gathered teams. At the end of five minutes each team submits in writing its estimate. The team with the estimate closest to the "correct answer" earns a point. In the case of two or more teams submitting an estimate within a factor of 2 of the accepted answer, each of those teams earns a point. After ten questions, the team with the most points wins.

The winner of the competition receives "The Coveted Klavier Stimmer Award," the title based on Fermi's legendary question, "How many piano tuners are there in Chicago?" The certificate presented to the members of the winning team reads, "In recognition of outstanding performance in order-of-magnitude estimates, the board of directors of the Klavier Stimmer Foundation proudly presents the annual Klavier Stimmer Award to the following team members."

The competition sparks considerable interest and generates vigorous debate. The placing of professors into the same arena with students seems to remove some of the barriers which often exist between students and teachers. Furthermore, topics for the questions can vary over a wide range, which hopefully increases the validity of the claim that skills learned in our discipline can be applied outside traditional physics topics.

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Quantum cyclotron

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In the study of quantum mechanics it is the analysis of physical systems for which the Schrödinger equation can be solved exactly that students often find most satisfying and illuminating. The systems commonly discussed are those of importance in both classical and quantum mechanics: a free particle, a confined particle, a simple harmonic oscillator and a particle in a central field. It is therefore surprising that the problem of a charged particle in a homogeneous magnetic field, "the quantum cyclotron," is rarely treated in textbooks on quantum mechanics. This system is of interest in both classical and quantum mechanics and has been studied extensively,[1–3] most recently by Thomson.[4]

The object of this article is to point out two properties of the system and its quantum-mechanical solution that may be useful in an elementary treatment of the problem: first, that the eigenfunctions can be simply expressed in terms of generalized Laguerre polynomials which are familiar from a solution of Schrödinger's equation for the hydrogen atom; second, that a calculation of the magnetic flux provides a useful connection between the classical and quantum theory.

A particle of charge e and mass m in a homogeneous magnetic field **B** in the z direction has eigenfunctions of the form[2,3]

$$\psi_{nl}(r,\theta)=e^{il\theta}R_{nl}(r),\quad l=0,\pm1,\pm2,\ldots, \tag{1}$$

when the Schrödinger equation is solved in cylindrical polar coordinates. The free particle motion along the field direction will be excluded from the discussion.

The radial function satisfies the differential equation

$$\rho(\rho R')'+(\lambda\rho^2-l^2-\rho^4)R=0 \tag{2}$$

where $\rho^2=eBr^2/2\hbar$, the prime signifies differentiation with respect to ρ, and the energy eigenvalues are given by

$$E=[(\lambda/2)-l]\hbar\omega_L/2. \tag{3}$$

$\omega_L=eB/m$ is the Larmor frequency and the constant λ arises in the usual way from separation of the Schrödinger equation.

The problem is therefore, for given l, to find λ and R that satisfy (2) and its boundary conditions. This equation also arises when the isotropic harmonic oscillator problem is solved in cylindrical polar coordinates,[5] and its solutions may be written in different forms.[3,5] The solutions can also be expressed simply in terms of generalized Laguerre polynomials.

If we make the substitutions

$$R(\rho)=e^{-\rho^2}\rho^{|l|}y(\rho^2),\quad x=\rho^2 \tag{4}$$

in Eq. (2), then $y(x)$ satisfies

$$xy''+y'(|l|+1-x)+ny=0, \tag{5}$$

where the prime now means differentiation with respect to x, and $\lambda=2(2n+|l|+1)$ or, using (3),

$$E_{nl}=(2n+|l|-l+1)\hbar\omega_L/2. \tag{6}$$

The solutions of (5) for which R is finite at infinity are, in the notation of Abramowitz and Stegun,[6]

$$y=L_n^{|l|}(x),\quad n=0,1,2,\ldots, \tag{7}$$

which are the generalized Laguerre polynomials. These are the familiar polynomials that occur in the radial part of the hydrogen atom eigenfunctions, though note that Pauling and Wilson[5] use a different suffix notation.

