PCT

WORLD INTELLECTUAL PROPERTY ORGANIZATION
International Bureau

INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(51) International Patent Classification 5 : G10D 13/08, G10K 1/06 | A1

(11) International Publication Number: WO 93/18503

(43) International Publication Date: 16 September 1993 (16.09.93)

(21) International Application Number: PCT/AU93/00101

(22) International Filing Date: 10 March 1993 (10.03.93)

(30) Priority data:
PL 1261 10 March 1992 (10.03.92) AU

(71) Applicant *(for all designated States except US):* COMMONWEALTH SCIENTIFIC AND INDUSTRIAL RESEARCH ORGANISATION [AU/AU]; Limestone Avenue, Campbell, ACT 2601 (AU).

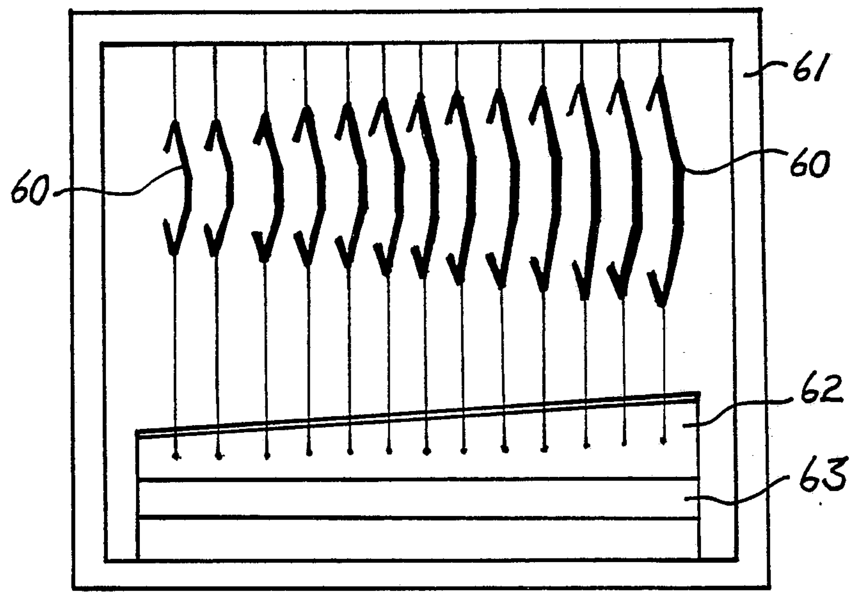
(72) Inventors; and
(75) Inventors/Applicants *(for US only)* : FLETCHER, Neville, Horner [AU/AU]; 30 Rosebery Street, Fisher, ACT 2611 (AU). HENDERSON, Moya, Patricia [AU/AU]; 10 Corniche Road, Church Point, NSW 2105 (AU).

(74) Agents: DUNCAN, Alan, David et al.; Davies Collison Cave, 1 Little Collins Street, Melbourne, VIC 3000 (AU).

(81) Designated States: AT, AU, BB, BG, BR, CA, CH, CZ, DE, DK, ES, FI, GB, HU, JP, KP, KR, KZ, LK, LU, MG, MN, MW, NL, NO, NZ, PL, PT, RO, RU, SD, SE, SK, UA, US, VN, European patent (AT, BE, CH, DE, DK, ES, FR, GB, GR, IE, IT, LU, MC, NL, PT, SE), OAPI patent (BF, BJ, CF, CG, CI, CM, GA, GN, ML, MR, SN, TD, TG).

Published
With international search report.

(54) Title: A MUSICAL PERCUSSION INSTRUMENT



(57) Abstract

A musical percussion instrument consists of more than three integrally formed, substantially linear, elongate sections. The sections are not colinear, but are substantially coplanar. The sections form a shape, and are made from a material (typically metal or a ceramic material) which has vibrational properties such that, when one section is struck with a mallet, the instrument emits a musically concordant sound. The mathematical analysis leading to alternative designs for such an instrument having five sections (termed a "pentangle"), made by bending mild steel rod, is presented. The sound produced by the instrument may be acoustically or electronically amplified. An array of "pentangles" (60), each tuned to different pitch, may be assembled to provide the equivalent of a keyboard instrument.

TITLE: "A MUSICAL PERCUSSION INSTRUMENT"

Technical Field

This invention relates to percussion instruments. More particularly, it concerns a musical percussion instrument containing a series of elongate, non-colinear, integrally formed sections which, when one section is struck with a mallet or beater, emits a musically pleasant sound. The quality of the emitted sound varies according to the nature of the mallet (soft or hard) and the way in which the instrument is struck. When struck with a hard beater or mallet, the instrument emits a bell-like sound containing partial tones in nearly harmonic relationship.

Background Art

The closest prior art to the present invention is the musical instrument commonly known as the "orchestral triangle" or the "percussion triangle". The traditional orchestral triangle produces a characteristic "triangle" sound of indefinite pitch. The triangle is not tuned to provide an harmonious sound. Although the relation between the mode frequencies of a percussion triangle could be varied to some extent by changing the base angles and corner curvatures of the triangle, the extent of such tuning is quite limited. It is possible to design a triangle having a nominal pitch fixed by its overall size, so as to bring two other mode frequencies into harmonic relation with this nominal pitch. However, the remaining inharmonic partials in such a "tuned" triangle are significantly prominent, and this is regarded by musicians as a limitation to the usefulness of the triangle.

Disclosure of the Invention

It is an object of the present invention to provide a musical percussion instrument having a series of elongate, non-colinear, integrally formed sections that is tuned to provide a musically pleasant sound when a section is struck by a mallet or beater.

This objective is achieved by providing a musical percussion instrument comprising a length or piece of metal or other suitable material (for example, a ceramic material) which is formed into a shape of more than three sections which, when one section is struck, will emit a sound having a frequency spectrum which is musically concordant. The sections of metal or other material are formed into a shape that is substantially planar, and preferably the frequency spectrum of the emitted sound is such that at least the first five in-plane modes are substantially harmonically related.

Thus, according to the present invention, there is provided a percussion instrument comprising a plurality of more than three integrally formed elongate sections, said sections being substantially co-planar, non-colinear, and formed from a material which, at room temperature, is rigid and has vibrational properties such that, when one of the sections is struck with a mallet, the instrument emits a musically concordant sound.

Preferably the instrument has five sections, which form a non-regular symmetrical shape. A particularly useful shape is one which is mirror symmetric about the centre point of the middle section, with its end sections of equal length, and with the intermediate sections, which are between the end sections and the central section, also of equal length. The present inventors have termed this structure a "pentangle" structure. In one realisation of the "pentangle" structure, the lengths of the sections are in the ratios 1.00 : 1.95 : 0.92 with two included angles of approximately 95° and the other two included angles being approximately 93°. In another realisation of this format, the lengths of the sections of the pentangle are in the ratios 1.00 : 1.85 : 0.97 and the included angles are approximately 146° and 10°.

The present invention also encompasses a percussion instrument which comprises an assembly or array of the individual instruments described above, each instrument in the assembly being tuned to a different pitch, to provide a desired musical scale. For example, a two octave scale (comprising 25 individual instruments) could be provided and played like a xylophone.

The instrument of the present invention may be associated with a sound radiator to increase the efficiency of the sound production. Such a radiator may be a resonant radiator or a non-resonant radiator, although it is preferred that a resonant radiator rather than a wide band, non-resonant radiator is used with an instrument comprising a single pentangle. Known radiator structures that may be used include tunable pipe or cavity (Helmholtz) resonators having a flexible diaphragm coupled to the instrument via a thin wire or cord. If the percussion instrument comprises an array of pentangles, a broad-band non-resonant soundboard backed by a cavity is preferred. Electronic amplification of the sound produced by instruments constructed in accordance with the present invention is also possible.

These and other features of the present invention will be discussed in more detail in the following description of examples of the present invention. In the following description, which is provided by way of example only, and which includes details of the derivation of suitable shapes for the integrally formed sections of an instrument constructed according to the present invention, reference will be made to the accompanying drawings.

Brief Description of the Drawings

Figure 1 illustrates a simplified pentangular shape which has been used as the starting point in the mathematical modelling of the preferred shapes of the present invention.

Figure 2 is a graph showing variation of the frequencies of the first few in-plane modes of an instrument constructed in accordance with the present invention, from a thin rod bent to form five sections.

Figure 3 shows solution surfaces in a 3-dimensional configuration space, which is referred to in the explanation of the derivation of a suitable shape for the present invention.

Figure 4 illustrates contours in the {θ, Φ} subspace, which is also referred to in the explanation of the derivation of a suitable shape for the present invention.

Figures 5 and 6 illustrate the pentangular shapes of two instruments constructed in accordance with the present invention.

