ler could operate. This would explain the admixture of low frequency (1–100 kc) components in the spectrum of the spurious oscillations, although at the time it was unfortunately impossible to embark on the direct investigation of the dependence of those oscillations on the pressure within the tube.

Summing up, it seems reasonable to assume that the validity of Cutler's work is much wider than would be generally inferred from the simple boundary conditions of his experiments.

ACKNOWLEDGMENT

Acknowledgment is made to the Admiralty for permission to publish this letter.

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Inductive AC Admittance of Junction Transistor*

It has been customary to take the small signal ac admittance of the transistor as capacitive. An experimental study reveals, however, that both the collector and the emitter admittances become inductive under certain conditions.

A typical plot of grounded-base open-circuit collector admittance against collector current for a *p-n-p* alloy junction transistor is shown in Fig. 1. It should be noted that the emitter current is held constant to a high value of 10 ma. It was measured at angular frequency of 3×10^5 radians/sec (47.7 kc) using a bridge specially designed to cover a wide range of variation of admittance. The amplitude of the ac signal across the transistor was held below 10 mv. It can be seen that, besides the variation of conductance with collector current, there is a remarkable variation of susceptance. At high collector current the susceptance is capacitive as usual, but it decreases almost linearly with the collector current until it becomes inductive. A change of emitter current shifts the intercept, where the susceptance is zero, but does not modify the slope greatly.

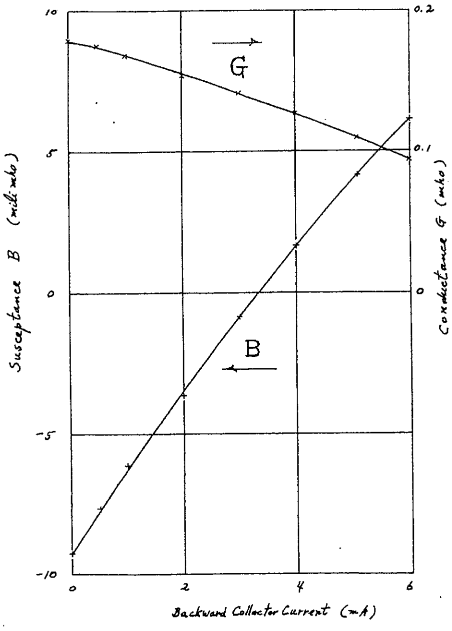

Fig. 1—Collector conductance (G) and susceptance (B) vs backward collector current for *p-n-p* alloy junction transistor, for emitter current of 10 ma.

Fig. 2 shows the relation between the collector and the emitter currents at which the collector admittance (curve C) or the emitter admittance (curve E) have zero susceptance. To avoid confusion we call the junction of the larger area *collector*, and another *emitter*, regardless of bias direction.

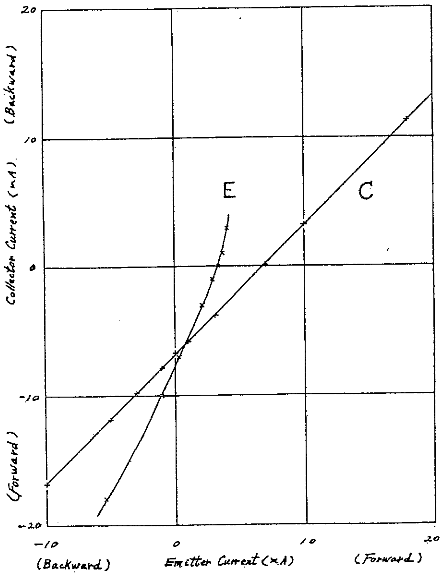

Fig. 2—Relation between the collector and the emitter currents at which the collector admittance (curve C) or the emitter admittance (curve E) have zero susceptance for *p-n-p* alloy junction transistor.

In the first quadrant of Fig. 2 the transistor is biased as in the ordinary operation, while in the third quadrant, as in the inverted operation, the collector is forward-biased and the emitter is backward-biased. It can be seen that both junctions may have inductive admittance in both operations when the forward-biased current is high. In the fourth quadrant both junctions are forward-biased.

Fig. 3 is a plot similar to Fig. 2 for a *n-p-n* grown junction transistor. A marked difference from Fig. 2 is that inductive admittance does not occur for the backward-biased junction, though it does for the forward-biased junction.

Measurements of other parameters reveal that grounded-base short-circuit admittances may become inductive as well as open-circuit admittances. However, no change of sign of imaginary part has been observed for the current amplification factors when the bias currents vary.

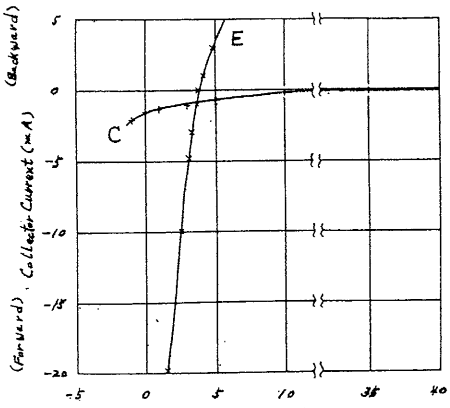

Fig. 3—Plot similar to Fig. 2 for *n-p-n* grown junction transistor.

We are indebted to Profs. N. Takagi, M. Imaoka, and Y. Adachi for many stimulating suggestions.

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* Received by the IRE, July 2, 1956.

