Sa1: Sound waves in a vacuum – bell in a bell-jar

The jar is evacuated and the sound of the bell becomes clearer as air is allowed in.
Introduction to Sound Waves

- Sound waves are longitudinal waves.
- Travel through any material medium.
- Their wave speed depends on the properties of the medium.
- *The mathematical description of sinusoidal sound waves is very similar to sinusoidal waves on a string.*

Categories of Sound Waves

- **Audible waves** are within the sensitivity of the human ear.
  - Approximately 20 Hz to 20 kHz.
- **Infrasonic waves** have frequencies below the audible range.
- **Ultrasonic waves** have frequencies above the audible range.

Sound Waves: Compression waves

- Compressible gas, initially uniform density.
- Piston suddenly moved to the right:
  - Gas in front is compressed.
- Piston comes to rest, but compression region continues to move:
  - Corresponds to a longitudinal pulse travelling through the tube with speed $v$.
  - Speed of the piston is *not* the same as the speed of the wave.

Speed of Sound Waves

- The speed of sound waves in a medium depends on the compressibility and the density of the medium.
- The speed of all mechanical waves is found to follow a general form:
  $$ v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} $$

- c.f. $$ v = \sqrt{\frac{Tension}{\text{Mass per unit length}}} $$ for a spring.
Speed of Sound in Liquid or Gas

The bulk modulus of the material is $B$

$B = \frac{\text{Volume Stress}}{\text{Volume Strain}} = \frac{\text{Increase in pressure}}{\text{Fractional change in volume}} = \frac{\Delta P}{\Delta V/V}$

- The density of the material is $\rho$
- The speed of sound in that medium is given by

$$v = \sqrt{\frac{B}{\rho}}$$

[see HRW §17.1 for the proof: relate the impulse on a volume element of gas caused by the increased pressure to the change in momentum of the element. Note that $B = \gamma P$ in an ideal gas.]

Speed of Sound in a Solid

- The Young’s modulus of the material is $Y$

$$Y = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} = \frac{\text{Increase in pressure}}{\text{Fractional change in length}} = \frac{\Delta P}{\Delta L/L}$$

- The density of the material is $\rho$
- The speed of sound in the rod is

$$v = \sqrt{\frac{Y}{\rho}}$$

- e.g. in a metal rod $v = \sqrt{\frac{7.0 \times 10^9 \text{ Pa}}{2.70 \times 10^3 \text{ kg/m}^2}} = 5090 \text{ m/s}$

Speed of Sound in Air

- The speed of sound also depends on the temperature of the medium
- Particularly important with gases
- For air, the relationship between the speed and temperature is

$$v = (331 \text{ m/s}) \sqrt{1 + \frac{T_C}{273}} \text{ C}$$

- 331 m/s is the speed at 0° C
- $T_C$ is the air temperature in Celsius
- Exercise for student: show that $v = (331 \text{ m/s}) \sqrt{\frac{T_C}{273}}$ where $T_C$ is temperature in Kelvin

### Speed of Sound in Various Media in m/s

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<thead>
<tr>
<th>Gases</th>
<th>1286</th>
<th>972</th>
<th>345</th>
<th>331</th>
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<tr>
<td>Hydrogen (0°C)</td>
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<td>Rubber</td>
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*Note the Temperatures given*
Sound Waves

- A compression moves through a material as a pulse, continuously compressing the material just in front of it.
- The areas of compression alternate with areas of lower pressure and density called rarefactions.
- These two regions move with the speed equal to the speed of sound in the medium.

Periodic Sound Waves: Displacement

- Each element of the medium moves with SHM parallel to the direction of the wave.
- Its displacement from equilibrium is:
  \[ s(x, t) = s_{\text{max}} \cos (kx - \omega t) \]
  - \(s_{\text{max}}\) is the maximum position from the equilibrium position.
  - Often called the displacement amplitude of the wave.