Figure 7 is a partly schematic illustration of an assembly of individual pentangular instruments, constructed in accordance with the present invention.

Detailed Description of Illustrated Embodiments

Because (i) the present invention can be regarded as a significant improvement of the orchestral triangle, although used for a different musical purpose, (ii) the triangle is normally constructed by bending a metal rod, and (iii) it is expected that the present invention will also be constructed by bending a metal rod, the following description will be mainly directed to this construction technique. However, it should be appreciated that the sections of the integral body which constitutes the present inventive concept can be formed by casting a metal or a metal alloy, or by pressure moulding and firing a ceramic material. Casting techniques, and construction using a mechanically strong ceramic material having suitable vibrational properties (for example, certain oxide ceramics), are expensive when compared with the bending of a metal rod, but (a) they may enable possible problems associated with the choice of a radius of curvature for a bend in a rod to be avoided, and (b) top quality orchestral instruments are never inexpensive.

If the percussion instrument of the present invention is to be made from a ceramic material, any suitable ceramic fabrication technique may be used. Most ceramic bodies, however, are constructed using the following steps:

(a) a finely ground powder of at least one ceramic material is mixed with a fugitive binder;
(b) the mixture so formed is moulded to the required shape and pressed (for example, using isostatic pressing techniques) to form what is known as a "green" body;
(c) the green body is then fired to a temperature at which the ceramic material is sintered (during the early stages of the heating to the firing temperature, the fugitive binder is evaporated from the green body); and
(d) the sintered ceramic body is allowed to cool to room temperature at a cooling rate which ensures that large cracks in the body are not created.

In the prototypes of the present invention (all having a pentangular construction), the present inventors have mainly used mild steel rod having a diameter of 12.7 mm, with the length of rod in individual pentangles varying from about 0.5 m to 1.5 m. Mild steel rod is not expensive, is easily worked, and has appropriate vibrational properties as far as internal damping and the mechanical admittance of the finished article is concerned. It will be appreciated that other metals or metal alloys may be used. If the instrument is to be made by metal casting techniques, bronze is a particularly useful material.

Traditionally, musical instrument design has evolved without the use of higher mathematics. Percussion instruments constructed in accordance with the present invention may be designed by trying different combinations of the variables associated with the instrument. However, mathematical modelling and analysis has enabled the present inventors to construct useful implementations of the present invention in a relatively short time. Details of the mathematical analysis will now be provided.

Figure 1 illustrates, in a simplified form, a pentangular shape formed by bending a thin rod. The independent dimensional parameters are (i) the lengths of the sections which make up the pentangle (a_1, a_2 and a_3) and (ii) the included angles between adjacent sections of the pentangle (θ and Φ). The dimensions of the metal rods used in the construction of the prototype instruments suggest that a thin-rod approximation is valid. (This is the usual approximation for the behaviour of beams that is implemented in finite-element packages.) The next assumption (simplification) made for the purpose of the mathematical modelling is that the instrument will be played using a hammer blow having a velocity only in the plane of the sections forming the pentangle. Such an impulse should excite only modes lying in the plane of the instrument. In non-ideal cases, when other vibration modes are excited, the amplitudes of the in-plane modes will be much greater than those of the non-planar modes.

One further simplification that is made for the initial mathematical analysis is the assumption that the corners of the pentangle structure are sharp corners. This assumption eliminates one parameter, the corner curvature, and allows an analytic solution for the straight-rod sections, which can be joined by appropriate boundary conditions at the corners.

Now the propagation of transverse elastic waves along a thin rod is described by the equation

$$\frac{\partial^4 y}{\partial x^4} = -\frac{4\rho}{Ea^2}\frac{\partial^2 y}{\partial t^2} \qquad (1)$$

where y is the displacement normal to the rod, x is the co-ordinate measuring length along the rod, ρ is the density and E the Young's modulus of the rod material, and a is the radius of the rod. This equation, appropriately supplemented for longitudinal motion as shown below, describes the behaviour of each straight section of rod in the pentangle, and simply leaves appropriate conditions to be imposed (i) at the bends where two rods meet and (ii) at the free ends of the pentangle. In this way an analytic solution to this approximate representation of the real problem can be achieved in such a way as to allow simple and rapid calculation of the normal mode frequencies.

Inspection of Equation (1) shows that its general solution can be written in the form

$$y_n(x) = \alpha_n \cos kx + \beta_n \cosh kx + \gamma_n \sin kx + \delta_n \sinh kx \qquad (2)$$

where k is the wave number, given, from Equations (1) and (2), in terms of the angular frequency ω by the relationship:

$$k^4 = \frac{4\omega^2\rho}{Ea^2} \qquad (3)$$

In Equation (2), which refers to a section of rod labelled by the subscript n, the quantities α_n, β_n, γ_n and δ_n are constants, the values of which are determined by the boundary conditions at the two ends of this section of rod.

Equation (2), however, describes only displacements normal to the axis of the rod. To complete the description of the vibrations, the possibility of displacement parallel to the axis of the rod must be allowed. For the section n of the rod, the symbol ϵ_n is used to denote a parallel displacement. Each ϵ_n is taken to be constant along the length of the relevant rod, which means that the possibility of longitudinal waves in the rod material is ignored. This is physically justified, since the frequencies of the normal modes associated with longitudinal waves are much higher than those of bending modes, and they are therefore outside the frequency range in which lie the modes to be tuned.

Since there are five straight sections of rod in the pentangle structure, there are 25 unknown coefficients.

Note, however, that the pentangle structure has a plane of mirror symmetry, shown by the axis OC in Figure 1, and this implies that the modes must be either symmetric or antisymmetric in relation to this plane. Applying this condition reduces the number of unknown coefficients to 15.

These 15 coefficients, however, are not sufficient to describe the dynamics of the problem. In particular, an appropriate balancing of forces and moments at the corners of the pentangle structure is required. The bending moments are properly described in terms of (i) the elastic moduli, (ii) the rod radius, and (iii) the second derivative of the normal displacement y. The shear forces similarly involve the elastic moduli and the third derivatives of the normal displacement. A description of the tension forces in the rod, however, requires the introduction of tension forces T, which vary along the length of each rod. Being concerned only with matching conditions at the corners, therefore, introduces a further 5 independent quantities. These can be designated, with reference to Figure 1, in terms of the symbol used for the corner (or the centre O) and the rod number, as T^O_1, T^A_1, T^A_2, T^B_2, T^B_3. The tension at the free end C clearly vanishes. Symmetry considerations allow tensions for one half of the pentangle only to be specified.

Adding the 5 tension quantities to the 15 independent displacement parameters gives the total of 20 independent parameters necessary to specify the dynamics of the simplified thin-rod model. Thus 20 linearly independent equations relating these quantities are now required. Once these have been written down and solved, there is presented a nonlinear equation in the wave number k, or equivalently in the frequency ω, the solutions of which are the mode frequencies for the pentangle.

Now for each of the two bends A and B (see Figure 1), it is required that the rods or sections join together in a continuous manner (2 equations) and that their slopes $\partial y/\partial x$ match (1 equation), so that the bend angle is not distorted (distortion of the bend angle would take an infinite moment about the join point). Furthermore, consideration of a tiny element of rod at the bend shows that, if its motion is to remain finite, the bending moments in the two rods at the join must be equal, implying continuity of $\partial^2 y/\partial x^2$ (1 equation). Finally, the forces exerted on the element by the two rods must balance in two orthogonal directions in the plane (giving 2 equations involving $\partial^3 y/\partial x^3$, T, and the bend angle). This gives 6 equations at A and a further 6 equations at B, making 12 equations in all.

At the free end C, the bending moment and the shear force must both vanish, giving $\partial^2 y/\partial x^2 = 0$ and $\partial^3 y/\partial x^3 = 0$ (a further 2 equations). The tension force has already been set equal to zero by not including it among the unknowns.

The longitudinal motion of each rod or section under the influence of the difference between the tensions at its two ends is now considered. This difference in tensions is equal to the product of the mass and acceleration of the rod, and is thus proportional to the displacement quantity

(ϵ), the rod length, cross-section and density, and the square of the frequency (3 equations).

This gives a total of 17 linearly independent equations. The remaining 3 equations needed to determine the parameters necessary to specify the dynamics of the thin-rod model are derived for the point 0 on the symmetry plane, and are different for symmetric and antisymmetric modes. For the symmetric modes case, clearly $\partial y/\partial x = 0$ and $\partial^3 y/\partial x^3 = 0$, while the necessity for a stationary centre of mass requires that $T_1^0 = 0$. For the antisymmetric modes case, symmetry dictates that $y = 0$, $\partial^2 y/\partial x^2 = 0$, and $\partial^4 y/\partial x^4 = 0$. In either case, 3 additional equations are provided.