Note on "The Variation of Junction Transistor Current Amplification Factor with Emitter Current"*

In 1954, W. M. Webster in the above paper[1] and E. S. Rittner[2] published theories describing the variation of α with emitter current for junction transistors. While the assumptions made were approximately equivalent, the limiting forms of the current dependence at high currents given by the two treatments differed by a factor of two. This disagreement has been noted[3] but it seems worthwhile to point out its source in detail and to show how rectification of an error in Webster's treatment brings the two theories into concord. This is the more necessary since Webster's treatment is that usually referred to by transistor engineers. We shall use Webster's notation throughout and refer to his paper for definition of the symbols used.

Webster's (15) describes the variation of emitter efficiency with current by

$$\frac{\partial I_{Ee}}{\partial I_{Ep}} \approx \frac{\sigma_b W}{\sigma_e L_e}(1+Z) \tag{1}$$

whereas Rittner's (70) can be manipulated to give, in Webster's notation,

$$\frac{\partial I_{Ee}}{\partial I_{Ep}} \sim \frac{\sigma_b W}{\sigma_e L_e}\cdot\frac{Z}{2} \tag{2}$$

for $Z \gg 1$ and $1-\gamma \ll 1$.

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* Received by the IRE, June 11, 1956.

[1] PROC. IRE, vol. 42, pp. 914–920; June, 1954.

[2] E. S. Rittner, "Extension of the theory of the junction transistor," *Phys. Rev.*, vol. 94, pp. 1161–1171; June, 1954.

[3] Toshio Misawa, "Emitter efficiency of junction transistor," *J. Phys. Soc. Japan*, vol. 10, pp. 362–367; May, 1955.

Webster's error lies in ignoring his own warning[4] and simply substituting

$$\sigma_b\left(1+\frac{Z}{2}\right)$$

for σ_b in his (6). While the majority carrier density in the base near the emitter is modified by this factor, or more properly by $(1+hZ)$, where

$$h(Z)=\frac{1}{Z}\int_0^Z g(Z)dZ=\frac{1}{Z}\cdot\frac{p_e}{N_d} \qquad (3)$$

with a consequent increase in I_{Ee}, there is also an increase in I_{Ep} due to the increase in the effective diffusion coefficient of holes in the base region because of the potential gradient. This is described[5] by writing $D_p/h(Z)$ for D_p in the equations from which Webster's (6) is derived. The result is then

$$\frac{I_{Ee}}{I_{Ep}}=\frac{\sigma_b W}{\sigma_e L_e}(1+hZ)h. \qquad (4)$$

Now, following Webster, we find

$$\frac{\partial I_{Ee}}{\partial I_{Ep}}=\frac{\sigma_b W}{\sigma_e L_e}(1+2hZ)g \qquad (5)$$

which, since both $g(Z)$ and $h(Z)$ approach $\frac{1}{2}$ for large Z, is seen to agree with Rittner's result (2).

A similar disagreement occurs in the two treatments of volume recombination. Rittner's treatment neglects variation of τ with injection level, but his (74) shows that if transport loss only is considered, then

$$1-\alpha\approx\frac{1}{2}\left(\frac{W}{L_b}\right)^2 \quad \text{for} \quad Z\rightarrow 0 \qquad (6)$$

and

$$1-\alpha\approx\frac{1}{4}\left(\frac{W}{L_b}\right)^2 \quad \text{for} \quad Z\rightarrow \infty. \qquad (7)$$

This is a consequence of the doubling of the effective diffusion coefficient of minority carriers in the base region at high currents.

Webster has again neglected this effect in deriving his (16) which states

$$\frac{\partial I_{VR}}{\partial I_{Ep}}=\frac{1}{2}\left(\frac{W}{L_b}\right)^2(1+Z). \qquad (8)$$

Since the $(1+Z)$ factor relates only to variation of τ,[6] which was neglected by Rittner, (8) is clearly at variance with (6) and (7).

When this effect is included in Webster's treatment, we find, in place of (8),

$$\frac{\partial I_{VR}}{\partial I_{Ep}}=\frac{1}{2}\left(\frac{W}{L_b}\right)^2(1+2hZ)g. \qquad (9)$$

Webster's result for surface recombination loss is correct (on the basis of the assumption that s does not depend on injection level) and gives

$$\frac{\partial I_{SR}}{\partial I_{Ep}}=\frac{sWA_s}{D_pA}g(Z). \qquad (10)$$

The final result is thus

$$\frac{1}{\alpha_{cb}}\approx\frac{sWA_s}{D_pA}g(Z)+\left[\frac{\sigma_b W}{\sigma_e L_e}+\frac{1}{2}\left(\frac{W}{L_b}\right)^2\right](1+2hZ)g(Z) \qquad (11)$$

and differs from Webster's original result in that the "fall-off factor" is now

$$(1+2hZ)g\approx\tfrac{1}{2}(1+Z) \qquad (12)$$

rather than

$$f(Z)=1+(g+h)Z\approx 1+Z. \qquad (13)$$

This correction to the theory alters in a corresponding way (by the introduction of a factor $\frac{1}{2}$ into the fall-off factor at high levels) the results of other derivations based upon it.[7] It does not affect the observed agreement of Webster's expression with experiment, since comparisons have generally been made[1,8] by choosing values of s and $\sigma_e L_e$ yielding best fit.

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[4] Webster, *op. cit.*, footnote 4.

[5] To an approximation similar to that used in other parts of Webster's treatment.

[6] Recent work indicates that the variation of τ with injection level may be more complicated than this assumed linear variation (private communication from Webster), as also does the Hall-Shockley-Read recombination theory. A different fall-off factor may therefore prove necessary for the volume recombination term.

[7] N. H. Fletcher, "Self-bias cut-off effect in power transistors," Proc. IRE, vol. 43, p. 1669; November, 1955.

[8] L. J. Giacoletto, "Variation of junction transistor current amplification factor with emitter current," Proc. IRE, vol. 43, p. 1529; October, 1955.