Forces of Friction

- When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion.
  - This is due to the interactions between the object and its environment.
- This resistance is called the force of friction.
Forces of Friction, cont.

- Friction is proportional to the normal force
  - \( f_s \leq \mu_s \ n \) and \( f_k = \mu_k \ n \)
  - \( \mu \) is the coefficient of friction
- These equations relate the magnitudes of the forces, they are not vector equations
- For static friction, the equals sign is valid only at impeding motion, the surfaces are on the verge of slipping
- Use the inequality if the surfaces are not on the verge of slipping
Forces of Friction, final

- The coefficient of friction depends on the surfaces in contact.
- The force of static friction is generally greater than the force of kinetic friction.
- The direction of the frictional force is opposite the direction of motion and parallel to the surfaces in contact.
- The coefficients of friction are nearly independent of the area of contact.
Static Friction

- Static friction acts to keep the object from moving.
- If $F$ increases, so does $f_s$.
- If $F$ decreases, so does $f_s$.
- $f_s \leq \mu_s n$.
  - Remember, the equality holds when the surfaces are on the verge of slipping.
Kinetic Friction

- The force of kinetic friction acts when the object is in motion.
- Although $\mu_k$ can vary with speed, we shall neglect any such variations.
- $f_k = \mu_k n$
Explore Forces of Friction

- Vary the applied force
- Note the value of the frictional force
  - Compare the values
- Note what happens when the can starts to move
# Some Coefficients of Friction

## Table 5.1

<table>
<thead>
<tr>
<th>Coefficients of Friction</th>
<th>$\mu_s$</th>
<th>$\mu_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rubber on concrete</td>
<td>1.0</td>
<td>0.8</td>
</tr>
<tr>
<td>Steel on steel</td>
<td>0.74</td>
<td>0.57</td>
</tr>
<tr>
<td>Aluminum on steel</td>
<td>0.61</td>
<td>0.47</td>
</tr>
<tr>
<td>Glass on glass</td>
<td>0.94</td>
<td>0.4</td>
</tr>
<tr>
<td>Copper on steel</td>
<td>0.53</td>
<td>0.36</td>
</tr>
<tr>
<td>Wood on wood</td>
<td>0.25–0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>Waxed wood on wet snow</td>
<td>0.14</td>
<td>0.1</td>
</tr>
<tr>
<td>Waxed wood on dry snow</td>
<td>—</td>
<td>0.04</td>
</tr>
<tr>
<td>Metal on metal (lubricated)</td>
<td>0.15</td>
<td>0.06</td>
</tr>
<tr>
<td>Teflon on Teflon</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>Ice on ice</td>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>Synovial joints in humans</td>
<td>0.01</td>
<td>0.003</td>
</tr>
</tbody>
</table>

*Note: All values are approximate. In some cases, the coefficient of friction can exceed 1.0.*
Friction in Newton’s Laws

Problems

- Friction is a force, so it simply is included in the $\sum \vec{F}$ in Newton’s Laws.
- The rules of friction allow you to determine the direction and magnitude of the force of friction.
Friction Example, 1

- The block is sliding down the plane, so friction acts up the plane.
- This setup can be used to experimentally determine the coefficient of friction.
- \( \mu = \tan \theta \)
  - For \( \mu_s \), use the angle where the block just slips.
  - For \( \mu_k \), use the angle where the block slides down at a constant speed.
Friction, Example 2

- Draw the free-body diagram, including the force of kinetic friction
  - Opposes the motion
  - Is parallel to the surfaces in contact
- Continue with the solution as with any Newton’s Law problem
- This example gives information about the motion which can be used to find the acceleration to use in Newton’s Laws
Friction, Example 3

- Friction acts only on the object in contact with another surface
- Draw the free-body diagrams
- Apply Newton’s Laws as in any other multiple object system problem
Analysis Model Summary

- **Particle under a net force**
  - If a particle experiences a non-zero net force, its acceleration is related to the force by Newton’s Second Law
  - May also include using a particle under constant acceleration model to relate force and kinematic information

- **Particle in equilibrium**
  - If a particle maintains a constant velocity (including a value of zero), the forces on the particle balance and Newton’s Second Law becomes $\sum \vec{F} = 0$
Chapter 6

Circular Motion
and
Other Applications of Newton’s Laws
Uniform Circular Motion, Acceleration

- A particle moves with a constant speed in a circular path of radius $r$ with an acceleration:

  $$ a_c = \frac{v^2}{r} $$

- The centripetal acceleration, $\vec{a}_c$, is directed toward the center of the circle.

- The centripetal acceleration is always perpendicular to the velocity.
Uniform Circular Motion, Force

- A force, $\vec{F}_r$, is associated with the centripetal acceleration.
- The force is also directed toward the center of the circle.
- Applying Newton’s Second Law along the radial direction gives

$$\sum F = ma_c = m\frac{v^2}{r}$$
Uniform Circular Motion, cont

- A force causing a centripetal acceleration acts toward the center of the circle.
- It causes a change in the direction of the velocity vector.
- If the force vanishes, the object would move in a straight-line path **tangent** to the circle.
  - See various release points in the active figure.
Conical Pendulum

- The object is in equilibrium in the vertical direction and undergoes uniform circular motion in the horizontal direction.
  - $\sum F_y = 0 \rightarrow T \cos \theta = mg$
  - $\sum F_x = T \sin \theta = m a_c$
- $v$ is independent of $m$
  - $v = \sqrt{Lg \sin \theta \tan \theta}$

(divide first 2 equations and use $\tan = \sin / \cos$)
Motion in a Horizontal Circle

- The speed at which the object moves depends on the mass of the object and the tension in the cord.
- The centripetal force is supplied by the tension.

\[ v = \sqrt{\frac{Tr}{m}} \]
Horizontal (Flat) Curve

- The force of static friction supplies the centripetal force.
- The maximum speed at which the car can negotiate the curve is

\[ v = \sqrt{\mu_s gr} \]
- Note, this does not depend on the mass of the car.
Banked Curve

- These are designed with friction equaling zero
- There is a component of the normal force that supplies the centripetal force

\[ \tan \theta = \frac{v^2}{rg} \]
The banking angle is independent of the mass of the vehicle.

If the car rounds the curve at less than the design speed, friction is necessary to keep it from sliding down the bank.

If the car rounds the curve at more than the design speed, friction is necessary to keep it from sliding up the bank.
Loop-the-Loop

- This is an example of a vertical circle
- At the bottom of the loop (b), the upward force (the normal) experienced by the object is greater than its weight

\[ \sum F = n_{bot} - mg = \frac{mv^2}{r} \]

\[ |n_{bot}| = mg \left( 1 + \frac{v^2}{rg} \right) \]
Loop-the-Loop, Part 2

- At the top of the circle (c), the force exerted on the object is less than its weight

\[ \sum F = -n_{top} - mg = \frac{mv^2}{r} \]

\[ |n_{bot}| = mg \left( 1 - \frac{v^2}{rg} \right) \]