Novel approach to description of spin liquid phases in low-dimensional quantum antiferromagnets

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Abstract

We consider quantum spin systems with dimerization, which at strong coupling have singlet ground states. To account for strong correlations, the $S = 1$ elementary excitations are described as dilute Bose gas with infinite on-site repulsion. This approach is applied to the two-layer Heisenberg antiferromagnet. Our analytic results for the triplet gap, the excitation spectrum and the location of the quantum critical point are in excellent agreement with numerical calculations. We also discuss singlet and triplet bound states of the elementary triplets. It has been demonstrated that the existence of such bound states is an intrinsic property of the dimerized spin liquid.

Keywords: Spin liquid; Elementary excitations; Critical point

A variety of quantum spin models have been introduced in connection with the high-$T_c$ cuprates and other recently discovered compounds. Examples include the Heisenberg ladder [1], the two-layer Heisenberg model [2], and many other models which favor singlets formation. The two-layer Heisenberg model as well as other similar models, except for the ladder, exhibit a quantum phase transition from a disordered dimer phase to a collinear Néel phase as the dimerization decreases.

In this work we develop an effective analytical method to find excitation spectrum in a dimerized spin liquid and locate the critical point. For definiteness we consider the two-layer Heisenberg model:

$$ H = J \sum_{\langle i,j \rangle} S_{1i} \cdot S_{1j} + J \sum_{\langle i,j \rangle} S_{2i} \cdot S_{2j} + J_\perp \sum_i S_{1i} \cdot S_{2i}. $$

All the spins are $\frac{1}{2}$ and the couplings are antiferromagnetic ($J, J_\perp > 0$). The summation in each plane is over nearest neighbors on a square lattice. At large $J_\perp > J$ the ground state consists of inter-chain spin singlets. Since each singlet can be excited into a triplet state it is natural to introduce a creation operator $t_a^i$ for this excitation:

$$ D_{i}^a, 0 = t_a^i D_{i}, a T_{i}^a, x, y, z. $$

Using the representation of the spin operators in terms of $t_a^i$ one finds [1]:

$$ H = \sum_{i,a} J_\perp t_a^i t_{a}^i + J \sum_{\langle i,j \rangle, a} \{ t_a^i t_{a}^j + t_{a}^i t_a^j \} + h.c. \} $$

$$ + \sum_{\langle i,j \rangle, a\beta} \{ t_a^i t_{b}^j t_{b}^i + t_{b}^i t_a^j t_{b}^i \}. $$

At quadratic level this Hamiltonian can be easily diagonalized. However the location of the critical point at this level of approximation ($J_\perp = 4J$) differs significantly from the recent numerical result 2.54, for details on the dimer series expansion see Ref. [3]. We find, in agreement with previous work [1], that the effect of the quartic term on the spectrum is quite small and therefore cannot explain the numerical results. We treat this term perturbatively.

The dominant contribution to the renormalization of the spectrum comes from the constraint that only one of the triplet states can be excited on every site: $t_a^i t_{b}^j = 0$. The central idea of our approach is to take this condition
The results of our calculations are presented in Fig. 1. It shows the triplet gap $\Delta$ as a function of the interlayer coupling. The agreement between the analytic method and the dimer series results is excellent.

The developed technique allows us to demonstrate that there are numerous bound states of the elementary triplets. Typical spectrum includes the elementary triplet, composite triplet and composite singlet. For details see our paper [5]. There are also multi-particle bound states which are considered in the work [6].

References