The 20 equations are homogeneous, since no external forces are involved, and the necessary and sufficient condition that they have a real solution is that the determinant of the matrix of their coefficients should vanish. This determinant is complicated, for it will be seen from Equation (2) that the coefficients involve quantities such as cos ka and cosh ka. The present inventors used one of the computer programs published in the book by W H Press, B P Flannery, S A Teukolsky and W T Vetterling, entitled "Numerical Recipes" (Cambridge University Press, New York 1986, page 39), for evaluating a determinant once its elements are given numerical form, by choosing a value of k. The selected program was used to search for those values of k for which the determinant vanishes. Equation (3) was then be used, with values of the elastic constants inserted, to convert these k values to frequencies. A separate computer program to perform this operation was written. It gave the first 6 or 7 mode frequencies to good precision in only a few minutes on an AT-compatible microcomputer using Microsoft QuickBasic. The speed of this part of the analysis could have been further improved by first using algebraic manipulation to reduce the rank of the determinant. There was no problem about requiring extra constraints and eliminating rigid-body modes as there is in some implementations of the corresponding finite-element calculation.

The results of a calculation using this analytic approach are given in Figure 2, which shows the variation of the first 6 mode frequencies as a 1 m length of 12.7 mm diameter steel rod is progressively bent into a rectangular shape and then unbent in the opposite order. In Figure 2, the angles θ and Φ are as defined in Figure 1 and the section lengths are chosen so that

$$R_{21} = a_2/a_1 = 2,$$
$$R_{31} = a_3/a_1 = 1.$$

Clearly there are large changes in the relative frequencies of the modes, suggesting that there is a strong likelihood that a shape might be achieved which gives a nearly harmonic relationship between some appreciable number of the frequencies of the modes. Having determined a suitable shape to first order, using this analytic approximation and the tuning philosophy outlined below, it is a relatively straightforward exercise to use a finite-element package to refine the shape by including the finite curvature of the corners.

As noted above, by restricting the instrument of the present invention to symmetrical shapes and leaving aside the possibility of changing the curvatures at the bends, it is clear from Figure 1 that five parameters (a_1, a_2, a_3, θ and Φ) are available for tuning the instrument. This suggests that it is possible to tune five modes or, more usefully, a basic pitch and four mode-frequency ratios relative to it. This is essentially the number of modes explicitly tuned in a church bell or a carillon bell.

In the pentangle shape illustrated in Figure 1, the basic pitch is determined by the overall length of the combined sections ($a_1 + 2a_2 + 2a_3$). It is convenient to take the parameter set, for tuning this instrument, to be (R_{21}, R_{31}, θ, Φ), where $R_{21} = a_2/a_1$ and $R_{31} = a_3/a_1$.

In percussion instruments of the bell or gong family, the sound can be listened to in two ways, known as holistic listening and analytical listening. In holistic listening, the perception is of a well-defined musical pitch and a characteristic musical timbre or tone-quality. In analytical listening, the perception is of the set of individual partials making up the sound. For a successful musical instrument, the relationship between the partials has to be such as to encourage holistic listening, and this is most readily achieved if the most prominent partials have frequencies in integral (harmonic), or nearly integral frequency relationship.

Such a relationship can be written as a product of primes $2^n3^m5^s$ ..., where n, m, s ... are small positive or negative integers. The degree of accuracy of the required tuning is highest if only the factor 2 is involved (that is, when the tuning produces prominent partials in octaves). The degree of accuracy is fairly critical if both 2 and 3 occur (that is, the prominent partials are in fifths and fourths), and it is much less critical if 2, 3 and 5 occur, to include major and minor thirds. Tuning of intervals involving 7, or higher primes, is very uncritical as far as consonance is concerned. The exact sequence of partial tones in the sound, and their relative strengths, has a great bearing on the sound quality, as does also the strength and general frequency distribution of the untuned higher partials which, generally, are not heard analytically.

Table 1 sets out the harmonic (or "just") frequency ratios for the musical pitches of concern in tuning the pentangle instrument illustrated in Figure 1.

Table 1

Harmonic or "Just" Pitch Ratios
(First line relative to C_1, second line relative to C_3)

C_1	E_1	F_1	G_1	C_3	E_3	G_3	C_4	E_4	G_4	C_5	Eb_5	E_5	G_5
1	5/4	4/3	3/2	4	5	6	8	10	12	16	96/5	20	24
.25	.3125	.3333	.375	1	1.25	1.5	2	2.5	3	4	4.8	5	6

In Table 1, the subscripts to the pitch symbols refer to the octave in which they occur, C_4 being "middle C" and C_1 being the lowest C on the piano keyboard. The notes of a keyboard instrument, such as the piano, are tuned to "equal temperament", in which all the fifths are flattened (tempered) by about 0.1 per cent so that all twelve notes of the scale have the same frequency ratio $2^{1/12}$ to their neighbours. This results in major and minor thirds which differ from the harmonic ratios 5/4 and 6/5 by about 1 per cent.

To produce a satisfying bell-like sound with a pentangle instrument of the present invention, the aim is to tune at least four prominent partials into small-integer ratios with the particular partial tone - generally the strongest low partial - that is taken as the nominal pitch of the bell. If the aim is to produce a sound like a church bell, then it is also highly desirable to include a minor-third interval (6:5 or one of its octaves) relative to this nominal.

For a straight rod of radius a and length L with free ends, the frequency of the nth mode is given by

$$f_n = A\frac{a}{L^2}(n+\frac{1}{2})^2 \qquad (5)$$

where A is a constant depending upon the density of the rod material and its Young's modulus. The frequencies of the modes thus have ratios close to a sequence which can be most helpfully written 0.36 : 1.00 : 1.96 : 2.56 : 3.24 :

... The lowest frequency is well removed from the others and is not radiated very efficiently, so that it is logical to take the frequency of the second mode as defining the nominal pitch. From Figure 2 it can be seen that the lowest mode of a bent rod is similarly isolated in relative frequency from the upper modes, so that the same nominal pitch assignment may be adopted for the instrument of the present invention. It should also be noted that the second mode is also generally taken as defining the pitch of a church bell, the first mode being called the "hum" or undertone.

The next step in the selection of a tuned configuration of a pentangle, therefore, is to explore the 4-dimensional parameter space $\{R_{21}, R_{31}, \theta, \Phi\}$ and find configurations for which the frequency ratios, relative to the second mode as nominal, have the required simple form. This task is potentially very extensive numerically, but it can be simplified greatly by proceeding one mode at a time and by adding parameters one at a time, as follows.

The sub-nominal first mode frequency is of little importance, since a sound resonator (radiator) coupled with the instrument can be tuned to the second mode and will then radiate little at this lower frequency. This sub-nominal frequency, therefore, can be neglected and only the higher mode frequencies relative to mode 1 considered. Assuming reasonable values for two of the parameters, say the side-length ratios R_{21} and R_{31}, allows numerical exploration of the 2-parameter $\{\theta, \Phi\}$ configuration space by computing mode frequencies over a grid of about 5 x 5 points and drawing (θ, Φ) contours along which the required harmonic frequency relationships are met. The base plane of the 3-dimensional configuration space shown in Figure 3 is an example of such contours for modes 3 and 4 at acceptable frequency ratios such as 2:1 or 3:1 relative to mode 2. If a solution to the tuning problem exists, then these two contours must cross at a point A within the accessible {θ, Φ} space.

The phase space may now be extended to three dimensions by calculating a similar set of acceptable (θ, Φ) contours for additional values of one of the remaining parameters - for example, R_{21}. This allows surfaces in the 3-dimensional {θ, Φ, R_{21}} space to be drawn, corresponding to acceptable values of the frequency ratios for modes 3 and 4, as shown in Figure 3. These two surfaces will intersect in a curve AB, if a solution indeed exists.

A third surface can be drawn in the space of Figure 3 corresponding to an appropriate ratio for the frequency of mode 5. If this surface cuts the solution curve AB for modes 3 and 4, for example at the point S, then this point represents a satisfactory solution for modes 3, 4 and 5 relative to mode 2. The process can then be continued by including the remaining parameter, extending point S into a curve, and seeking an intersection with the surface for mode 6 at an acceptable frequency ratio.

The advantage of this procedure is that it limits the amount of configuration space that must be explored at each step to that near a previously established curve or surface, and thus greatly reduces the calculation time involved. In fact, it is an effective procedure, when the approximate location of the solution point in configuration space has been identified, to search for the solution iteratively in the orthogonal {θ, Φ} and {R_{21}. R_{31}} sub-spaces in turn. Thus Figure 4 shows such a section in {θ, Φ} space for R_{21} = 2, R_{31} = 1, which is close to, but not coincident with, the exact solution ratio. It is clear that there are three regions in this sub-space, marked S_I, S_{II} and S_{III} on Figure 4, which are close to multiple intersections of the individual solution surfaces for particular modes. Exploration for a solution can then be limited to the immediate vicinity of these regions.

