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Scaling Rules for Organ Flue Pipe Ranks

by N. H. Fletcher

Department of Physics, University of New England, Armidale N.S.W. 2351, Australia

Summary

The scaling problems involved in designing a rank of organ flue pipes which is satisfactorily coherent in tonal balance and matched in loudness level are investigated from the viewpoint of acoustical theory. The suggested solution from the viewpoint of tonal balance, that the pipe diameter should vary as the 5/6 power of pipe length, leads to a rank which is too loud in the bass. The usual practical solution, which is to make the diameter vary as the 3/4 power of length and to make further voicing adjustments, is seen to give an approximately balanced loudness and an increase in harmonic development towards the bass of the rank. Such a solution is similar in effect to that adopted for other musical instruments.

Skalierungsregeln für Lippenpfeifen-Orgelregister

Zusammenfassung

Die Skalierungsprobleme, die beim Entwurf eines in der tonalen Balance befriedigend kohärenten und im Pegel angepaßten Registers von Lippenpfeifen für Orgeln auftreten, werden vom Standpunkt der akustischen Theorie untersucht. Die aus dem Gesichtspunkt der tonalen Balance heraus vorgeschlagene Lösung, daß nämlich der Pfeifendurchmesser sich mit der 5/6. Potenz der Pfeifenlänge ändern sollte, führt zu einem Register, das im Baß zu laut ist. Man erkennt, daß die übliche, praktische Lösung, bei der sich der Durchmesser mit der 3/4. Potenz der Länge ändert, und bei der weitere, zusätzliche Stimmenjustierungen durchgeführt werden, eine näherungsweise ausgeglichene Lautstärke und ein Anwachsen der Harmonischen zum Baß des Registers hin liefert. Eine solche Lösung ist in der Wirkung den bei anderen Musikinstrumenten angewandten Lösungen ähnlich.

Règles d'établissement des rangs de tuyaux d'orgue

Sommaire

On étudie, du point de vue de la théorie acoustique, les problèmes d'établissement d'une rangée de tuyaux d'orgue qui soit bien cohérente dans son équilibre tonal et en même temps adaptée en niveau de sonie. En ce qui concerne l'équilibre tonal, on a proposé de choisir un diamètre de tuyau variant comme la puissance 5/6 de sa longueur: mais on obtient ainsi un jeu trop sonore dans les basses. La solution pratique usuelle est de prendre un diamètre variant comme la puissance 3/4 de la longueur, et d'ajuster ensuite les voix: la sonie est alors à peu près équilibrée, et on a d'avantage d'harmoniques du côté des basses: cette solution est, en fait, semblable à celle qu'on adopte pour d'autres instruments de musique.

1. Introduction

The pipe organ differs from woodwind and brass orchestral instruments (which, in the nineteenth and early twentieth century it tried to imitate) by the fact that it has a separate acoustic generator (pipe) for each note of each stop instead of producing the whole range of notes from a single acoustic system. In this way it is analogous to the harpsichord or piano among stringed instruments but differs from the bowed strings of the violin family.

This distinction and analogy may seem trivial or forced from some points of view but is important for our present concern. In a flute or trumpet the designer must effect many compromises to achieve a satisfactory musical balance for the instrument as a whole; in a rank of organ pipes each pipe may be designed separately with a good deal of freedom to achieve any effect the builder wishes. The same is true for harpsichord or piano, though the freedom is limited by the use of a common soundboard for all strings.

The design of a rank of organ pipes to produce a musically satisfactory result involves control of many variables for each pipe. Pitch, loudness, harmonic structure and initial transient are the most important of these and each is influenced both by initial design considerations and by final voicing adjustments. The degree of success achieved is difficult to measure in an objective physical manner and rests on qualitative judgements related to human auditory physiology. Fortunately there is a good amount of agreement among organ builders and musicians on what constitutes a satisfactory result and also on the means of achieving it.

It is the purpose of this paper to examine the physical characteristics of generally accepted pipe rank designs in the light of recent developments in the understanding of the mechanism of pipe speech and to relate this discussion to the musical criteria involved. We limit ourselves to flue pipe ranks though similar considerations apply to reed pipes. In a subsequent paper we shall apply similar considerations to harpsichord design.

