Halo

Geometry: predominantly spherical "cloud" of stars on randomly distributed elliptical orbits. Stellar density that falls off as $\sim 1/r^3$.

Stellar Halo : is revealed by (1) globular clusters, and (2) as a population of "high velocity" stars in the solar neighbourhood (Since Halo stars are on predominantly "radial" orbits they are "left behind" by the circularly orbiting stars of the disk, including the Sun).



Stars in globular clusters and the halo (and the bulge) are old (ages in the range 10-14Gyr) and have much lower "metallicities" than that seen for stars in the Solar neighbourhood.

Metallicity is "astronomer speak" (i.e. historical and a bit silly) for the relative abundance of heavy elements in a star, compared to the amount of hydrogen. The elements in the Universe heavier than Li have all been formed in the cores of stars, and returned to the interstellar medium via stellar winds or supernovae explosions.

So very old stars will tend to have lower metallicities because their formation material has been through less cycles of enrichment. Metallicity is usually paramatrised via the Fe/H abundance ratio written as [Fe/H], which refers to the logarithmic Fe/H ratio relative to that of the Sun.

A star with [Fe/H] = -4.0 (i.e. Fe abundance 1/10,000th that of the Sun) is considered quite metal poor, and the current record for the lowest metallicity star known is ~ [Fe/H] < -7 (Keller et al. 2014, Nature, 506, 463). The overall density of this Halo population is *very* small compared to that of the disk (~1/10000th) at the Galactic radius of the Sun.

ulge" is readily apparent – the "Halo" is not. It's ily seen via Globular clusters, and "high velocity" stars near the Sun.

The Bulge/Spheroid and the Stellar Halo

Bulge / Spheroid : similar geometry (and density profile) to the Halo, but is much more obvious in the Galactic central region because its density is much higher. It is not clear whether it and the Halo are the same population (i.e. the halo is an outer extension of the bulge), or distinct ones. Similar bulges are seen in almost every spiral galaxy.

It is believed that many (if not most) galaxy bulges are host to a massive black hole ...

In our own Galactic centre, the use of very high resolution imaging techniques at infrared wavelengths allows the stars near the Galactic centre to be monitored. These proper motions allow the orbits of objects to be tracked, which reveals the presence of a dark, compact, massive object at the Galactic centre.



Left: An HKL-band colour mosaic of the region around the black hole at the Galactic centre: $H(1.8 \ \mu m) =$ blue, K'(2.2 \ \mu m) = green, L'(3.8 \ \mu m) = red.

Right: Blow-up of the 0.8"×0.8" region around the position of the supermassive black hole (labelled Sgr A*).

Early data : 1992-1998

Later data!





A 2.2 micron animation of the stellar orbits in the central parsec. Images taken from the years 1995 through 2011 are used to track specific stars orbiting the proposed black hole at the centre of the Galaxy.

Astrometric positions and orbital fits for 2 stars, within the central 0."8 ×0."8 of the Galaxy, that show significant deviation from linear motion for measurements obtained at the Keck telescopes between 1995 and 2012.

Positions are plotted in the reference frame in which the central dark mass is at rest. Overlaid are the best fitting simultaneous orbital solutions, which assume that all the stars are orbiting the same central point mass.

These two stars have orbital periods of ~ 16 and ~ 11 years.

These orbits, and a simple application of Kepler's Laws, provide the best evidence yet for a "supermassive" black hole, which has a mass of 4 $\times 10^{6} M_{\odot}$.

Fig. 2. The orbits of SO-2 (black) and SO-102 (red). RA. right ascension: DEC. declination. The data points and the best fits are shown. Both stars orbit clockwise. The dashed lines represent the parts of the orbits that have been observed with Speckle data: the solid lines indicate AO observations. The data points for SO-2 range from the year 1995 to 2012, and S0-102's detections range from 2000 to 2012. The connecting lines to the best fit visualize the residuals. Although the best-fit orbits are not closing, the statistically allowed sets of orbital trajectories are consistent with a closed orbit. SO-102 has an or-



bital period of 11.5 years, which is 30% shorter than that of S0-2, the shortest-period star previously known.

from Meyer et al. 2012, Science, 338, 84

Our Milky Way has a fairly *small* black hole at its centre ... as we will see later, other galaxies have much larger ones, and it may well be that almost all galaxies harbour black hole at their centre.