Using the methods outlined above, these solutions were refined for the idealised sharp-corner tuning problem. Other solutions may exist for greatly different side-length ratios, since the exploration was not completely exhaustive. Only the solution associated with region S_I was essentially exact. In the case of regions S_{II} and S_{III}, the solution surfaces do not all pass through the solution point, but simply approach closely to it. Details of the initial solutions are given in Tables 2 and 3.

Table 2

Initial Configurations

	R_{21}	R_{31}	θ	Φ
Solution I	1.95	0.92	96°	93°
Solution II	2.20	1.07	135°	46°
Solution III	1.85	0.97	146°	10°

Table 3
Initial Mode Frequency Ratios

Solution I	(0.35)	1.00	2.00	3.00	4.80	6.00
Pitches	($F\sharp_1$)	C_3	C_4	G_4	$E\flat_5$	G_5
Solution II	(0.32)	1.00	1.51	1.98	2.99	4.80
Pitches	(E_1-F_1)	C_3	G_3	C_4	G_4	$E\flat_5$
Solution III	(0.31)	1.00	1.25	1.46	2.49	4.04
Pitches	(E_1)	C_3	E_3	G_3	E_4	C_5

With Solution I (see Table 2), the top angles are closed so much that the two free ends of the pentangle structure overlap. This means that the pentangle structure must be bent slightly out of plane so that there is adequate separation of the free ends. Modes 2 to 6 can be tuned exactly, in the sharp-corner approximation, and give a well spread set of modes, including a minor third at mode 5. The nominal pitches of these notes are indicated, taking the nominal pitch of the pentangle structure as a whole to be tenor-C (C_3). Unfortunately, the sub-nominal mode 1 has a dissonant pitch close to $F\sharp_2$, but this can be ignored, for the reasons given above.

Solution II gives an instrument of "coat hanger" shape which, from Table 3, has a well distributed set of mode frequencies. The sub-nominal, in this case, is well located near a harmonic frequency. Once again there is a minor third (this time at mode 6) and, since mode 3 has the frequency ratio 1.5, there may be an implied fundamental at frequency ratio 0.5, an octave below the nominal pitch for psychophysical reasons.

In Solution III, which has a very flattened shape because of the small value of Φ, the mode frequencies are densely clustered in the range 1.0 to 2.5 and contain no less than three major thirds relative to the nominal pitch. The subjective pitch may again be below the nominal pitch because of the close spacing of these mode frequency ratios. The shape of this pentangle, however, is not satisfactory for practical reasons, particularly when rounded corners come to be considered. For this reason, Solution III was not pursued by the present inventors.

The final step in the mathematical modelling exercise, which would not be required if the percussion instrument is cast or moulded with sharp corners, is to modify the Solutions I and II to include the effects of corner rounding. For this part of the design exercise, the finite-element package "Strand5" (produced by G & D Computing, Suite 307, 3 Smail Street, Ultimo, NSW 2007, Australia) was used, again on an AT compatible microcomputer. This package is particularly suitable for this calculation because its structure allows access to all the files and executable modules, so that it is a relatively easy task to write a batch program to perform the necessary exploration of configuration space in the immediate vicinity of the initial Solutions I and II. It does not require the inclusion of external constraints on the pentangle. Optimisation of the pentangle design for corner rounding effects, using this approach, typically takes only a few hours.

When corner curvature effects are to be included in the mathematical modelling, it is necessary to define how the curvatures are to be measured. The choice is (i) between centres of curvature, (ii) between the intersections of the axes of the rod segments, and (iii) in some other way. The choice will have a significant influence upon the results when the corner radii are large or the bend angles are large.

For steel rod, the minimum reasonably achievable bend radius corresponds to bending the rod around itself, giving a neutral-section bend radius about equal to the rod diameter, so that corrections to straight-side lengths of at least this magnitude are involved. Since the bend radius introduces an absolute scale into the problem, it is necessary to define the total length of the rod, which was taken to be 1000 mm. If the bend radius is taken as 15 mm for 12.7 mm diameter rod, then little change in the shape of the pentangle is required.

However this leaves no flexibility in bend radius for simply-scaled smaller pentangles. Accordingly, the neutral-section bend radius r was taken to be 26 mm for a 1000 mm rod (corresponding to an internal bend radius of 20 mm), thus allowing for tighter bends in smaller pentangles made from the same rod stock. With these assumptions, Solutions I and II were refined as indicated above. It was found that the 26 mm radius adopted required significant changes in both segment lengths and bend angles. The final Solutions are shown in Table 4, which gives the straight-line section lengths, or distances between centres of curvature for the corners, together with the bend angles.

Table 4

Final Practical Designs

Dimensions	a_1(mm)	a_2(mm)	a_3(mm)	θ	Φ	r(mm)
Design Solution I	103	258	109	90°	90°	26
Design Solution II	113	250	100	136°	19°	26

In the case of Solution I, rounding of the corners requires only small adjustments to the straight-side lengths and a reduction in the inside angles to approach again the design mode frequencies to an accuracy of better than 2 per cent. The resultant shape is shown in Figure 5. It is essentially rectangular, with a large side overlap. The adjusted form of Solution II is illustrated in Figure 6. In this case, the rounding of the lower corners produces a considerable change in mass distribution. This necessitates a considerable reduction of the angle Φ relative to the sharp-corner configuration. Nevertheless, the original calculated mode frequencies are regained to better than 1 per cent. The extent of this angular change, however, further supports the view that a pentangle corresponding to Solution III probably could not be made by simple bending of a rod.

By relaxing the planar constraints applied to the finite-element solution, it is possible, although of limited practical importance, to calculate the frequencies of the out-of-plane vibration modes in addition to the planar modes. The existence of these inharmonic out-of-plane modes allows the performer a degree of control of the timbre of the instrument, since it can be struck to minimise or to maximise the amplitude of these modes relative to the harmonic in-plane modes.

The pentangles as designed above require no hand-tuning, their mode frequencies being defined by their basic shape. The same is true to some extent of traditional church bells, but it is almost universal practice to fine-tune the mode frequencies of church bells by turning small amounts of metal off the interior surface of the bell on a lathe, following recipes which have been established by long experience. Clearly the same sort of procedure could be used with pentangles, both to reduce the residual tuning discrepancies of the first six modes and perhaps also to tune some of the higher modes.

A practical approach to this tuning problem for the case of bells has been developed R G J Mills, and is described in his paper entitled "Tuning of Bells by a Linear Programming Method" which was published in the <u>Journal of the Acoustical Society of America</u>, Volume 85, pages 2630-2633 (1989). This approach involves evaluating the effect on the tuning of all modes of the removal of a small amount of metal from each of a large number of bands along the interior surface of the bell. A linear programming procedure is then used to define a metal-removal schedule that will produce optimal tuning. Such a procedure may be readily implemented for a bent rod pentangle by using the finite-element package to determine the effect of filing metal off the rod in various locations. With cast metal or moulded ceramic pentangles, fine tuning can also be effected by varying the cross-sectional size and shape of the sections of the pentangle, or by including a wide-section "weight" at an appropriate location on a section. It is also theoretically possible to fine tune a metal pentangle by welding a "weight" to it. However, it is the belief of the present inventors that, in practice, fine-tuning will be generally unnecessary.

Once a suitable design has been achieved for a pentangle of some assumed size, it is possible to scale this design to the sizes necessary to produce a required set of nominal pitches for a musical scale. There are three possible approaches.

Conceptually the most straightforward approach is simply to scale all the dimensions of the pentangle (rod length, diameter and bend radius) uniformly. From Equation (4), the mode frequencies will then all vary inversely with the scale factor. The practical difficulty with this approach is that it requires a different rod diameter and bend radius for each pentangle of the set.

At the other extreme, both the rod diameter and the bend radius might be kept constant and only the lengths of the rod sections scaled. This would require performing a separate finite-element optimisation to incorporate corner rounding for each different member of the set, and the limitations imposed by a fixed bend radius might mean that the overtone structure of the resulting pentangles might have to change at certain nominal pitches. This is not very satisfactory.

The most practically appealing scaling approach, therefore, is to use the same diameter rod for all pentangles, scaling rod length and bend radius, and to use once more the scaling law Equation (4), which shows that the mode frequencies in this case vary inversely as the square of the scale factor. This has economic advantages, even though a different bending die has to be made for each size of pentangle.

Table 5 shows the measured mode frequency ratios of the first two pentangle instruments constructed by the present inventors according to the calculated curved corner designs. The measured deviations from the calculated frequencies can be ascribed in large measure to small deviations from the desired geometry of the pentangle, since the rod was bent by hand in a simple jig. Agreement is certainly adequate to validate the design principles.

