Mechanics of flight

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The study of mechanics is a basic part of all senior school physics courses and of nearly all first year physics courses at tertiary level. In elementary mechanics, more than in most other branches of physics, the presentation usually consists of a large selection of carefully chosen examples illustrating the application of the relatively small number of physical principles involved. To many students the mechanics part of the course is dull. We do our best with the examples, and linear and rotational air tracks have revolutionized the laboratory part of the course, but still much of the exposition fails to excite interest.

In an attempt to improve this situation we have developed a presentation of related applications of physical principles based on the theory of flight (or elementary aerodynamics). This is a subject involving a large range of mechanical principles which can be fairly easily dissected out, it is of considerable interest to young people, it is of obvious practical relevance, and useful laboratory experiments can be devised. Biology students appreciate its application to the flight of birds and the structure of wings.

It is, of course, usually necessary to introduce the basic physical ideas and equations in a more formal manner and perhaps to consolidate them with simple examples to emphasize their very general applicability. The attractiveness of the present approach is that we can then apply many of these principles to a single more complex and interesting system in order to understand its behaviour. Kermode (1972) is an excellent book covering a good deal of this material (and in SI units), while a more practical book for background reading for the enthusiast is van Sickle (1971). Both these books, however, are much too detailed for what is really only a small part of the mechanics course and I have found it more satisfactory to write a small booklet to supplement standard physics texts.

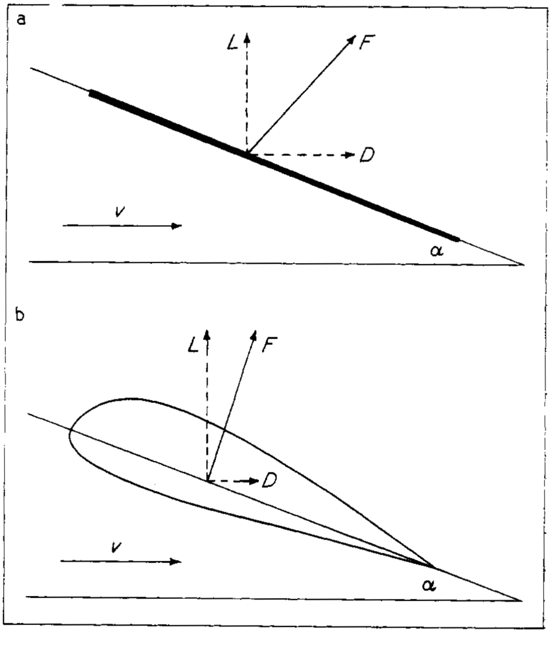
This article originally appeared, in a shorter version, in The Australian Physicist.

Aerofoils and lift

The basic theory of aerofoils is usually introduced following a derivation of Bernoulli's theorem and its application to discussion of the venturi and the Pitot tube, and this is often a good plan. The dynamic pressure $\frac{1}{2}\rho v^2$, for a fluid of density ρ moving with velocity v relative to the observer, is however a somewhat difficult concept and we can derive the basic equations for aerofoils in a simpler way. At the same time this approach makes it clear that the important thing about an aerofoil (say an aircraft wing) is not so much that its upper surface is humped and its lower surface nearly flat, but simply that it moves through the air at an angle. This also avoids the otherwise difficult paradox that an aircraft can fly upside down!

Figure 1(a) shows in section a plane rectangular plate of area A moving through the air with velocity v in such a way that it meets the airstream at an angle α, the angle of attack. For convenience we suppose the plate to be stationary and the air to be moving past it. The moving air will exert some force F on the plate and, because of the asymmetry of the situation, we would not expect F to be parallel to v but directed at some angle as in the figure. F can then be resolved into two components: D (the drag) parallel to v, and L (the lift) perpendicular to v. The symmetry of the problem then tells us quite a deal about the behaviour of these two force components. Fairly clearly the

Figure 1 (a) A flat plate and (b) an aerofoil section in an airstream of velocity v, showing angle of attack α and the resultant force F resolved into lift L and drag D



drag is always positive and goes from a minimum when the plate is parallel to the flow ($\alpha=0$) to a maximum when it is perpendicular to the flow ($\alpha=90°$). The lift, however, must be zero for the two positions $\alpha=0$ and $90°$ and must change sign when α changes sign. Very roughly, then, we might expect

$$L \sim L_0 \sin 2\alpha \quad (1)$$

$$D \sim D_0(a - \cos 2\alpha) \quad a > 1 \quad (2)$$

The real behaviour is quite like this, as we shall see later, though the angular terms are much more complicated.