2. History of pipe scaling

The basic physics of organ pipes is well known, in an elementary sense, and treated in most physics texts. More detailed discussions are given in texts on musical acoustics [1], [2] and on organ building [3], [4], [5] and in general articles on the subject [6]. We shall not repeat this here. The general construction of a flue pipe, together with the main technical terms is shown in Fig. 1.

In addition to this general information, a recently published translation of a monograph by Mahrenholz [7] deals specifically with the calculation of organ pipe scales from the middle ages to the mid-nineteenth century. Most of the information on which the present section is based is drawn from Mahrenholz [7] and from the book by Andersen [5], though our analysis is rather different from both of these.

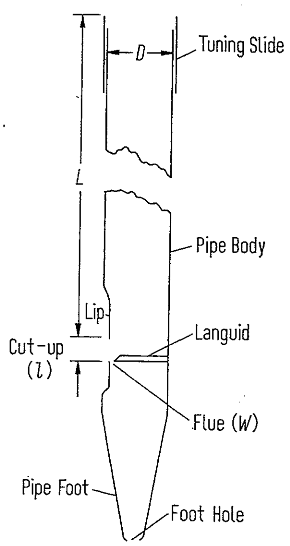

Fig. 1. Cross-sectional view of a typical organ open flue pipe.

The organ is an instrument with a very long history, and recognisable instruments date back to at least the second century A.D. Up to the middle ages organs had a compass limited to about two octaves and all pipes were of the same diameter ("the size of a pidgeon's egg" or about 25 to 30 mm), the longest pipe having a length to diameter ratio of about 8 : 1. In these organs the design problem was largely one of determining the pipe lengths to give an appropriate pitch, and the practical solutions were well known. This length problem is, of course, ultimately bound up with the problem of musical temperament [2], [7], [8] but the common temperaments are all generally within the range of tuning adjustment for normal pipes so this need not concern us further here.

The uniform scale of the middle ages suffered the major defect that it could not be extended much beyond a compass of two octaves without the bass pipes becoming too narrow in scale (giving first a weak "stringy" sound and ultimately only overblown sound in a higher mode) and the treble pipes too wide (giving a dull sound and again ultimately no sound at all). For this reason, in about the thirteenth century, a new pipe scaling system was introduced in which all pipes were graded in diameter in proportion to their length. This scaling (called "doubling on the octave" or 2 : 1 scaling) gave a workable pipe rank, but the lower pipes were now too wide and gave a loud dull sound, while the higher pipes were too narrow and hence soft and stringy. These judgments were made, of course, strictly on the base of aural assessment.

Many theorists of pipe scaling developed in succeeding centuries and, though many of them were practical and successful organ builders, their theoretical writings often sought a "perfect" scaling formula in terms of numerology or the properties of simple geometrical constructions. These are more easily discussed algebraically.

Suppose D is the diameter of a pipe of length L sounding fundamental frequency ν_1, and let D_0, L_0 and ν_0 refer to some reference pipe within the rank (often the longest pipe). If we include end corrections Δ, which amount to about $+D$ for an open pipe when both end and mouth are considered, then

$$L = (L_0 + \Delta_0)(\nu_0/\nu_1) - \Delta \,. \tag{1}$$

The mediaeval pipe scaling is given by

$$D = D_0 \tag{2}$$

while the later scale doubling on the octave is

$$D = D_0(\nu_0/\nu_1) \,. \tag{3}$$

Because this scaling makes bass pipes too broad and treble pipes too narrow, many builders adopted the simple correction

$$D = D_0(\nu_0/\nu_1) + \sigma \tag{4}$$

and it was this relation which was used, with appropriate choice of D_0 and δ, by many of the great builders of the seventeenth and eighteenth centuries.

Another type of scaling law which developed later is represented by the formula

$$D = D_0(\nu_0/\nu_1)^x \tag{5}$$

where, if $x < 1$, the scaling deficiencies noted above can also be corrected. Because these scalings were developed in an arithmetical rather than an algebraic manner, x was chosen so that 2^x was a rational fraction or some other "magic" number. In this way octave diameter ratios of $5:3 = 1.667$, $\sqrt[4]{8}:1 = 1.682$ and 1.618 (the "Golden Section") were all suggested. It is clear that the differences between these scalings in practice is probably close to being negligible.