The Disk (2) - Rotation

The Galactic disk does not rotate as a solid body, but rather differentially rotates – it rotates faster at smaller Galactic radii. This can be observed in even the Solar Neighbourhood by looking at the mean motions of stars in the disk.

If we assume we are a 'standard of rest', then stars exterior to us are seen to lag behind, while those interior to us advance ahead. Stars at the same Galactocentric radius have the same velocity and appear not to move.

In practice, stars do not have perfectly regular, circular orbits. So while they do tend to stay near the same Galactocentric radius, they do move both radially and above and below the disk. The result is an apparent "random" fluctuation in motion compared to the "bulk flow" of the disk – what are known as *peculiar velocities*.

We define a "Local Standard of Rest" (or LSR) in our Galactic co-ordinate system which is the velocity an "ideal" star would have were it to have no peculiar velocity. (The Sun's peculiar velocity relative to the LSR is ~13km/s.)

How can we measure the extent of that differential rotation?



Figure 12.9. The pattern of motions to be expected for the differential mean motions of stars within a few thousand light-years of the Sun.

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Fig 12.9 from Shu
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The Disk (2) – Oort's Constants

Consider a star in the mid-plane of the disk with Galactic longitude *l* at a distance *d* from the Sun. Assume that both the star and the Sun have circular orbits around the centre of the Galaxy at radii of *R* and R_0 from the Galactic centre, and rotational velocities of *V* and V_0 respectively. The motion of the star observed from the position of the Sun along our line of sight (i.e. its radial velocity *Vobs*,*r*), and motion of the star across the plane of the sky, (or transverse velocity *Vobs*,*t*), are then:

$$V_{\text{obs, r}} = V_{\text{star, r}} - V_{\text{sun, r}} = V \cos(\alpha) - V_0 \sin(l)$$
$$V_{\text{obs, t}} = V_{\text{star, t}} - V_{\text{sun, t}} = V \sin(\alpha) - V_0 \cos(l)$$

Where α is the angle the star's velocity makes to the line of sight. For circular motions, we can convert those linear velocities to angular ones ($v = \Omega r$), to get

$$V_{\text{obs, r}} = \Omega R \cos(\alpha) - \Omega_0 R_0 \sin(l)$$
$$V_{\text{obs, t}} = \Omega R \sin(\alpha) - \Omega_0 R_0 \cos(l)$$

From the geometry in the figure, one can see that the triangles formed between the galactic center, the Sun, and the star share a side or portions of sides, so the following relationships hold and substitutions can be made:

$$R\cos(\alpha) = R_0 \sin(l)$$
$$R\sin(\alpha) = R_0 \cos(l) - d$$

to get

$$V_{\text{obs, r}} = (\Omega - \Omega_0) R_0 \sin(l)$$
$$V_{\text{obs, t}} = (\Omega - \Omega_0) R_0 \cos(l) - \Omega d$$

What we really want, however, is that expression in terms of observable quantities (l,d) rather than angular velocities.



The Disk (2) – Oort's Constants

To do that we take advantage of the Taylor expansion in Ω - Ω_0 about the position R_0 . Recall that for a function *f* near value *a*, one can write

$$f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \cdots$$

So using just the first two terms we can expand $\Omega(r)$, and rearrange to get

$$(\Omega - \Omega_0) = (R - R_0) \frac{d\Omega}{dr}|_{R_0} + \dots$$

 $R - R_0 = -d \cdot \cos\left(l\right)$

And in addition as long as we are studying local stars (so $d \ll R$ and R_0).

So

$$V_{\text{obs, r}} = -R_0 \frac{d\Omega}{dr}|_{R_0} d \cdot \cos\left(l\right) \sin\left(l\right)$$
$$V_{\text{obs, t}} = -R_0 \frac{d\Omega}{dr}|_{R_0} d \cdot \cos^2\left(l\right) - \Omega d$$

Using the sine and cosine half angle formulae $(sin(2A) = 2 sin A cos A, cos(2A) = 2 cos^2 A - 1)$ these velocities may be rewritten as functions of 2*l*:

$$V_{\text{obs, r}} = -R_0 \frac{d\Omega}{dr} |_{R_0} d\frac{\sin(2l)}{2}$$
$$V_{\text{obs, t}} = -R_0 \frac{d\Omega}{dr} |_{R_0} d\frac{(\cos(2l)+1)}{2} - \Omega d = -R_0 \frac{d\Omega}{dr} |_{R_0} d\frac{\cos(2l)}{2} + \left(-\frac{1}{2}R_0 \frac{d\Omega}{dr}|_{R_0} - \Omega\right) dr$$