We can go even further than this and note that the forces D and L both arise because the direction of the airstream changes as it flows over the plate, so that it experiences a change in momentum which necessitates a reaction force on the plate. The mass of air interacting with the plate in unit time is proportional to $\rho v A$, where ρ is the air density. Since the air is deflected through a fixed angle by the plate, the change in velocity is proportional to v, so that the force is proportional to $\rho v^2 A$. We can therefore write

$$L = \tfrac{1}{2}\rho v^2 C_L A \quad (3)$$

$$D = \tfrac{1}{2}\rho v^2 C_D A \quad (4)$$

where C_L and C_D are called the coefficients of lift and of drag respectively and the factor $\frac{1}{2}$ is included by convention. The angular behaviour suggested by equations (1) and (2) is included in C_L and C_D.

A real wing is, of course, not generally a thin plate but has the more complex aerofoil section shown in figure 1(b). This shape increases the lift coefficient C_L without greatly increasing the drag coefficient C_D, thus giving a more efficient wing. The behaviour is no longer symmetrical and the wing works better right-way-up than inverted, but the lift force still reverses its direction when the angle of attack becomes more than a few degrees negative.

Several general characteristics of aerofoils are of importance and can be discussed semiquantitatively. The dependence of C_L and C_D on angle of attack α leads to a maximum lift/drag ratio at an attack angle near 5° for real aircraft wings, the C_L value being in the range 0·5 to 1·0 depending on the wing section. The C_D value near this optimum depends on the length of the wing because of parasitic vortices around the wing tips. For a wing of infinite span, C_D is about 0·1 near this maximum.

Stall of an aerofoil is, of course, of great importance in practice but, as far as the isolated performance of an aerofoil is concerned, it is manifest only by the fact that the lift first increases with the angle of attack, reaches a maximum, and then decreases. The more spectacular aspects of stall are shown only by complete aircraft structures. In a later section of this article we describe a laboratory experiment which allows measurement of the properties of aerofoils and verification of the formulae given above.

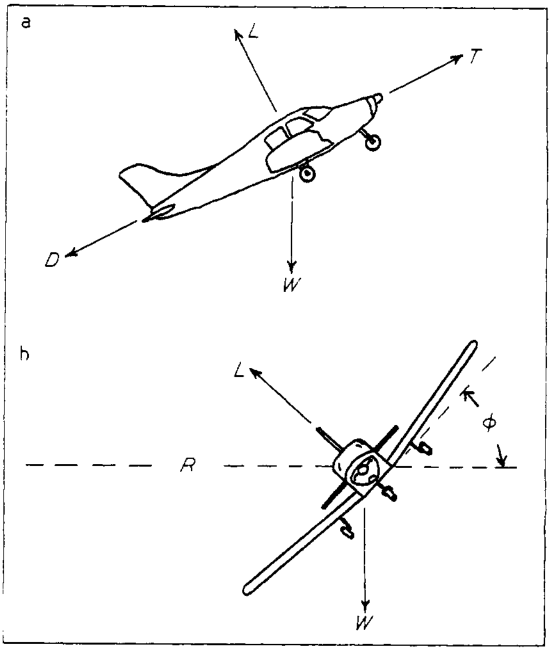


Figure 2 The forces of thrust T, drag D, lift L and weight W acting on an aircraft (a) in a steady climb and (b) in a banked turn

Aircraft structures

Once the general behaviour of aerofoils is appreciated, the next step is a simple discussion of aircraft design—and this may include the morphology of birds, which behave like simple aircraft when gliding. Clearly several things must be taken into account. The most important necessities are to provide lift, balance and control. One large aerofoil (the wing) is set at a positive angle of attack and provides lift sufficient to sustain the aircraft weight. The centre of lift of this wing must be near the centre of mass of the aircraft and this consideration together with some assessment of the relative masses of aircraft engines, fuel (stored in the wings) and passengers explains much of the observed variation in aircraft shape.

Control in the vertical plane is achieved by a small aerofoil (the horizontal stabilizer) set well away from the centre of mass, to give a large moment, and adjusted to a very small positive or even a negative angle of attack for reasons of stability which need not concern us here. Either the whole horizontal tailplane or else its rear section (the elevators) can be controlled in angle of attack by the pilot to change its lift and provide a pitching moment. The tail fin or vertical stabilizer provides similar stability and control in the horizontal plane, while the ailerons which act in opposite directions on each wing control the rolling motion of the aircraft. All these features are well known to anyone who has built a model aeroplane.