Periodic Sound Waves: Pressure

- The variation in gas pressure, \(\Delta P\), is also periodic:
  \[ \Delta P = \Delta P_{\text{max}} \sin (kx - \omega t) \]
- \(\Delta P_{\text{max}}\) is the pressure amplitude:
  \[ \Delta P_{\text{max}} = \rho v_0 s_{\text{max}} \]
  - \(k\) is the wave number.
  - \(\omega\) is the angular frequency.
  - We will now prove this.
Proof of $\Delta P = \Delta P_{\text{max}} \sin (kx - \omega t)$

[see HRW §17.2]

Consider a thin, disk-shaped element

We have $\Delta P = -B \frac{\Delta V}{V} = -B \frac{A \Delta s}{A \Delta x}$

where $\Delta s$ is the difference in displacement amplitude across $\Delta x$

i.e. $\Delta P = -B \frac{\partial s}{\partial x}$ with $s = s_{\text{max}} \cos (kx - \omega t)$

So $\Delta P = Bs_{\text{max}} k \sin (kx - \omega t)$

Since $B = \rho v^2$ and $k = \omega / v$ we have

$\Delta P = \rho v s_{\text{max}} \sin (kx - \omega t)$

So $\Delta P = \Delta P_{\text{max}} \sin (kx - \omega t)$

where $\Delta P_{\text{max}} = \rho v s_{\text{max}} = Bs_{\text{max}} k$

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Periodic Sound Waves: Pressure vs. Displacement

- A sound wave may be considered either a displacement wave or a pressure wave

- Since $s(x, t) = s_{\text{max}} \cos (kx - \omega t)$ and $\Delta P = \Delta P_{\text{max}} \sin (kx - \omega t)$ the pressure wave is $90^\circ$ out of phase with the displacement wave

- The pressure is a maximum when the displacement is zero, etc.

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PHYSCLIPS 4.3: Pressure & Density in a Sound Wave

If you blow across the top of an empty soft-drink bottle, a pulse of sound travels down through the air in the bottle. At the moment the pulse reaches the bottom of the bottle, what is the correct description of the displacement of elements of air from their equilibrium positions and the pressure of the air at this point?

1. The displacement and pressure are both at a maximum.
2. The displacement and pressure are both at a minimum.
3. The displacement is zero, and the pressure is a maximum.
4. The displacement is zero, and the pressure is a minimum.
A vibrating guitar string makes very little sound if it is not mounted on the guitar body. Why does the sound have greater intensity if the string is attached to the guitar body?

1. The string vibrates with more energy.
2. The energy leaves the guitar at a greater rate.
3. The sound power is spread over a larger area at the listener’s position.
4. The sound power is concentrated over a smaller area at the listener’s position.
5. The speed of sound is higher in the material of the guitar body.
6. None of these answers is correct.

**Calculation of Energy: I**
- The speed of the element of air is the time derivative of its displacement
  - \( s(x,t) = s_{\text{max}} \cos(kx - \omega t) \)
  - \( v(x,t) = \frac{\partial}{\partial t} s(x,t) = \omega s_{\text{max}} \sin(kx - \omega t) \)

  \[ \Delta \mathcal{K} = \frac{1}{2} \left( \Delta m \right)^2 = \frac{1}{2} \rho A \Delta x (\omega s_{\text{max}})^2 \sin^2 kx \]

  (note: we have dropped the \( \omega t \) term for simplicity as this applies at any moment of time) where \( \Delta m = \rho A \Delta x \)

**Energy of Periodic Sound Waves**
- The piston transmits energy to the element of air in the tube
- This energy is propagated away from the piston by the sound wave
- We now calculate the KE + PE of an oscillating element to give the total energy of the wave

Remember \( \cos^2 \theta = \frac{1}{2} (1 + \cos(2\theta)) \) and \( k = 2\pi / \lambda \)

**Calculation of Energy: II**
- Making use of \( \frac{1}{2} \int \sin^2(kx)dx = \frac{\lambda}{2} \) then total KE in one wavelength is
  \[ K_\lambda = \frac{\lambda}{4} \rho A (\omega s_{\text{max}})^2 \lambda \]
  (as for a wave on a string)
- Total PE for one wavelength is the same as the KE
  - As each element undergoes SHM
- Thus, the total mechanical energy is
  \[ E_\lambda = K_\lambda + U_\lambda = \frac{\lambda}{2} \rho A (\omega s_{\text{max}})^2 \lambda \]
**Power of a Periodic Sound Wave**

- $E_j$ is the energy that passes by a given point during one period of oscillation.
- The rate of energy transfer is the **power** of the wave.

\[ P = \frac{\Delta E}{\Delta t} = \frac{E_j}{T} \quad \text{over 1 complete wavelength} \]

So \[ P = \frac{1}{2} \rho Av (\omega s_{\text{max}})^2 \] since \( v = \frac{\lambda}{T} \).