We can write the velocities in terms of the measurable quantities and two coefficients A and B :

$$V_{\text{obs, r}} = Ad\sin(2l) \qquad \qquad A = -\frac{1}{2}R_0\frac{d\Omega}{dr}|_{R_0}$$
$$V_{\text{obs, t}} = Ad\cos(2l) + Bd \qquad \qquad B = -\frac{1}{2}R_0\frac{d\Omega}{dr}|_{R_0} - \Omega$$



The Disk (2) – Oort's Constants

So we have the "Oort constants" A and B expressed in terms of angular rotation velocities. These can then be transformed into linear velocities by differentiating $\Omega = v/r$ and substituting

$$A = -\frac{1}{2}R_{0}\frac{d\Omega}{dr}|_{R_{0}} \implies A = \frac{1}{2}\left(\frac{V_{0}}{R_{0}} - \frac{dv}{dr}|_{R_{0}}\right)$$
$$B = -\frac{1}{2}R_{0}\frac{d\Omega}{dr}|_{R_{0}} - \Omega \qquad B = -\frac{1}{2}\left(\frac{V_{0}}{R_{0}} + \frac{dv}{dr}|_{R_{0}}\right)$$

What is the physical meaning of these 'constants'?

- A is a measure of the *shear* (i.e. how much angular velocity changes with radius) in the Solar Neighbourhood.
- If A is positive, then this implies the Galaxy's angular velocity is *decreasing* with *increasing* Galactocentric radius in the Solar Neighbourhood.
- If A is zero, then there is no shear and one would have solid body rotation (i.e. $V=\Omega R$). In which case, B can be seen to just be the magnitude of the angular velocity.
- B describes the angular momentum gradient in the solar neighbourhood, and is also referred to as vorticity.

The Disk (2) – Oort's Constants

As noted above ...

$$V_{\text{obs, r}} = Ad\sin(2l)$$
$$V_{\text{obs, t}} = Ad\cos(2l) + Bd$$



Which can be rearranged to give A and B solely in terms of measurables (radial velocity, tangential velocity, distance, longitude)

$$A = \frac{V_{\text{obs, r}}}{d \sin(2l)}$$
$$B = \frac{V_{\text{obs, t}}}{d} - A \cos(2l)$$

The Disk (2) – Oort's Constants

So, if you can measure distances, radial velocities and longitudes, and plot $V_{obs,r}/d$ as a function of *l*, you can determine A and B observationally.

$$A = \frac{V_{\text{obs, r}}}{d \sin(2l)}$$
$$B = \frac{V_{\text{obs, t}}}{d} - A \cos(2l)$$



Figure 19–8 Observed galactic rotation. The radial velocities of nearby Cepheids are plotted as a function of galactic longitude. These are motions with respect to the LSR. The solid curve is the expected motions in the Oort model.

Using Cepheids (a class of stars for which distances can be measured – more on this later) you can apply an Oort model and start to understand the *observed* differential rotation of the Galaxy. A is indeed found to be non-zero, which tells us that the Galaxy is differentially rotating, and not rotating as a solid body.

Modern values for A and B (Feast et al. 1997)

 $A = 14.8 \pm 0.8$ km/s/kpc $B = -12.4 \pm 0.6$ km/s/kpc

The Disk (2) – Oort's Constants

One can then take potential models for the rotation curve of our Galaxy (e.g. solid body, Keplerian, flat), determine what their Oort constants would be, and ask whether they agree with what we see (which is A = 14.8 ± 0.8 km/s/kpc, B = -12.4 ± 0.6 km/s/kpc).