Aircraft possess, in addition to these basic surfaces, a number of refinements. The trailing edge of the wing, for example, can be extended by one or more flaps to increase both lift and drag and facilitate flight at low speed. The leading edge, too, may be fitted with a raised slat or a drooping section. These features are familiar to all passengers. It makes an interesting exercise to compare these man made devices with the almost identical appendages developed on birds' wings for the same purpose.

One final use of the aerofoil in aircraft structures is in the propeller which is essentially a twisted aerofoil. The reason for the twist is obvious since the tip moves tangentially at a high speed while the whole propeller moves forward at the aircraft speed v. For a propeller angular speed ω, the twist angle at a distance r from the hub should be about $\tan^{-1}(v/r\omega)$ on top of the set required to give the necessary angle of incidence. In sophisticated aircraft the pitch of the propeller (ie the initial angular set of the blades) can be varied to match flight conditions.

Aircraft performance

It is from a consideration of aircraft performance that most material of use in an elementary physics course is derived. The aircraft is considered as a body acted on by four forces as shown in figure 2(a). To a sufficient approximation, the aircraft moves through the air along its long axis and the forces of thrust T and drag D act in opposite directions along this axis. Lift L acts normal to this direction and weight W is, of course, directed vertically downwards. The situation is now rather similar to that of a particle sliding on an inclined plane, but the practical situation is much more interesting.

For horizontal flight at constant speed, $T=D$ and $L=W$. At the minimum flying speed v_0, the maximum lift developed by the wings (at just less than the stall angle) must be sufficient to support the aircraft weight, while at the standard cruising speed v_c this lift should be developed at an angle of attack giving maximum lift/drag ratio and hence best efficiency. The power of the engine must be adequate to move the aircraft at constant speed against the drag in each case. If this is done, then, at speeds between v_0 and v_c, the engine will be able to produce more power than is required for level flight and the excess is available to increase potential energy by climb to a higher altitude.

If, on the other hand, the aircraft tries to fly too slowly, the angle of attack must be increased. This is satisfactory until maximum lift is achieved but any further increase in angle of attack leads to stall of the aerofoil and its lift decreases. The lift developed is then no longer adequate to support the aircraft weight and it begins to fall, leading to further increase in angle of attack and more severe decrease in lift. A stalled aircraft therefore pitches downward and essentially falls from the sky.

For gliding flight on a path descending at angle θ, $T=0$ and we readily establish that

$$\tan\theta = D/L = C_D/C_L \qquad (5)$$

so that, for best gliding distance, the pilot trims his aircraft to a speed and angle of attack which maximizes C_L/C_D, the lift/drag ratio. For an ordinary light aircraft this maximum is about 10 so that $\theta \simeq 5°$, while for a high performance glider which has long thin wings to minimize parasitic drag from wing tip vortices, the ratio may be as high as 50, giving $\theta \simeq 1°$.

Climbing flight can be considered similarly and a great variety of practically interesting problems treated. All this can be further complicated if desired (and this is always the case in practice) by remembering that the motion of the aircraft is being considered relative to the air and that the wind will normally be moving the whole air mass steadily over the earth. This presents several useful problems in relative velocities.

The motion of an aircraft in a turn is particularly interesting. What happens is that the wings are banked an angle ϕ away from the horizontal as shown in figure 2(b) so that the lift, as well as balancing the weight, provides the necessary component towards the centre of the circular path. If the aircraft mass is m, its velocity v and the turn radius R, then we immediately have

$$mv^2/R = L\sin\phi \qquad (6)$$

$$W = mg = L\cos\phi \qquad (7)$$

whence

$$\tan\phi = v^2/Rg. \qquad (8)$$

The bank angle is thus uniquely determined for a given turn and interesting examples can be devised about optimum speeds for race circuits around pylons, etc.

Another interesting interpretation of equation (7) comes from writing it in the form

$$L = mg\sec\phi = mg' \qquad (9)$$

where

$$g' = g\sec\phi \qquad (10)$$

so that the effective value for g for an aircraft in a balanced turn is increased by a factor $\sec\phi$, which means a factor of two for a 60° bank angle. This increased 'g loading' must be supported by the aircraft structure and endured by the pilot, a matter of considerable practical importance.

The range of material covered can, if we wish, be extended in several ways. One which is fairly obvious is to use aircraft navigation problems with a constant wind velocity as exercises in triangles of vectors, problems which are in fact solved routinely by all pilots when navigating under visual flight rules.