Another way of employing eq. (5) is to choose $x = 12/n$ where n is an integer. Since the frequency ratio for an equal-tempered semitone is $2^{1/12}$, this formula will ensure that the pipe diameter doubles over an interval of n semitones. Values of n in the range 15 to 18 give practically acceptable scalings, and $n = 16$ leads again to the octave ratio $1:\sqrt[4]{8}$ which was recommended on a semi-theoretical basis by Töpfer in 1833 and has been fairly widely accepted.

Examination of the pipe work on organs built by the great master builders of the sixteenth, seventeenth and eighteenth centuries [5] shows that their scaling practices differed slightly from all these simple models but it is not clear to what extent these deviations were deliberate and consistent in the work of a single builder. These organs were generally judged to be excellent in their time and remain so to modern ears. We therefore conclude that a scaling law of the form eq. (5) with x close to 0.75 gives satisfactory tonal homogeneity and balance to a normal flue pipe rank. This appears to be true for diapasons (principals), which have medium scale, for narrow-scaled string-tone ranks and for wide-scaled flutes. A scaling law of generally similar nature is used by most organ builders today.

Before we go on to examine the physical basis for the success of these scaling rules we must examine two other aspects of pipe design: the dimensions of the mouth and flue and the blowing pressure.

The principal variables available for pipe mouth design are the mouth width B, the lip cut-up l, and the width of the flue opening W. These are shown in Fig. 1. Normal practice has always been to specify the mouth width as a fraction of the pipe circumference, the fraction usually being near 1/4 for diapason pipes, and then to specify the lip cut-up as a fraction of mouth width, again about 1/4 for diapason pipes. Thus

$$B = \alpha \pi D, \tag{6}$$

$$l = \beta B \tag{7}$$

with $\alpha \cong \beta \cong 0.25$ for diapason ranks. For string-toned ranks where a quieter tone is desired the scaling is narrower and typically $\alpha \cong 0.25$, $\beta \cong 0.3$. For wide-scaled flutes $\alpha \cong 0.2$ and $\beta \cong 0.4$, the large cut-up further reducing the harmonic development. In all cases, however, these parameters may vary somewhat depending on the precise tone quality sought.

The flue width W is also usually made proportional to mouth width

$$W = \gamma B \tag{8}$$

with γ typically about 0.03, though again this may vary with the loudness desired by the builder.

In most organs it is customary for all pipes of a given rank to stand on the same wind chest and to be blown with air at the same pressure. Exceptions to this practice occur in some large romantic organs but are not important for our present discussion. The pressure in the pipe foot is not identical with the wind-chest pressure because of pressure drops across the pallet, key-chamber and pipe-foot opening. The pipe-foot opening is normally adjusted during the voicing process to achieve even speech and loudness across the rank, but to a first approximation we may take the actual blowing pressure in the pipe foot to be constant for all pipes in a rank.

The effects of some of these voicing adjustments on the acoustic behaviour of organ pipes has been described in some detail, from a practical point of view, by Bonavia-Hunt [3] and by Mercer [9], and we shall make use of their experience as general background to our discussion. For the most part, however, it will be based on rather more recent investigations which we shall now discuss.

3. Physical basis of pipe scaling

The acoustical problems involved in understanding the speech of organ flue pipes have been investigated by many physicists during the past 150 years. It is only recently, however, that this understanding has begun to be nearly quantitative, and the discussion which follows is based largely on work by Cremer and Ising [10], Coltman [11], Benade and Gans [12], Elder [13] and the present author [14], [15], [16]. A somewhat different approach has been provided by the work of Sundberg [17], though he does not reach any general conclusions of the type we are seeking here.

The acoustic output of an organ pipe is determined by a complex non-linear feedback mechanism which couples the pipe modes to propagating disturbances on the air jet and hence to each other. Details have been discussed elsewhere [14], [16] and can be summarized in a form suitable for our present discussion in terms of the following scaling law.

If two pipes blown with the same pressure but of different lengths L have their diameters D scaled in such a way that the anharmonicities $(\nu_N - N\nu_1)/\nu_1$ and quality factors Q_N of corresponding resonances ν_N are the same, and if flue dimensions are scaled proportionally to D while the lip cut-up is proportional to L, then the velocity amplitudes of corresponding modes in the two pipes will be equal.

This law, which is a modified form of that given previously [16], rests on some approximations about wave behaviour on the jet but appears to be sufficiently accurate for our present purpose. We can also go a little further [18] to note that sound pressure at a given distance on the pipe axis is proportional to the cross-sectional area of the pipe, to the velocity amplitude and to the frequency. The angular distribution of sound energy varies somewhat with νD but we shall neglect this here.