**Intensity of a Periodic Sound Wave: I**

- The **intensity** $I$ of a wave is defined as the power per unit area.
- This is the rate at which the energy transported by the wave passes through a unit area, $A$, perpendicular to the direction of the wave.

\[ \text{Intensity} = \frac{\text{Power}}{\text{Area}} \]

i.e. \[ I = \frac{P}{A} \]

**Intensity of Sound Wave: II**

- So, for air, \[ I = \frac{1}{2} \rho v (\omega s_{\text{max}})^2 \]
  - square of the displacement amplitude
  - square of the angular frequency
- In terms of the pressure amplitude
  - $\Delta P_{\text{max}} = \rho v \omega s_{\text{max}}$, then

\[ I = \frac{\Delta P^2}{2 \rho v} \]

**Intensity of a Point Source**

- A **point source** will emit sound waves equally in all directions.
- This results in a **spherical wave**.
- The power will be distributed equally through the area of a sphere around the source.

\[ I = \frac{P_{\text{Average}}}{A} = \frac{P_{\text{Average}}}{4\pi r^2} \]

- This is an inverse-square law.
  - The intensity decreases in proportion to the square of the distance from the source.
PHYSCLIPS 3.3
Radiation & Intensity

The range of intensities, $I$, detectable by the human ear is very large.

It is convenient to use a logarithmic scale to determine the intensity level, $\beta$:

$$\beta = 10 \log \left( \frac{I}{I_0} \right)$$

- $I_0$ is called the reference intensity.
  - It is taken to be the threshold of hearing: $I_0 = 1.00 \times 10^{-12} \text{ W/m}^2$, $\beta = 0 \text{ dB}$
  - $\beta$ is in decibels (dB).
  - Threshold of pain: $I = 1.00 \text{ W/m}^2$; $\beta = 120 \text{ dB}$

### Sound Level, Example

What is the sound level that corresponds to an intensity of $2.0 \times 10^{-7} \text{ W/m}^2$?

$$\beta = 10 \log \left( \frac{2.0 \times 10^{-7}}{1.0 \times 10^{12}} \right) \text{ W/m}^2$$

$$\beta = 10 \log 2.0 \times 10^5 = 53 \text{ dB}$$

### Typical Sound Levels

<table>
<thead>
<tr>
<th>Source of Sound</th>
<th>$\beta$ (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nearby jet airplane</td>
<td>150</td>
</tr>
<tr>
<td>Jackhammer; machine gun</td>
<td>130</td>
</tr>
<tr>
<td>Siren; rock concert</td>
<td>120</td>
</tr>
<tr>
<td>Subway; power mower</td>
<td>100</td>
</tr>
<tr>
<td>Busy traffic</td>
<td>80</td>
</tr>
<tr>
<td>Vacuum cleaner</td>
<td>70</td>
</tr>
<tr>
<td>Normal conversation</td>
<td>50</td>
</tr>
<tr>
<td>Mosquito buzzing</td>
<td>40</td>
</tr>
<tr>
<td>Whisper</td>
<td>30</td>
</tr>
<tr>
<td>Rustling leaves</td>
<td>10</td>
</tr>
<tr>
<td>Threshold of hearing</td>
<td>0</td>
</tr>
</tbody>
</table>
Increasing the intensity of a sound by a factor of 100 causes the sound level to increase by what amount?

\[ \beta = 10 \log \left( \frac{I}{I_0} \right) \]

1. 100 dB
2. 20 dB
3. 10 dB
4. 2 dB

Loudness

- Sound level in decibels relates to a *physical measurement* of the strength of a sound
- We can also describe a *psychological measurement* of the strength of a sound
- Our bodies “calibrate” a sound by comparing it to a reference sound
  - In practice, we relate this to the threshold of hearing at a frequency of 1000 Hz
- Rule of thumb: A doubling in the “loudness” is approximately equivalent to an increase of 10 dB

Loudness and Frequency

- There is a complex relationship between loudness and frequency
- The lower curve of the white area shows the threshold of hearing
- The upper curve shows the threshold of pain
Sa3: Range of Hearing

An audio oscillator drives a speaker via an amplifier. Frequency can be varied from 10 Hz to 25 kHz. Microphone + CRO shows that sound is still present even when not audible.