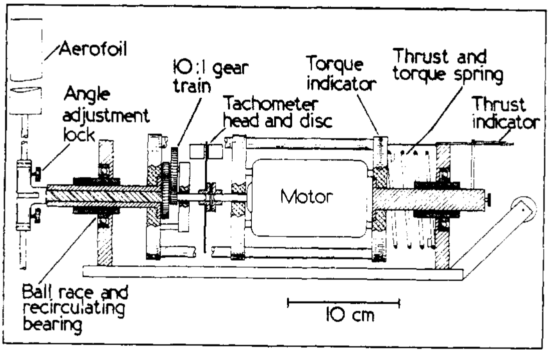


Figure 3 Construction of the experimental equipment. Calibrated scales read the forces exerted by the reaction spring

Another interesting extension is to the behaviour of gyroscopes which are vital in aircraft instruments indicating the aircraft altitude (the artificial horizon), the heading flown (the gyro compass) and the rate of turn (in the turn and balance indicator). Discussion of gyroscopic principles (including precession) for these instruments emphasizes that the gyroscope is technically vital and not just a laboratory toy.

Laboratory experiments

A variety of laboratory experiments related to the topics covered in this part of the mechanics course is possible. Many laboratory courses already include experiments on the venturi and these gain interest when it is realized that a venturi is often used on simple aircraft to provide a pressure difference to run air turbine driven gyro flight instruments. Similarly the Pitot tube can be introduced as the primary means of measuring airspeed in all aircraft.

A small windtunnel for laboratory experiments on aerofoils is unfortunately out of the question for large undergraduate classes but the behaviour of aerofoils is of such basic importance that an experiment is needed. We have therefore devised an experiment in which two matched aerofoils are mounted on the ends of light rods and rotated about an axle. There are obvious defects from a quantitative point of view—the airspeed varies about 30% from the inner to the outer end of the aerofoil and the fan effect reduces the real angle of incidence, for example—but the results are good enough to bring out all the physics we need.

The equipment is shown diagramatically in figure 3. The aerofoils, each about 25 cm long and 5 cm wide, are made of balsa wood and mounted on light aluminium tubes. The angle of attack is adjustable as shown. The driving motor is an ordinary sewing machine motor and speed control is by means of a

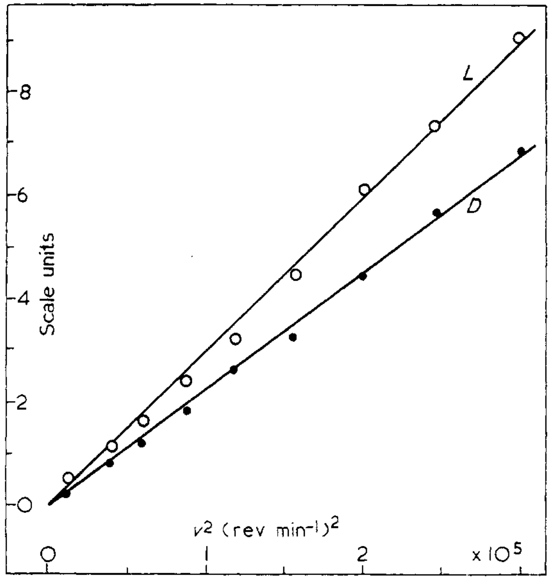
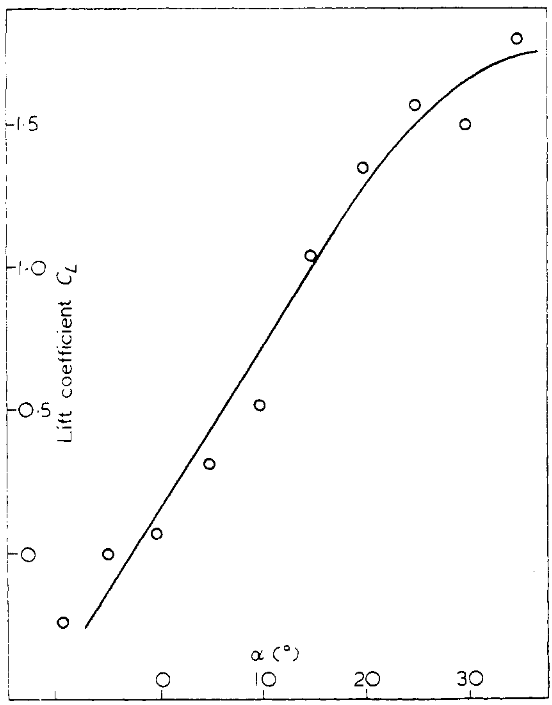


Figure 4 Measured values of lift L and drag D, showing square law dependence on the airspeed v

small autotransformer, although the sewing machine foot control would probably also suffice. Two stage gearing provides a 10:1 speed reduction to the drive shaft and speed is measured with a simple