The eigenvalues of the present problem are therefore given by Eq. (6) and the eigenfunctions are

$$\psi_{nl}(r,\theta)=e^{il\theta}\rho^{|l|}e^{-\rho^2/2}L_n^{|l|}(\rho^2),\quad \rho^2=eBr^2/2\hbar. \quad (8)$$

The eigenfunctions that describe the classical circular motion about the origin are those for $n = 0$ and l negative.[4] Since $L_0^{|l|}(x) = 1$ the eigenfunctions (8) are those given by Thomson[4] in this case. The form (8) of the eigenfunctions is particularly useful since the mathematical properties of the associated Laguerre functions have been extensively studied.

As an example of the use of this form of the eigenfunctions consider the expectation value of the quantum mechanical magnetic flux Φ.

$$\langle\Phi\rangle=\langle\pi r^2B\rangle$$
$$=(2\pi\hbar/e)\{\int_0^\infty e^{-x}x^{|l|+1}[L_n^{|l|}(x)]^2\,dx\}$$
$$\times\{\int_0^\infty e^{-x}x^{|l|}[L_n^{|l|}(x)]^2\,dx\}^{-1} \quad (9)$$

with the relevant integrals written in terms of the variable $x=\rho^2$.

This expression can be evaluated by the use of the recurrence and orthogonality relations[6]

$$(n+1)L_{n+1}^l(x)-(2n+l+1-x)L_n^l(x)+(n+l)L_{n-1}^l(x)=0$$
$$\int_0^\infty e^{-x}x^lL_n^l(x)L_{n'}^l(x)\,dx=[(l+n)!/n!]\delta_{nn'}$$

The numerator of (9) can then be reduced to $(2n + |l| + 1)$ times the denominator so that

$$\langle\Phi\rangle=(2n+|l|+1)2\pi\hbar/e. \quad (10)$$

The case $n = 0$ corresponds to the classical motion in a circle about the origin so that for these eigenfunctions

$$\langle\Phi\rangle=(2\pi/e)(\langle L_z\rangle+1), \quad (11)$$

where L_z is the z component of angular momentum. The eigenfunctions for $n \neq 0$ are related to circular motions about some other center[4] and in this general case the expectation value of the flux is the expression (10).

The classical result can be derived as follows.

$$\Phi=\pi r^2B=(\pi/e)mvr, \quad (12)$$

where r is the radius and v the velocity of the particle in the classical orbit. Now the generalized momentum $\mathbf{p}$ is given by

$$m\mathbf{v}=\mathbf{p}-e\mathbf{A}, \quad (13)$$

where $\mathbf{A}$ is the vector potential, and so

$$mvr=L_z+eBr^2/2 \quad (14)$$

using the cylindrical vector potential $\mathbf{A} = (\mathbf{B} \times \mathbf{r})/2$. Comparing (12) and (14) gives

$$\Phi=(2\pi/e)L_z. \quad (15)$$

The difference between (12) and (15) emphasizes the importance of the distinction in Hamiltonian mechanics between momentum and the mass–velocity product.

Comparing the expressions (11) and (15) we see that the classical and quantum results agree apart from a zero point flux.

Finally, it is interesting to note that the eigenvalues of the system are not of the form

$$E=(n_x+\tfrac{1}{2})\hbar\omega+(n_y+\tfrac{1}{2})\hbar\omega,$$

which would be the case for two orthogonal harmonic oscillators. The zero point energy is $\hbar\omega/2$ and not $\hbar\omega$ since the circular motion is a combination of two *coupled* harmonic oscillators.

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[6]M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions* (Dover, New York, 1965), pp. 778, 781–783.

Erratum: "Precision timing applied to a driven mechanical oscillator." J. Morris Blair [Am. J. Phys. **43**, 1076 (1975)].

On p. 1077, in Sec. B, the second and third paragraphs should read as follows:

> With a single supporting spring, there is a tendency for the moving system to wobble at some frequencies. Much smoother vibrations are obtained if two springs are used, one above the mass M and one below it. With two springs the value of K is the sum of the constants of the two springs and the effective driving amplitude is the amplitude of the motion applied to the top of the upper spring multiplied by $K_1/(K_1+K_2)$, where K_1 is the constant of the upper spring and K_2 is that for the lower one.
>
> For convenience and accuracy in determining the amplitude of motion of the mass M, a simple cathetometer using an open sight-bar instead of a telescope has been found useful.