Test your own hearing at:

Sa12: Doppler Effect – the Doppler Ball

The Doppler Effect

The Doppler effect is the apparent change in frequency (or wavelength) of a wave that occurs because of the motion of the source and/or the observer.

- When the relative speed between the observer and the waves is higher than the wave speed, the frequency appears to increase
  - Usually occurs as source and observer approach
- When the relative speed between the observer and the waves is lower than the wave speed, the frequency appears to decrease
  - Usually occurs as source and observer separate

We need to consider whether the observer is moving, the source is moving, or both, when analysing the situation.
Doppler Effect: Observer Moving I

- The observer moves with a speed of $v_o$
- Assume a point source that remains stationary relative to the air
- It is convenient to represent the waves with a series of circular arcs concentric to the source
  - These surfaces are called a wave front

Doppler Effect: Observer Moving II

- Take the speed of the sound as $v$, the frequency as $f$, and the wavelength as $\lambda$, with $v=\lambda f$
- When the observer moves toward the source at speed $v_o$, the speed of the waves relative to the observer is $v' = v + v_o$
  - The wavelength is unchanged
  - Thus $f' = \frac{v + v_o}{\lambda} = f \frac{v + v_o}{v}$

Doppler Effect: Observer Moving III

- i.e. The frequency heard by the observer, $f'$, appears higher when the observer approaches the source
  \[ f' = \left( \frac{v + v_o}{v} \right) f \]
- Similarly, the frequency heard by the observer, $f''$, appears lower when the observer moves away from the source
  \[ f'' = \left( \frac{v - v_o}{v} \right) f \]

Doppler Effect: Source Moving I

- Active Figure 17.09
- Consider the source being in motion while the observer is at rest
- As the source moves toward the observer, the wavelength appears shorter; i.e. A
- As the source moves away from the observer, the wavelength appears longer; i.e. B
Doppler Effect: Source Moving II

- Between successive wavefronts the source will move a distance $v_s/f$
- Thus, the distance between wavefronts, $\lambda''$, is now $\lambda' - \frac{v_s}{f}$
- Hence $\lambda'' = \frac{\lambda - v_s}{f} = \frac{v - v_s}{f}$

So that $f'' = \frac{v'}{\lambda''} = f \frac{v}{v - v_s}$

Doppler Effect: Source Moving III

- Thus, when the source is moving toward the observer, the apparent frequency is higher
  \[ f'' = f \frac{v}{v - v_s} \]

- And similarly, when the source is moving away from the observer, the apparent frequency is lower
  \[ f'' = f \frac{v}{v + v_s} \]

Doppler Effect: General – Both Source and Observer Moving

- Combining the motions of the observer and the source, when both approaching each other
  \[ f' = f \frac{v + v_0}{v - v_s} \]

- In general, we have $f' = f \frac{v \pm v_0}{v \mp v_s}$
- The signs depend on the direction of the velocity
  - If motion towards each other choose signs to increase the frequency: i.e. $+v_0$ and/or $-v_s$
  - If motion away from each other choose signs to decrease the frequency: i.e. $-v_0$ and/or $+v_s$

When Medium is also moving

- Simply replace $v$ with $v + v_m$ where $v_m$ is the speed of the medium.
- i.e. $f' = f \left( \frac{v + v_m}{v + v_m \mp v_s} \right) f$

with the same sign convention as before
Doppler Effect: final comments

- Convenient rule for signs
  - The word *toward* is associated with an *increase* in the observed frequency
  - The words *away from* are associated with a *decrease* in the observed frequency
- The Doppler effect is common to all waves
- The Doppler effect does not depend on distance
You stand on a platform at a train station and listen to a train approaching the station at a constant velocity. While the train approaches, but before it arrives, what do you hear?