Figure 5 Measured lift coefficient as a function of angle of attack, showing approach to stall at high attack angles



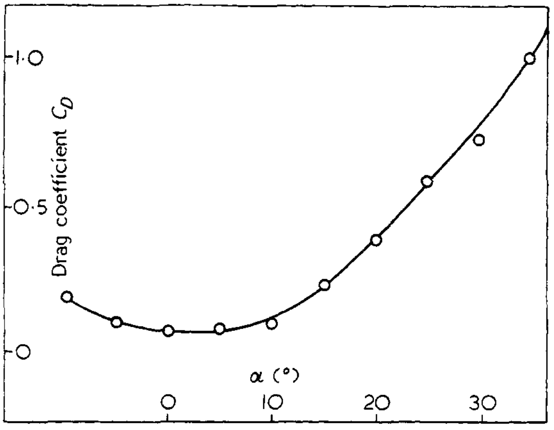
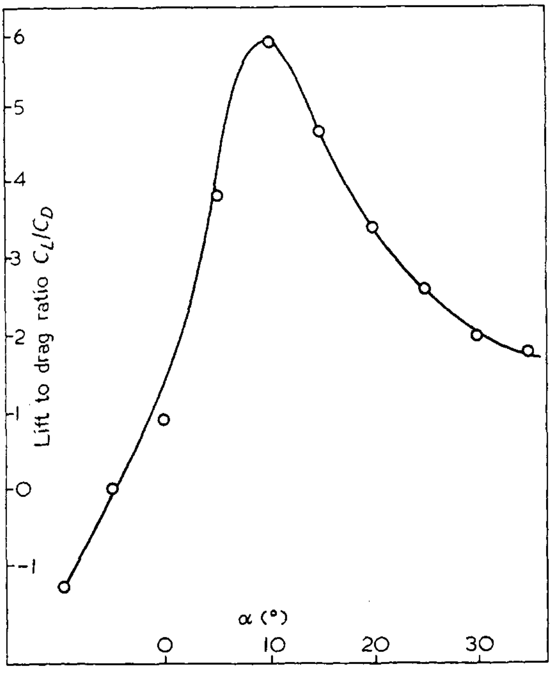


Figure 6 Measured drag coefficient as a function of angle of attack

electric tachometer. The motor is mounted in a cage supported on recirculating ball bearings inset into ball races (the only really critical part of the equipment) and both thrust (lift) and torque (drag) are taken up in a helical spring and measured with simple mechanical indicators.

After calibration of the reaction spring by means of a spring balance or other load applied to the aerofoils, the equipment can be used to verify equations (3) and (4) and to determine C_L and C_D as functions of the angle of attack α. As examples of the sort of results obtainable, figures 4–7 show the measured dependence of L and D on v^2 and of C_L, C_D and C_L/C_D on α for a typical aerofoil. Part of the scatter of points in the curve for C_L seems to represent a real departure from the simple curve drawn, but this is probably best ignored.

Figure 7 Resulting lift to drag ratio as a function of angle of attack, showing the characteristic maximum for a small attack angle



The development of further experiments and demonstrations with gliders and model aircraft is obviously possible and one can envisage the enthusiasm which might be generated by a demonstration or class experiment involving radio controlled model aircraft! Even without such delights there is a great deal of interest to be gained from watching aircraft or bird manoeuvres which are more freely accessible.

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Kermode A C 1972 *Mechanics in Flight* (8th edn) (London: Pitman)

van Sickle N D 1971 *Modern Airmanship* (New York: Van Nostrand Reinhold)

Technology for teachers

'Technology for teachers' is a new Open University postexperience course which aims to show teachers how to help children come to terms with a future that will be poorer in natural resources. In this way it is hoped that technology will be brought into a perspective alongside the human priorities of housing, food, communications, transport and industry.

The course is one of 14 postexperience courses offered in 1976, and the application period runs until 24 October 1975.

Further details about this and about all OU courses can be obtained from the Postexperience Student Office, Open University, PO Box 76, Milton Keynes MK7 6AN.

Fluid flow projects

The Department of Aeronautics, Imperial College, London, has recently produced a 150 page manual called *Fluid Dynamics and Structural Mechanics Projects for Schools.* Suggestions for fluid flow experiments range from a vacuum cleaner air jet and a simple water channel to detailed instructions for building and using a wind tunnel with a 30 cm square working section; a wide range of structural principles is illustrated by means of balsa wood models. The manual is self contained, and intended for sixth form use. After a limited free distribution, copies will be supplied by the Department of Aeronautics at the cost price of about £1·50.