To determine an appropriate scaling rule for a pipe rank we must try to achieve two objectives: an equal or appropriately graded harmonic development and an equal loudness throughout the rank. Let us consider these in turn.

The simple scaling law given above assumes that the Q values and anharmonicities are the same for corresponding resonances of all pipes in the rank, but this can certainly not be achieved in general. The pipe damping, which determines the Q value, has two components which differ in frequency dependence. The first is due to radiation losses from the mouth and open end and the second is caused by viscous and thermal losses to the pipe walls. To sufficient accuracy for our present purposes the derived Q for a resonance of frequency ν can be written [19] as

$$Q \cong (5 \times 10^{-5} \nu D^2 L^{-1} + 1.4 \nu^{-1/2} D^{-1})^{-1} \quad (9)$$

where the numerical coefficients assume L and D to be given in centimetres. Clearly the first term, representing radiation losses, is dominant for wide pipes at high frequencies and the second wall-loss term for narrow pipes at low frequencies.

For the Nth harmonic of a pipe with fundamental frequency ν_1 we have $\nu = N\nu_1$ and $L = c/2\nu_1$, where c is the velocity of sound. From eq. (9) the Q for the Nth resonance of this pipe is thus

$$Q_N \cong (3 \times 10^{-9} N \nu_1^2 D^2 + 1.4 N^{-1/2} \nu_1^{-1/2} D^{-1})^{-1} \quad (10)$$

and we can examine the behaviour of this quantity for various values of x in the scaling rule (5). Fig. 2

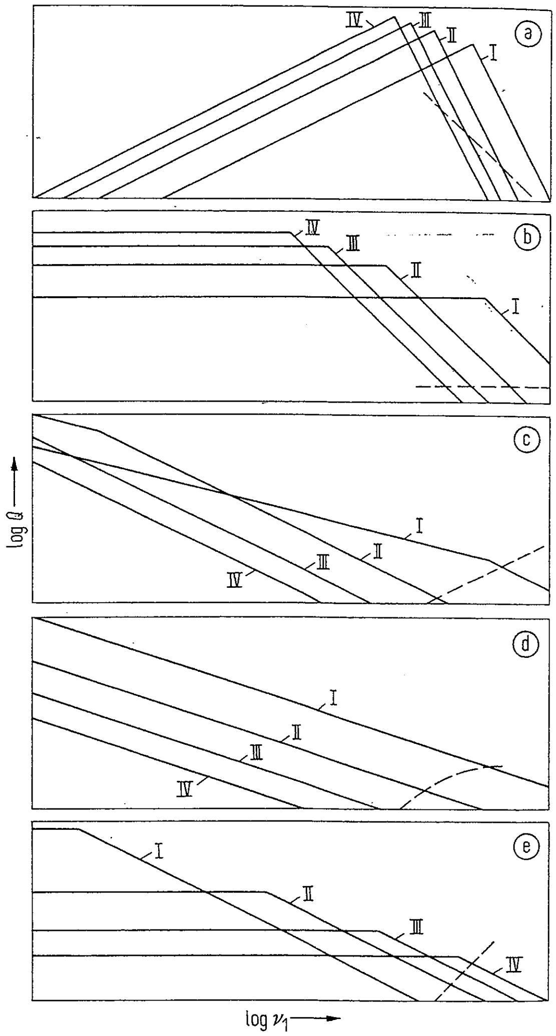

Fig. 2. Schematic diagrams for the behaviour of the Q values for the first four pipe modes (I, II, III, IV) as a function of pipe fundamental frequency ν_1. Asymptotic behaviour only is shown and the real curve for each mode lies smoothly below the asymptotes, for which the slopes are given in Table I. The broken line indicates the cut-off of each mode at ν^*. Values of the scaling parameter x are (a) 0, (b) 0.5, (c) 0.75, (d) 0.83, (e) 1.00.

shows this in schematic form; obviously the details differ according to the diameter D_0 chosen for the standard pipe ν_0. The particular x values given in the figure are $x = 0$ (a rank with mediaeval constant diameter), $x = 1$ (a rank with diameter proportional to length), $x = \frac{1}{2}$ (a rank with cross sectional area proportional to length) $x = 5/6$ (a special case to be discussed later) and the standard modern scaling $x = 3/4$.