Table 5

Calculated and Measured Frequency Ratios

Mode Numbers	1	2	3	4	5	6
Design Solution I	0.35	1.00	1.96	3.05	4.82	6.13
Pentangle I (measured)	0.35	1.00	2.01	3.05	4.79	5.93
Out-of-plane (measured)	0.39	1.43	2.40	?	4.97	9.57
Design Solution II	0.32	1.00	1.50	1.99	3.04	4.76
Pentangle II (measured)	0.33	1.00	1.49	1.96	3.05	4.75
Out-of-plane (measured)	0.83	1.06	1.42	2.57	4.03	5.17

Also shown in Table 5 are the frequencies of the out-of-plane modes, which form an inharmonic series interlacing those of the planar modes. In practice, and despite the lack of planarity of the pentangle of Figure 5 made necessary by overlap of the ends in Solution I, it is easily possible to execute the strike so as to excite almost exclusively the in-plane or the out-of-plane modes. The large inharmonicity of the out-of-plane modes creates a very different sound, and thus places some interesting effects in the hands of the performer.

In judging musical effectiveness, the view can be taken that simulation of the sound of a traditional western European church bell is being attempted, or it can simply be required that the sound be pleasant, a matter which can be judged only subjectively.

For a traditional church bell, the first five modes are (i) the hum or undertone, with frequency 1:2 relative to the second mode, (ii) the fundamental or prime, which is the reference frequency, (iii) the tierce or minor third (6:5), (iv) the quint or perfect fifth, and (v) the nominal or octave (2:1). Clearly neither of the first two pentangles constructed by the present inventors comes close to this set of frequencies. Solution I contains the correct frequency ratios, including the minor third, to within powers of 2, except in the case of the first mode, but they are spread over several octaves rather than being concentrated. Solution II has more closely clustered mode frequencies and again has a minor third. Solution III, it will be noted, has mode frequencies clustered more like those of a church bell but, as explained above, this design has not been implemented for practical reasons. The musical effectiveness of the present invention, therefore, cannot be a matter of exact simulation but must rely upon the production of an appropriate subjectively bell-like sound.

To summarise, a method for optimising the shape of a pentangular framework in order that its first few modes should form a series with harmonically related frequencies has been described above. The method is relatively straightforward and can be generalised to tune either a different set of modes for the pentangle, or to tune a set of modes in a framework of different geometry, although the number of available parameters increases rapidly if many more sections are used.

Since making the first two "pentangles", the present inventors have constructed a percussion instrument with thirteen "pentangles", as shown schematically in Figure 7. Each pentangle 60 is constructed in the form shown in Figure 6 and is suspended from a frame 61 using a nylon thread. The pentangles are also connected to a broad-band, non-resonant soundboard 62, backed by a cavity or sound box 63, to enhance the sound transmission of the instrument. Both the soundboard and the sound box are tapered from the bass to the treble end, and are coupled to the pentangles by elastic cords (which are attached to the soundboard at points along a non-central line). The soundboard is braced by ribs glued to its back face, as in a guitar or harpsichord. The distribution of these ribs and the volume of the backing cavity or box are such that there is an appropriately shaped radiation response over the playing range of the instrument. The thirteen graded size pentangles were "tuned" to enable the instrument to play a full scale.

This instrument produces a mellow concordant sound when the pentangles are excited by a blow from a hard mallet. When a soft beater is used by the percussionist, the instrument produces a warm, harp-like sound. Increasing the hardness of the beater or mallet increases the higher frequency content of the sound produced, so that a bell-like sound is produced when the pentangles are struck with a hard mallet. A more percussive sound is produced if a pentangle is hit on one side. The fact that out-of-plane vibration modes are not adjusted to harmonic relationship has been found to give a useful degree of tonal freedom to the performer, a possibility that could be enhanced by making each pentangle from a rectangular steel bar instead of from a steel rod.

A number of microphones have been installed within the soundbox of the instrument illustrated in Figure 7. These microphones enable the instrument to be used with the acoustic resonator, with an electronic amplifier, or with both acoustic and electronic amplification of the sound which is produced.

For the sake of completeness, an example of the scaling of a pentangle (used by the present inventors to test the scaling approach subsequently adopted) will now be given, together with information about the use of sound radiators with the "pentangles" of the present invention.

Scaling Example

The aim of this scaling example was to produce an instrument of the Solution I shape (see Figure 5) for the note E_1 = 41.2 Hz. Solution I was chosen for the pentangle shape since it is more compact and uses less rod material than the shape shown in Figure 6 (Solution II). Because the pentangle is rather large, 14 mm diameter steel rod was used to give adequate weight and rigidity.

The original Solution I pentangle was rectangular in shape with θ = Φ = 90°. It had a total rod length of 813 mm. The section lengths, measured for the straight sections, were a_1 = 85 mm, a_2 = 212 mm, a_3 = 89 mm. There was a neutral section bend radius of 26 mm at each corner. When made from 12.6 mm diameter rod, this pentangle gave an internal bend radius of approximately 20 mm. The pentangle had reasonably good tuning and a measured frequency of about 185 Hz. This pentangle was to be scaled to produce the required lower pitch for the note E_1.

Assuming that the original instrument had a rod length l_1, a bar diameter d_1 and a frequency f_1, while the new instrument has a bar diameter d_2 and a design frequency f_2, then the rod length l_2 for the new instrument is given by the scaling algorithm

$$l_2 = \sqrt{\frac{f_1}{f_2} \cdot \frac{a_2}{a_1}}$$

The new pentangle was successfully constructed using this relationship for the rod length, with an equivalent inner bend radius of 38 mm, and with the interior bend angles remaining at 90°.

Any one of a number of different sound radiators may be matched to the percussion instrument of the present invention to enhance sound transmission, but on the basis of experience a resonant structure is preferred to a wide band radiator when the instrument comprises a single pentangle. The simplest resonant structure is a diaphragm coupled pipe resonator. However, a tube-loaded cavity resonator or an air-loaded resonant diaphragm may be used. These alternatives are considered below.

At 41.2 Hz, the normal sound wavelength is about 8.25 metres, so that a quarter wave pipe resonator, driven as a high impedance, will have an acoustic length of 2060 mm. This is long enough to require folding, but may still be practicable. Suppose the resonator is made from pipe of internal diameter d, then its physical length L should be

$$L = 2060 - 0.3d \ mm$$

where d is also in millimetres. If the pipe is bent back along itself using two right angle bends, then the length should be measured roughly along the centre line of the pipe. It is best to make the pipe too long by perhaps 300 mm and to cut a slot about one-third of the diameter in width along the length of the excess section. This slot can then be covered over progressively to tune the pipe (as in some organ pipes). Alternatively a tuning sleeve could be fitted outside the pipe for sliding over the pipe to increase its length. The diaphragm covering the driven end of the pipe should be only moderately taut, since the frequency at which it must vibrate is quite low. An optimal combination of diaphragm thickness and tension requires experimentation.

To design a Helmholtz cavity resonator consisting of a relatively large volume, the shape of which is not important, coupled to the environment through a tuning pipe and driven by means of a membrane in its base or side wall, take the volume of the cavity as V (in cubic metres) and assume that the coupling is effected by a pipe of length L and diameter d (both in metres). At the chosen frequency of 41.2 Hz, the pipe length of such a resonator is given by the relationship

$$L \approx \frac{2d^2}{V} - 0.6d$$

For a cavity with a volume of about 0.6 m^3 and a pipe diameter of 100 mm, the required pipe length is about 270 mm. Some of this length could protrude into the interior of the cavity. Again, it might be advantageous to provide a means for tuning the length of the pipe.

Another type of resonant radiator that may be used with a single pentangle is analogous to the membrane and kettle of the tympani or, in a simpler form, to the membrane of a bass drum. The membrane would be tuned, as in the tympani, to the nominal pitch of the pentangle. The attachment cord of the pentangle instrument should meet the membrane at about the point chosen for striking the tympani - that is, about one third of the way in from the edge - and not at the centre of the membrane.

The bass drum resonator is rather similar, though the tuning is much less critical and the attachment point might well be in the centre of the membrane, rather than off to one side. However, it would give a much less resonant sound than the tympani-type resonator, which has several modes in nearly harmonic relation.

Those skilled in the art of musical instrument manufacture and mathematical modelling will appreciate that the adoption of the term "pentangle" is perhaps inappropriate when the structure has the shape shown in Figure 6, with the fifth angle (of the "open corner") not readily apparent to non-mathematicians. However, the present inventors prefer to use this term in view of (a) the mathematical modelling approach used and (b) the relationship of the present invention to the orchestral triangle. It will also be appreciated that the invention described above is susceptible to variations and modifications other than those specifically described and illustrated in this specification, and it is to be understood that the invention includes all such variations and modifications that fall within the scope of the following claims. Included in those variations and modifications are the use of hard plastic and advanced materials to form the sections of the instrument, the provision of one or more arcuate or curved sections, tubular construction of the sections, production of several connected pentangles (or other multi-section instruments) by a multiple casting technique, and - in the case of an instrument of the type illustrated in Figure 7 - the inclusion of mechanical beaters, attached to or separate from the instrument. This list is not intended to be exhaustive.