1. the intensity and the frequency of the sound both increasing
2. the intensity and the frequency of the sound both decreasing
3. the intensity increasing and the frequency decreasing
4. the intensity decreasing and the frequency increasing
5. the intensity increasing and the frequency remaining the same
6. the intensity decreasing and the frequency remaining the same

Doppler Effect: Water Example

- A point source is moving to the right
- The wave fronts are closer on the right
- The wave fronts are farther apart on the left

Consider detectors of water waves at three locations A, B, and C, with the source moving at speed $V_s$, as shown. Which of the following statements is true?

1. The wave speed is highest at location A.
2. The wave speed is highest at location C.
3. The detected wavelength is largest at location B.
4. The detected wavelength is largest at location C.
5. The detected frequency is highest at location C.
6. The detected frequency is highest at location A.

Doppler Effect: Submarine Example question

- Submarine A (the source) travels at 8.00 m/s emitting at a frequency of 1400 Hz
- The speed of sound in water is 1533 m/s
- Submarine B (the observer) travels at 9.00 m/s
- What is the apparent frequency heard by the observer as the submarines approach each other? Then, as they recede from each other?
Doppler Effect: Submarine Example solution

- Approaching each other:
  - i.e. Choose both signs so frequency increases
  
  \[ f' = f \frac{v + v_s}{v - v_s} = \left( \frac{1533 \text{ m/s} + 9.00 \text{ m/s}}{1533 \text{ m/s} - 8.00 \text{ m/s}} \right) 1400 \text{ Hz} \]
  = 1416 Hz

- Receding from each other:
  - i.e. Choose both signs so frequency decreases

  \[ f' = f \frac{v - v_s}{v + v_s} = \left( \frac{1533 \text{ m/s} - 9.00 \text{ m/s}}{1533 \text{ m/s} + 8.00 \text{ m/s}} \right) 1400 \text{ Hz} \]
  = 1385 Hz

Shock Waves

- When \( v_s > v \)
  - The speed of the source, \( v_s \), can exceed the speed of the wave, \( v \)
  - The envelope of these wave fronts is a cone whose apex half-angle is given by \( \sin \theta = v / v_s \)
  - This is called the Mach angle

Mach Number

- The ratio \( v_s / v \) is referred to as the Mach number, \( M \)
- The relationship between the Mach angle and the Mach number is

  \[ \sin \theta = \frac{v}{v_s t} = \frac{v}{v_s} = \frac{1}{M} \]
An airplane flying with a constant velocity moves from a cold air mass into a warm air mass. The Mach number:

1. increases.
2. decreases.
3. stays the same

**PHYSCLIPS 5.6**

Shock Waves

WBM10VD1

Resonance in a Wine Glass

Figure P18.19
Sa5: Resonance - singing tubes

A piece of metal gauze is placed inside the tube towards one end. This gauze is heated with a bunsen burner. When the tube is removed from the burner a low pitched sound will be heard. When the tube is laid on its side the sound stops, demonstrating that the sound is caused by the hot air rising from the heated gauze.

Resonance

- A system is capable of oscillating in one or more normal modes
- If a periodic force is applied to such a system, the amplitude of the resulting motion is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system

Resonance, cont

- Because an oscillating system exhibits a large amplitude when driven at any of its natural frequencies, these frequencies are referred to as resonance frequencies
- The resonance frequency is symbolized by $f_0$
- The maximum amplitude is limited by friction in the system

Sa6: Resonance – resonating chambers

With two identical tuning forks, striking one will cause the other to vibrate. This vibration can be shown by placing a suspended ping-pong ball next to the second fork.
Standing Waves in Air Columns

Standing waves can be set up in air columns as the result of interference between longitudinal sound waves travelling in opposite directions.

The phase relationship between the incident and reflected waves depends upon whether the end of the pipe is opened or closed.

A closed end of a pipe is a displacement node in the standing wave.

- The wall at the end will not allow longitudinal motion in the air.
- Thus, the reflected wave is 180° out of phase with the incident wave.
- Creates a “standing wave”.