From Fig. 2 several important points are immediately apparent when it is remembered that, broadly speaking, the acoustic power associated with a particular pipe mode will vary directly with the Q of the resonance involved. Thus in Fig. 2a, for the constant diameter scaling $x = 0$, the upper pipes are "fluty" because resonance I predominates and, because of the sharp drop in Q with rising frequency, the treble pipes ultimately refuse to speak at all. In the bass, in contrast, the resonance curves cross over and the tone becomes "stringy" until extreme bass pipes refuse to speak in their fundamental mode and overblow to a higher mode. Across the usable part of the compass the tone quality varies with pitch. The fundamental frequency at which the mode Qs cross over is determined by the diameter D_0 of the standard pipe in eq. (5).

Somewhat similar effects occur for the cases $x = 0.5$ and $x = 0.75$ shown in Figs. 2b and c. The tone quality changes progressively across the rank, being somewhat fluty in the treble and stringy in the bass, but the shifts are more gradual and a satisfactory rank of at least five octaves compass can be built.

Fig. 2d, for the case $x = 5/6 = 0.83$, is a special case for which the relative Q values of all the resonances are the same for all pipes. There is no crossing point and change of the diameter D_0 of the reference pipe simply shifts the lines for the lower resonances relative to each other — for a sufficiently small D_0, for example, Q_{I} might lie below Q_{II} everywhere. For a rank with $x = 0.83$ the tone quality, as judged by physical rather than aural standards, will remain constant across any compass in this approximation.

For the case of proportional scaling, $x = 1$, as shown in Fig. 2e, the lower pipes are satisfactory but the upper pipes, being too narrow, become progressively more stringy in tone and ultimately will sound only a higher mode.

For reference we have collected in Table I the exponents of the slopes of the Q curves, both above and below the turnover point for all the cases shown in Fig. 2.

One further matter requires consideration in this discussion and that is the upper cut-off frequency for pipe modes. This is not properly treated by the approximate expression (9) and indeed we find that, once the wavelength becomes less than the pipe circumference, resonances effectively disappear. This cut-off frequency ν^* can be expressed as

$$\nu^* \cong 10^4/D \qquad (11)$$

Table I.
Exponents y of asymptotes $Q \propto \nu_1{}^y$, below (y_-) and above (y_+) the turnover points, for different scaling parameters x.

x	y_-	y_+
0	0.50	− 2.00
0.50	0.00	− 1.00
0.75	− 0.25	− 0.50
0.83	− 0.33	− 0.33
1.00	0.00	− 1.00

where D is in centimetres. From the scaling rule (5) we see that $\nu^* \propto \nu_1{}^x$ so that, since $x < 1$, the total number n^* of available resonant modes below ν^* decreases as the fundamental frequency ν_1 increases. In fact we have

$$n^* \propto \nu_1^{x-1}. \qquad (12)$$

The anharmonicity of the pipe resonances is determined by n^* and increases as ν_N approaches ν^*. This effect lowers the amplitudes of the corresponding pipe harmonics more than would be expected simply from the decrease in Q.

On the basis of all this discussion it is tempting to assume that a scaling parameter $x = 5/6$, giving an octave ratio of 1.78 and a rule of doubling diameter on the 14th or 15th semitone step, would be ideal for pipe scaling. Indeed it is quite close to the semi-empirical scaling rule $x = 0.75$. We must, however, look first at the question of loudness across the compass of the rank.

As we have noted before, the sound pressure p at a given distance on the pipe axis varies as $\nu_1 D^2$, and hence as ν_1^{1-2x}, provided the Q values of corresponding resonances are independent of ν_1. This will, of course, never be exactly the case and the non-linear theory [16] is sufficiently complex that no simple accurate generalizations can be made. It is, however, roughly true that the internal velocity amplitudes and hence also the external sound pressures will also be nearly proportional to Q, provided the Q values for all resonances vary similarly. We then have

$$p \propto Q\,\nu_1^{1-2x}. \qquad (13)$$

The only case for which this condition is fully met is that of the scaling rule $x = 5/6$ shown in Fig. 2c, for which $Q_N \propto \nu_1^{-1/3}$ for all N. Using these arguments we find immediately that for $x = 5/6$ the sound pressure varies as ν_1^{-1} giving a sound pressure level which falls at 6 dB per octave towards high frequencies.