CLAIMS

1. A percussion instrument comprising a plurality of more than three elongate sections, said sections being substantially co-planar, non-colinear, and formed integrally from a material which, at room temperature, is rigid and has vibrational properties such that, when one of the sections is struck with a mallet, the instrument emits a musically concordant sound.

2. A percussion instrument as defined in claim 1, having five sections.

3. A percussion instrument as defined in claim 1 or claim 2, in which the lengths of the sections and the angles included by adjacent sections are such that, when one of the sections is struck by a striker to excite vibrational modes lying in the plane of the sections of the instrument, the sound that is emitted by the instrument has a series of harmonically related frequencies.

4. A percussion instrument as defined in any preceding claim, in which each section is of substantially uniform cross-section, and the sections of the instrument have essentially the same cross-sectional shape and dimensions.

5. A percussion instrument as defined in claim 4, in which said sections are sections of a metal or metal alloy

rod or bar, and the instrument is formed by bending the metal or metal-alloy rod or bar.

6. A percussion instrument as defined in claim 5, in which the metal is mild steel.

7. A percussion instrument as defined in claim 1, claim 2, claim 3 or claim 4, in which said sections are formed by casting a molten metal.

8. A percussion instrument as defined in claim 7, in which the metal is bronze.

9. A percussion instrument as defined in claim 1, claim 2, claim 3 or claim 4, in which said instrument is formed by (a) pressure moulding a mixture of particles of at least one ceramic material and a fugitive binder to form a green body having the required shape of the instrument, (b) firing the green body at an elevated temperature, then (c) allowing the fired body to cool to room temperature at a rate sufficient to avoid the formation of large cracks in the fired body, thereby producing a percussion instrument of a mechanically strong ceramic material.

10. A percussion instrument as defined in any preceding claim, said instrument being made by bending a steel rod of radius 26 mm and having five sections, characterised in that

(a) the five sections form a shape which is mirror-symmetric relative to the central point of the central section;
(b) the central section has a length a_1;
(c) each section adjacent to the central section has a length a_2;
(d) each section remote from the central section has a length a_3;
(e) the included angle between the central section and each adjacent section is θ;
(f) the included angle between the sections of length a_2 and a_3 is Φ;
(g) the ratios $a_1:a_2:a_3$ are substantially 1.00:2.50:1.06; and
(h) θ and Φ are substantially 90°.

11. A percussion instrument as defined in any one of claims 1 to 9, said instrument being made by bending a steel rod of radius 26 mm and having five sections, characterised in that
(a) the five sections form a shape which is mirror-symmetric relative to the central point of the central section;
(b) the central section has a length a_1;
(c) each section adjacent to the central section has a length a_2;
(d) each section remote from the central section has a length a_3;
(e) the included angle between the central section and each adjacent section is θ;

(f) the included angle between the sections of length a_2 and a_3 is Φ;

(g) the ratios $a_1:a_2:a_3$ are substantially 1.00:2.21:0.88; and

(h) θ is substantially 136° and Φ is substantially 19°.

12. A percussion instrument as defined in any preceding claim, including means to support said plurality of sections.

13. A percussion instrument as defined in any preceding claim, in which said sections are acoustically coupled to a sound radiator, to increase the efficiency of sound production by the instrument.

14. A percussion instrument as defined in claim 13, in which said sound radiator is a resonant radiator.

15. A percussion instrument as defined in claim 13, in which said sound radiator comprises electronic amplification means.

16. A percussion instrument comprising an assembly of a plurality of percussion instruments as defined in any one of claims 1 to 11, each percussion instrument in said assembly having a respective characteristic pitch which is different from the characteristic pitch of each other percussion instrument in said assembly; each percussion instrument being mechanically and

acoustically coupled to an acoustic radiator or to a respective individual acoustic radiator.

17. A percussion instrument as defined in claim 16, including electronic amplification means associated with the or each acoustic radiator.

18. A percussion instrument as defined in claim 1 or claim 16, substantially as hereinbefore described with reference to the accompanying drawings.