Since the pressure and displacement are 90° out of phase, the closed end must correspond with a pressure antinode.
- i.e. it is a point of maximum pressure variation.
Standing Waves in Air Columns, Open End

- The open end of a pipe is a displacement antinode in the standing wave
- The compressed air is free to expand into the atmosphere
- The open end corresponds with a pressure node, since it is 90° out of phase with the displacement antinode
  - i.e. it is a point with no pressure variation; there must be continuity with the value of the pressure outside the column, which is constant.

Standing Waves in an Open Tube

- Both ends are displacement antinodes
- First resonance must have \( L = \frac{\lambda_1}{2} \)
- Thus, fundamental frequency is \( f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} \)
- Similarly, for 2\(^\text{nd} \) resonance, \( L = \frac{\lambda_2}{2} \), so \( f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1 \)
- Similarly, higher resonances given \( f_n = n f_1 = n \left( \frac{v}{2L} \right) \) where \( n = 1, 2, 3, ... \)

Standing Waves in Tube Closed at One End

- Closed end is a displacement node
- Open end is a displacement antinode
- For first resonance (or mode) we have \( L = \frac{\lambda_1}{4} \)
- Thus, fundamental corresponds to \( f_1 = \frac{v}{\lambda_1} = \frac{v}{4L} \)
- 2\(^{\text{nd}} \) mode, \( L = \frac{3\lambda_2}{4} \), so \( f_2 = \frac{v}{\lambda_2} = \frac{3v}{4L} = 3f_1 \)
- And so frequencies of higher modes given by
  - \( f_n = (2n-1)f_1 = (2n-1) \left( \frac{v}{4L} \right) \) where \( n = 1, 2, 3, ... \)
End Effects

- In practice, at the end of an open tube the antinode occurs a small distance, \( \varepsilon \), beyond the end of the tube.
- Must add to the length of the tube when considering resonance
  - e.g. in the tube above, open at both ends, the first resonance is given by \( L + 2\varepsilon = \lambda / 2 \)

Resonance in Air Columns, Example

- A method of finding the resonant frequencies for a tuning fork:
  - A tuning fork is placed near the top of the tube
  - When \( L \) corresponds to a resonance frequency of the pipe, the sound is louder
  - The water acts as a closed end of a tube
  - The wavelengths can be calculated from the lengths where resonance occurs

Sa8: Resonance – Organ Pipes

Several organ pipes are used to show the relationship between tube length, open or closed end and frequency.

A pipe open at both ends resonates at a fundamental frequency \( f_{\text{open}} \). When one end is covered and the pipe is again made to resonate, the fundamental frequency is \( f_{\text{closed}} \). Which of the following expressions describes how these two resonant frequencies compare?

1. \( f_{\text{closed}} = f_{\text{open}} \)
2. \( f_{\text{closed}} = \frac{1}{2} f_{\text{open}} \)
3. \( f_{\text{closed}} = 2 f_{\text{open}} \)
4. \( f_{\text{closed}} = \frac{3}{2} f_{\text{open}} \)
Standing Waves in Rods: I
- A rod is clamped in the middle
- It is stroked parallel to the rod
  - Longitudinal waves
- The rod will oscillate
- The clamp forces a displacement node
- The ends of the rod are free to vibrate and so will correspond to displacement antinodes

Standing Waves in Rods: II
- By clamping the rod at other points, other normal modes of oscillation can be produced
- Here the rod is clamped at distance $L/4$ from one end
  - i.e. $L/4 = \lambda/4$
- This produces the second normal mode
- e.g. musical instruments: triangles, xylophones, chimes etc.