The loudness of a sound is determined, however, not only by the sound pressure level but also by the characteristics of human hearing. For sounds in the frequency range 30 ⋯ 4000 Hz, which contains the major energy for all organ pipes, and at loudness levels of 60 to 80 phons, which is also typical for organ music, the equal loudness contours for human hearing fall towards high frequencies at an average rate of about 3 dB per octave. A similar characteristic is therefore necessary in an organ pipe rank to achieve equal subjective loudness across its compass. The discussion above shows, therefore, that a rank scaled with $x = 5/6$ will be too loud in the bass because of its 6 dB per octave slope.

Similar arguments applied to a pipe with $x = 0.5$ are more complicated because of the crossing of the Q_N curves. Above the turnover of the fundamental, Q_I varies as ν^{-1} giving a total fall of 6 dB per octave and weak fluty trebles. Below turnover the envelope of the set of Q curves has a slope of $\nu^{-1/3}$ and naive application of relation (13) suggests a fall rate of about 2 dB per octave which is satisfactory. The increasing stringiness of tone and instability in the bass may, however, cause problems towards the low end of the compass.

The modern scaling with $x = 0.75$ can have the diameter of the standard pipe chosen so that the fundamental is dominant over nearly the whole compass and displays a Q variation as $\nu^{-1/4}$. This leads, on approximate application of relation (13), to a fall rate of about 4.5 dB per octave for the fundamental and 6 dB per octave for the higher harmonics. Such a rank, without further adjustment, would still be subjectively louder in the bass than in the treble.

This discussion suggests that satisfactory scalings might range from $x = 0.83$ down to below $x = 0.75$, and indeed seventeenth century builders used scales from 0.67 to 0.86 [7]. Without further voicing adjustments or modifications to the scaling rule towards the top and bottom of the compass, however, all these ranks would sound louder in the bass than in the treble.

4. Discussion

We have considered, so far, only the scaling of the pipe itself, assuming mouth width and flue width to be proportional to pipe diameter D and lip cut-up to pipe length L. With all pipes speaking on the same wind pressure this gives an input power varying as ν^{-2x}. For the scaling rule with $x = 5/6$, for which output power varies as ν^{-2}, this implies that the pipes become more efficient as acoustic systems for lower fundamental frequencies. Because conversion efficiency is in any case usually less than 10 percent this does not lead to embarassment, though clearly the treatment requires modification for extremely low frequencies.

Some adjustment of power is available to the pipe voicer by changing the flue width or the size of the hole in the pipe foot, which effectively varies the blowing pressure at the flue. Pipes are also generally voiced with the lip cut-up proportional to mouth width, and hence to D rather than to L. This shortens the cut-up for bass pipes and lengthens it for treble pipes. Other things being equal, an increased lip cut-up decreases harmonic development, but it also allows a greater blowing pressure to be used before the pipe overblows to its second mode and, if the pressure is increased in this way, the acoustic output also increases [16]. The effect of all these adjustments is thus to increase the loudness of treble pipes and decrease that of bass pipes, thus remedying the deficiencies of scaling rules with x near 0.8.

A further effect of the variation of lip cut-up is progressively to increase the harmonic development of pipes at the bass end of the rank. There is, however, a limitation to what can be achieved in this way and extreme bass pipes of many ranks are distinguished virtually only by their loudness.

Tonal identity for a pipe rank resides largely in spectral components in the normal speech range, say 300 to 3000 Hz, for fairly obvious primitive human evolutionary reasons. For tonal coherence, individual pipes should produce similar spectral distributions in this important formant range. The extreme bass pipes of flue ranks produce little energy in this range and therefore tend to become indistinguishable, even as between stopped and open pipes. Extreme treble pipes have only one, two or at most three spectral components in this range, with the fundamental almost always greatly predominant, and again become indistinguishable: many builders replace the pipes of the uppermost octave of a reed rank with flue pipes with no noticeable change in tone quality.

Ideally, therefore, we should aim at increased harmonic development in the bass pipes of a rank to maintain tonal coherence with the important mid-range pipes of the same rank. This is achieved to some extent by the scaling rule which allows, by relation (12), the generation of a larger number of overtones for bass than for treble pipes. The effect is reinforced by the normal scaling rule (7) for lip cut-up, as discussed above. For more extreme developments of this same type, mixture ranks with appropriate breaks back are used [5].