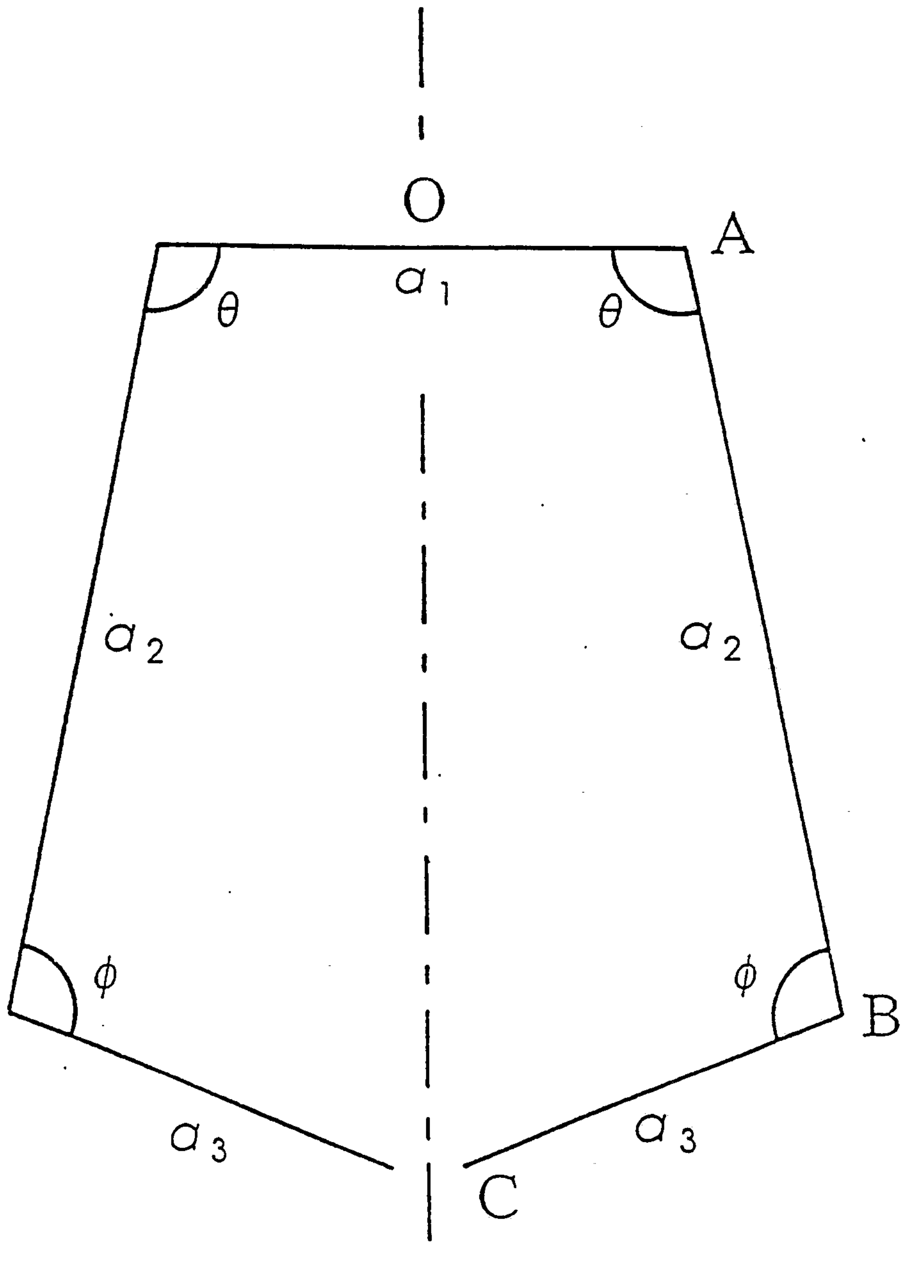


FIG. 1

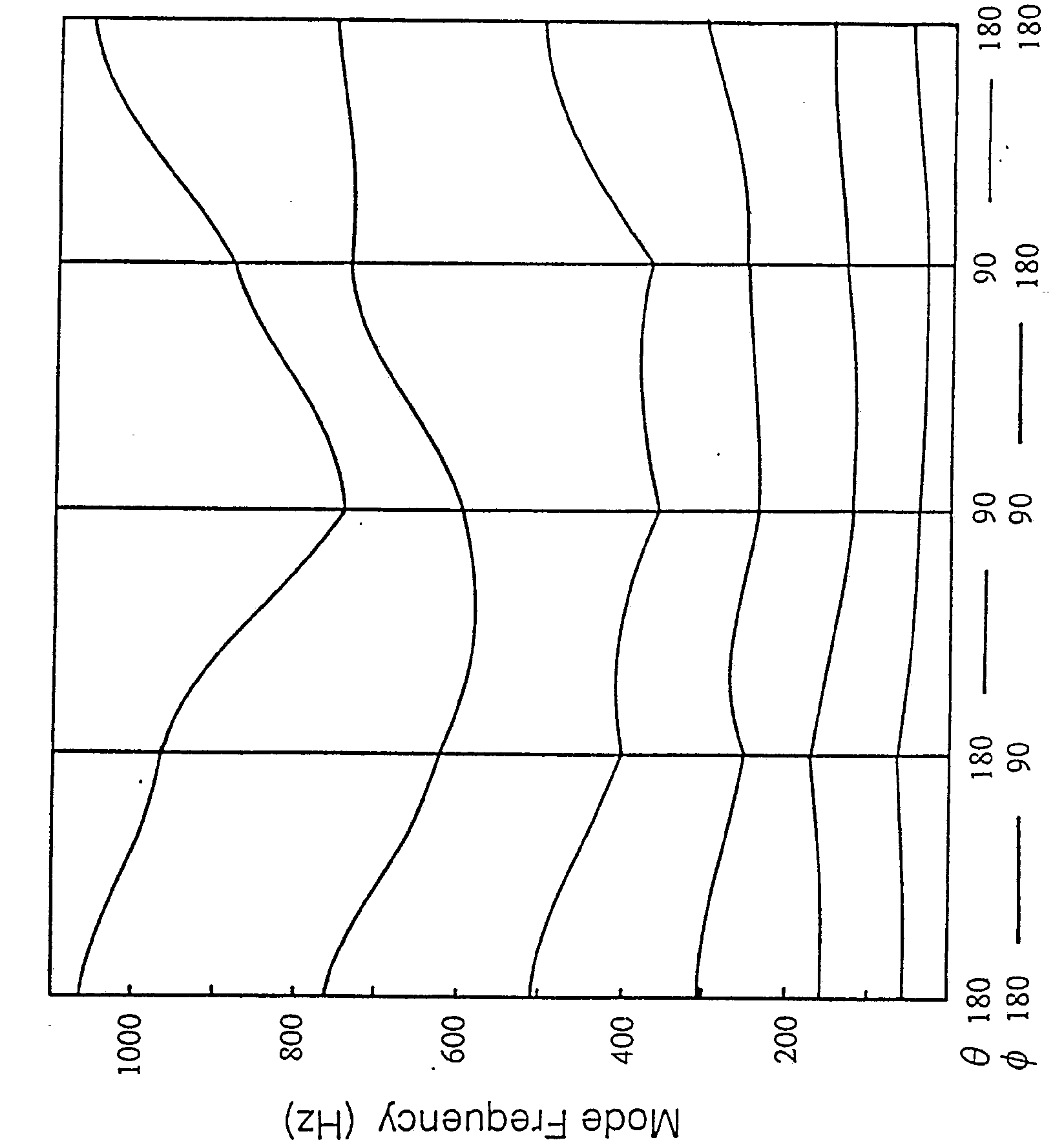

Mode Frequency (Hz)
1000
800
600
400
200
θ 180 ——— 180 ——— 90 ——— 90 ——— 180
φ 180 ——— 90 ——— 90 ——— 180 ——— 180