Standing Waves in Membranes
- Two-dimensional oscillations may be set up in a flexible membrane stretched over a circular hoop
- The resulting sound is not harmonic because the standing waves have frequencies that are not related by integer multiples
- The fundamental frequency contains one nodal curve

Sa16: Resonance – Chladni Figures
At several frequencies the plate can be heard to resonate, whereupon the sand will move until it reaches nodal lines.
Spatial and Temporal Interference

- Spatial interference occurs when the amplitude of the oscillation in a medium varies with the position in space of the element
  - e.g. standing waves on a string, in a pipe
- Temporal interference occurs when waves are periodically in and out of phase
  - There is a temporal alternation between constructive and destructive interference
  - Beats

Beats

- Temporal interference will occur when the interfering waves have slightly different frequencies
- Beating is the periodic variation in amplitude at a given point due to the superposition of two waves having slightly different frequencies
- Active Figure 18.17

Beats: additional of sinusoids

Suppose
\[ y_1 = A \cos \omega_1 t = A \cos 2\pi f_1 t \]
\[ y_2 = A \cos \omega_2 t = A \cos 2\pi f_2 t \]

Then \[ y = y_1 + y_2 = 2A \left[ \cos 2\pi \left( \frac{f_1 - f_2}{2} \right) t \right] \cos 2\pi \left( \frac{f_1 + f_2}{2} \right) t \]

(exercise for the student)

- i.e. wave with the mean frequency, \((f_1+f_2)/2\), modulated by time-varying amplitude \[ 2A \left[ \cos 2\pi \left( \frac{f_1 - f_2}{2} \right) t \right] \]

Sa9: Beats – between tuning forks

Two tuning forks, one 703 Hz the other 704 Hz are struck and beats can be heard. A microphone connected to a CRO will give visual reinforcement to what is being heard.
The number of amplitude maxima per second is the **beat frequency**

- Occurs when \( \cos 2\pi \left( \frac{f_1 - f_2}{2} \right) t = \pm 1 \); i.e. twice per cycle
- Thus it is the difference between the frequencies of the two sources, \( f_{\text{beat}} = |f_1 - f_2| \)
- The human ear can detect a beat frequency up to about 20 beats/sec

**Nonsinusoidal Wave Patterns**

- The wave patterns produced by a musical instrument are the result of the superposition of various harmonics
- The human perceptive response associated with the various mixtures of harmonics is the **quality** or **timbre** of the sound
- The human perceptive response to a sound that allows one to place the sound on a scale of high to low we call the **pitch** of the sound
  - Pitch is essentially the frequency of the sound, but can be perceived differently for different observers, and also depends on how the sound is produced.

**Quality of Sound – Tuning Fork**

- A tuning fork produces only the fundamental frequency

**Quality of Sound – Flute**

- The same note played on a flute sounds differently
- The second harmonic is very strong
- The fourth harmonic is close in strength to the first
Analyzing Nonsinusoidal Wave Patterns

- If the wave pattern is periodic, it can be represented as closely as desired by the combination of a sufficiently large number of sinusoidal waves that form a harmonic series.
- Any periodic function can be represented as a series of sine and cosine terms.
  - This is based on a mathematical technique called Fourier’s theorem.

Fourier Series

- A **Fourier series** is the corresponding sum of terms that represents the periodic wave pattern.
- If we have a function $y$ that is periodic in time, Fourier’s theorem says the function can be written as:
  \[ y(t) = \sum_{n} (A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t) \]
- $f_1 = 1/T$ and $f_n = nf_1$
- $A_n$ and $B_n$ are amplitudes of the waves.

Fourier Synthesis of a Square Wave: Active Figure 18.20

- Fourier synthesis of a square wave, which is represented by the sum of odd multiples of the first harmonic, which has frequency $f$.
- In (a) waves of frequency $f$ and $3f$ are added.
- In (b) the harmonic of frequency $5f$ is added.
- In (c) the wave approaches closer to the square wave when odd frequencies up to $9f$ are added.

We began this chapter by asking how the guitarist produces different notes on his guitar. From our knowledge of waves on strings, we know that the frequency of the resulting wave will depend on the length of strings and the waves produced in the string, which, in turn, depends on the linear mass density of each string and the tension in the string. By holding down the strings at different positions with his fingers, the guitarist varies the effective length of the strings and therefore the notes it plays. He plays different series of notes on different strings because they have different mass densities. He can tune the guitar by varying the tension in the strings. The same note played on different instruments sounds different because it consists of a different series of harmonics, even though the fundamental frequency—the note—is the same. Using Fourier synthesis, the keyboarder can produce a sound on the keyboard very much like that of a guitar, piano or other instrument.