Finally we note that the theory suggests [15], [16] that the initial speech transient of all pipes in a rank has a duration proportional to ν_1^{-1} provided the usual flue and mouth scaling rules (6) to (8) are approximately followed. Pipes can in fact be made to speak more slowly than this rule suggests, but not more quickly. Typical initial transients cover 20 to 40 cycles of pipe fundamental and steady speech is achieved adequately rapidly in the middle and upper part of the rank compass. The speech of lower frequency pipes, particularly for pedal flue ranks, may however be objectionably slow. There appears to be no simple scaling or voicing solution to this problem, but it is easily solved by coupling octave or mixture ranks of appropriate power which, because of their higher frequency, have correspondingly rapid speech.

As we hope to show in a subsequent paper, all these features have their counterparts in harpsichord string choir design, though the means of achieving the desired effects are very different.

5. Measurements

Pipe scaling is, of course, only the first step towards achieving a musically balanced pipe rank and the scaling rule simply sets limits to the extent to which the pipe sound can be adjusted by the voicer. It is nevertheless instructive to examine the acoustic output of a typical pipe rank in the light of our discussion. We have done this for the 8 ft Prestant rank of a modern (ca. 1970) organ by Flentrop in the chapel of St. Paul's College at Sydney University and the results are shown in Fig. 3. Measurements were made at two different microphone positions about 5 m from the organ and the results averaged

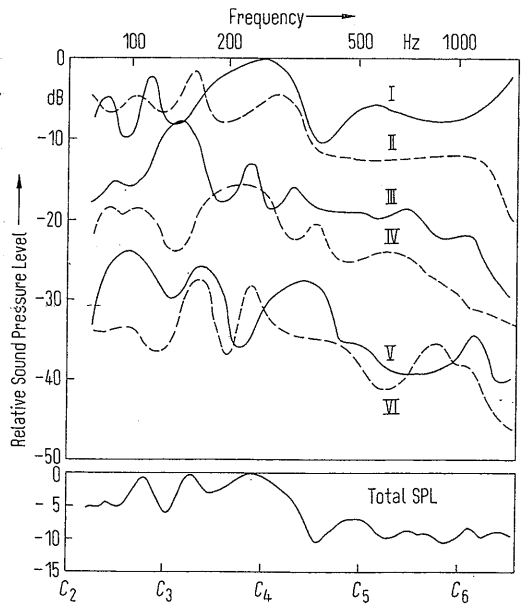

Fig. 3. Relative sound pressure levels for the first six harmonics (I, II, ..., VI) and total relative sound pressure level of the pipes of the Prestant rank of an organ by Flentrop. The oscillations in the curves are probably caused by standing waves in the building.

for neighbouring pipes to reduce the effects of standing waves in the building. The figure thus represents average trends across the rank rather than the detailed behaviour of individual pipes. The residual oscillations in the curves are probably due to standing waves.

The semi-quantitative agreement with the results to be expected from a standard scaling rule with $x = 0.75$ is immediately seen. We expect a decrease in sound pressure level of the fundamental of about 4.5 dB per octave with a slope of about 6 dB per octave for higher harmonics. Because the fundamental is dominant, the total sound pressure level should fall at about 4.5 dB per octave. In fact the measured trends are very close to these values except in the lowest octave, where the sign of the trend reverses, and in the top octave where the sound pressure level of the fundamental increases and that of the upper harmonics falls even more steeply. These are the sort of modifications we should expect from the variation of cut-up and effective blowing pressure which we discussed before. The fact that the curves for the first and second modes cross at low frequencies as in Fig. 2c is not significant until a more detailed investigation is made, both of the actual scaling rule for the pipe rank and of the adjustments made in voicing.

6. Conclusions

The organ flue pipe is a complex non-linear system whose detailed acoustic behaviour can be derived only following complex mathematical analysis. It is possible, however, to deduce a simple approximate treatment which is accurate enough to shed light on the physical principles underlying the semi-empirical scaling rules which have been developed by organ builders over several centuries of experience.

In no sense does our analysis lead to an "ideal" scaling system but simply provides a framework or standard scaling against which the effects of variations can be judged. The scaling rule appropriate to a particular rank will depend upon the building in which it is to sound, upon the tonal architecture of the organ as a whole, upon the structure of the music it is desired to perform and upon the musical taste of the builder.

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