For example if orbits in the local neighbourhood followed Keplerian orbits

$v = \sqrt{\frac{GM}{r}}$	which implies	$dv = 1 \int GM = 1 v$
		$\frac{1}{dr} = -\frac{1}{2}\sqrt{\frac{1}{R^3}} = -\frac{1}{2}\frac{1}{r}$

One can show that the Oort constants would be which for the known Galactic rotation and solar position would give A~20km/s/kpc and B~-7km/s/kpc. So this doesn't match what we see. $A = \frac{1}{2} \left(\frac{V_0}{R_0} + \frac{v}{2r} |_{R_0} \right) = \frac{3V_0}{4R_0}$ $B = -\frac{1}{2} \left(\frac{V_0}{R_0} - \frac{v}{2r} |_{R_0} \right) = -\frac{1V_0}{4R_0}$

What if the rotation curve was flat (ie. V independent of radius, or equivalently dv/dr = 0? This gives

$$A = \frac{1}{2} \left(\frac{V_0}{R_0} - 0|_{R_0} \right) = \frac{1}{2} \left(\frac{V_0}{R_0} \right)$$
$$B = -\frac{1}{2} \left(\frac{V_0}{R_0} + 0|_{R_0} \right) = -\frac{1}{2} \left(\frac{V_0}{R_0} \right)$$

and substituting the known solar rotation velocity and radius into that gives A~14km/s/kpc and B~-14km/s/kpc, which is remarkably close to the measured values.

The Disk (2) – Oort's Constants

So the Oort constants provide insight into the nature of the Galactic rotation curve (at least in the Solar Neighbourhood) ... the rotation curve is more similar to a flat one, than to a Keplerian one.

This is an insight we will come back to next time ... when we look at Galactic rotation curves for the whole Galaxy (and for other galaxies)



Are you interested in contributing to the improvement of teaching in Physics? Ensuring that the opinions of students are heard? Become a course representative.

What's a course representative?

•A student who acts as a liaison between the students in a course and the School of Physics Teaching Committee. This may include meeting with the Committee to provide general feedback; or passing on student problems or complaints.

•We would like one representative from every undergraduate physics course.

How to nominate:

•Send an email with your name, student number and course code to Sue Hagon <u>s.hagon@unsw.edu.au</u> by Sunday 2 August. If more than one student nominates in a course, we will organize a vote.

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References

- Feast, M.; Whitelock, P. (November 1997). "Galactic Kinematics of Cepheids from HIPPARCOS Proper Motions". MNRAS 291: 683. arXiv:astro-ph/9706293.
- Website of Andrea Ghez's Galactic Centre group at UCLA : http://www.astro.ucla.edu/~ghezgroup/gc/
- Also copies of the movies in this Lecture are available at the <u>PHYS2160 Part I Materials page</u>.
- Bibliography
 - Shu, F. The Physical Universe, Chapter 12 (can be found on google books)

Useful constants, units, and formulae:

Gravitational constant	G :	= 6	6.67	×	10^{-11}	${\rm N~m^2~kg^{-2}}$
Speed of light	c :	= 3	3.00	\times	10^{8}	${\rm m~s^{-1}}$
Planck constant	h :	= 6	6.626	\times	10^{-34}	Js
Boltzmann constant	k :	= 1	.38	\times	10^{-23}	$\rm J~K^{-1}$
Stefan-Boltzmann constant	σ :	= 5	5.67	\times	10^{-8}	$\mathrm{W}~\mathrm{m}^{-2}~\mathrm{K}^{-4}$
Mass of the hydrogen atom	m_H :	= 1	.67	\times	10^{-27}	kg
Solar mass	M_{\odot}	=	1.99	×	10^{30}	kg
Solar radius	R_{\odot}	=	6.96	×	10^{8}	m
Earth mass	M_{\oplus}	=	5.98	\times	10^{24}	kg
Equatorial radius of Earth	R_\oplus	=	6.378	×	10^{6}	m
Mass of moon	M_{moon}	=	7.3	\times	10^{22}	kg
Astronomical unit	AU	=	1.496	i ×	10^{11}	m
Parsec	\mathbf{pc}	=	3.086	i ×	10^{16}	m
Hubble's constant	H_0	=	70			$\rm km~s^{-1}~Mpc^{-1}$

Distance modulus	m - M	=	$5\log d - 5$	(d in pc)
Apparent magnitude	$m_2 - m_1$	=	$2.5 \log \frac{f_1}{f_2}$	
For small recession velocities	v/c	=	$\Delta\lambda/\lambda$	
Definition of redshift	(1 + z)	=	$\lambda_{obs}/\lambda_{rest}$	
Energy and frequency	E	=	h u	
Frequency and wavelength	c	=	$ u\lambda$	