FIG. 2

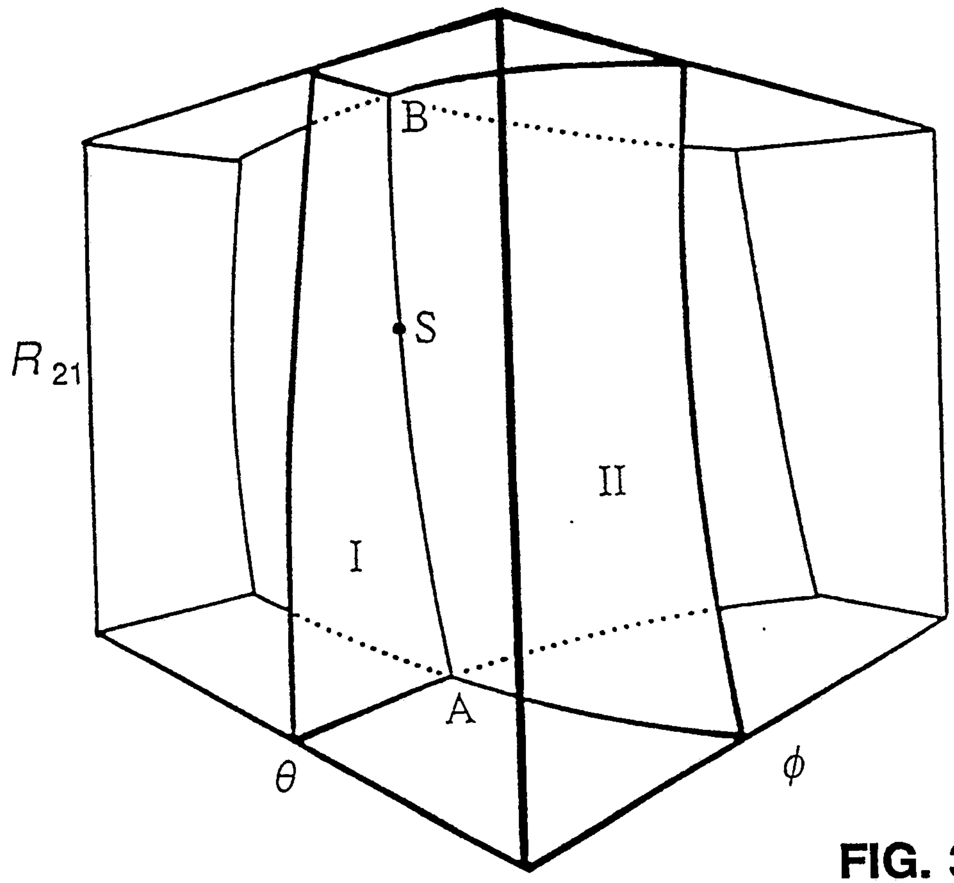


FIG. 3

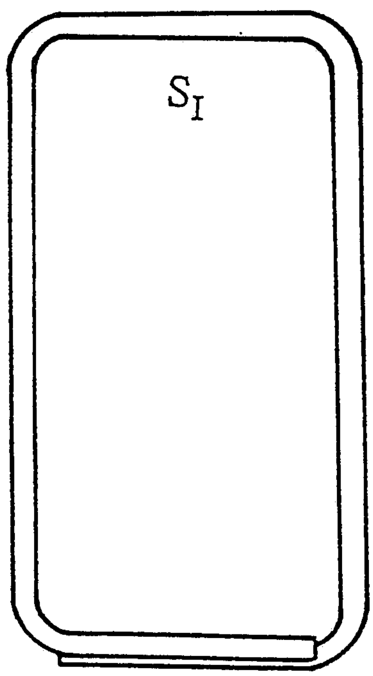


FIG. 5

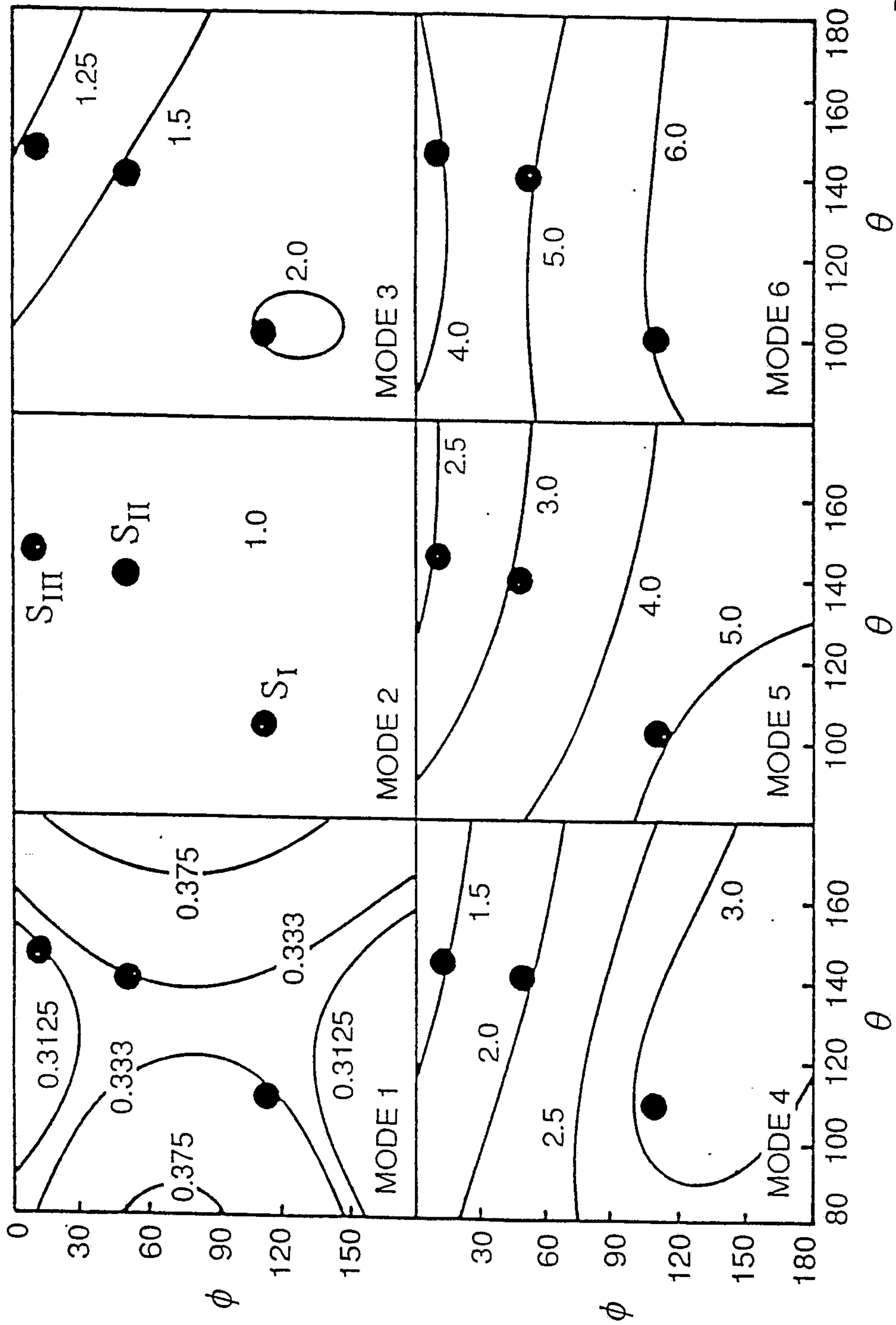

MODE 1
0.375
0.333
0.3125
MODE 2
S_I
S_II
S_III
1.0
MODE 3
1.25
1.5
2.0
MODE 4
1.5
2.0
2.5
3.0
MODE 5
2.5
3.0
4.0
5.0
MODE 6
4.0
5.0
6.0
θ
φ
0
30
60
90
120
150
180
80
100
120
140
160

FIG.4

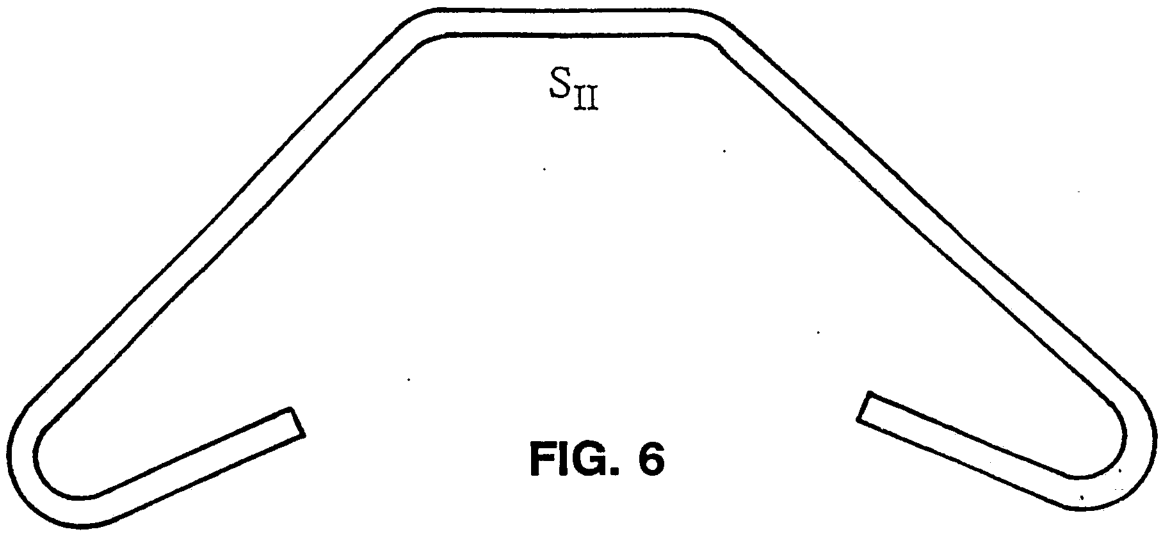


FIG. 6

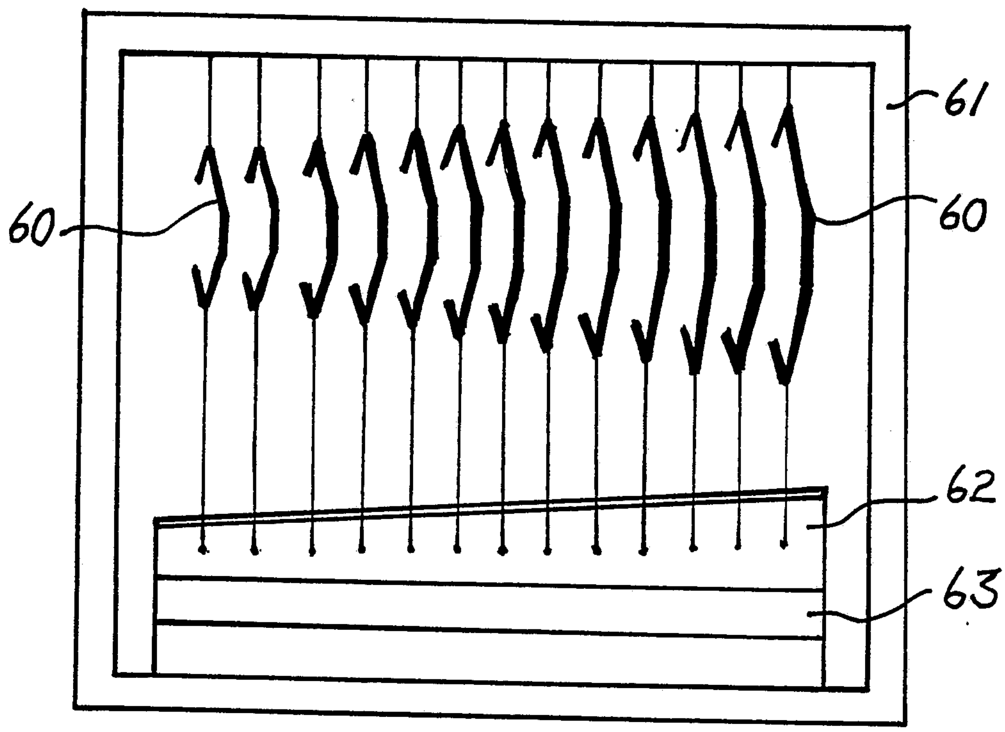


FIG. 7

A. CLASSIFICATION OF SUBJECT MATTER

Int. Cl.[5] G10D 13/08, G10K 1/06

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC G10D 13/00, 13/08, G10K 1/00, 1/06

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
AU : IPC as above

Electronic data base consulted during the international search (name of data base, and where practicable, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to Claim No.
A	US,A, 4805513 (ITO et al) 21 February 1989 (21.02.89) See the whole document	1-18
A	US,A, 4779507 (SHIMODA et al) 25 October 1988 (25.10.88) See the whole document	1-18

☐ Further documents are listed in the continuation of Box C.

☒ See patent family annex.

* Special categories of cited documents :

"A" document defining the general state of the art which is not considered to be of particular relevance
"E" earlier document but published on or after the international filing date
"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
"O" document referring to an oral disclosure, use, exhibition or other means
"P" document published prior to the international filing date but later than the priority date claimed

"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
21 June 1993 (21.06.93)	23 JUNE 1993 (23.06.93)

Name and mailing address of the ISA/AU	Authorized officer
AUSTRALIAN PATENT OFFICE PO BOX 200 WODEN ACT 2606 AUSTRALIA Facsimile No. 06 2853929	John Thomson J W THOMSON Telephone No. (06) 2832214

This Annex lists the known "A" publication level patent family members relating to the patent documents cited in the above-mentioned international search report. The Australian Patent Office is in no way liable for these particulars which are merely given for the purpose of information.

Patent Document Cited in Search Report		Patent Family Member					
US	4805513	DE	3743687	JP	63163397	NL	8703112
US	4779507	JP	63033788				

END OF ANNEX