Fourth-order calculation of the vacuum wave function and mass gap of SU(2) lattice gauge theory in 2+1 dimensions

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The long wavelength vacuum wave function and mass gap of SU(2) lattice gauge theory in 2+1 dimensions are calculated by a truncated eigenvalue equation method. The mass gap shows good scaling behavior at both third and fourth order.

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Recently, we developed a method for studying the long wavelength behavior of lattice gauge theory (LGT) [1]. The method is similar to Greensite's method of the truncated eigenvalue equation [2], but we use a different truncation scheme. We calculated the long wavelength ground state wave function of (2+1)-dimensional SU(2) LGT up to third order. The results at third order are in reasonable agreement with Monte Carlo results [3] and showed correct scaling behavior extending to the deep weak coupling region. We now present the fourth-order calculation for the vacuum wave function and also calculate the mass gap of the theory.

We use the notation of Ref. [1]. The Hamiltonian of SU(2) LGT is

$$H = \frac{g^2}{2a} \sum_i E_i^2 - \frac{2}{g^2 a} \sum_p \text{Tr} U_p.$$  

The ground state is represented as

$$|\Psi_0\rangle = e^{R(U)} |0\rangle,$$  

where $E_i^2 |0\rangle = 0$ and $R(U)$ consist of closed loops of $U_i$. The ground state eigenvalue equation is

$$\sum_i \left[ \{E_i, E_i^2\} + [E_i, R] [E_i, R] \right] - \frac{4}{g^4} \sum_p \text{Tr} U_p = w_0,$$  

where $w_0 g^2 / 2a$ is the ground state energy. $R(U)$ is decomposed according to the order of graphs. The lowest order term in $R$ is

$$R_1 = c_1 \square.$$  

Let $R_i$ and $R_j$ be graphs of order $i$ and $j$, respectively; then all new graphs generated by $\sum_i \{E_i, R_i\} [E_i, R_j]$ are defined as graphs of order $i + j$. The truncated eigenvalue equation at order $n$ is

$$\sum_i \left[ \{E_i, E_i^2\} \sum_{i=1}^n R_i \right] + \sum_{i+j \leq n} \{E_i, R_i\} [E_i, R_j] \right]$$

$$- \frac{4}{g^4} \sum_p \text{Tr} U_p = w_0.$$  

There are three graphs of second order, nine graphs of third order, and 57 graphs of fourth order. Up to fourth order, $R$ is a linear combination of 70 graphs:

$$R = \sum_{n=1}^{70} c_n g r_i.$$  

Substituting $R$ in the eigenvalue equation (5), we obtain 70 nonlinear equations for $c_i$. The equations are solved numerically.

Evaluating the long wavelength limit of each graph by the method of Ref. [1], we obtain

$$R = -\frac{\mu_0}{e^2} \int d^2 x \text{Tr} F^2 - \frac{\mu_2}{e^6} \int d^2 x \text{Tr} (DF)^2,$$  

where $\mu_0$ and $\mu_2$ are linear combination of the coefficients $c_i$. Results for $\mu_0$ and $\mu_2$ versus $\beta$ are plotted in Fig. 1.

As seen from the figure, the fourth-order results preserve the general trends of the third-order results, but the scaling behavior is not as good as the latter. Comparing the results at second, third, and fourth orders, it is probable that the higher order results for $\mu_0$ will oscillate and a limiting curve with the correct scaling behavior.

Among the fourth-order graphs, there are four peculiar ones:

![Graphs](image)

It is interesting to note that, if we discard these four graphs, the results for $\mu_0$ and $\mu_2$ as shown in Fig. 1 show much better scaling behavior. At present, we have no compelling reason for discarding these graphs. A possible reason is that these graphs introduce large short wavelength components in the wave function, and hence they may be suppressed in higher order calculations. Thus there remains the possibility of a different truncation scheme for a faster approach to the continuum limit.

We now turn to mass gap calculations. We take the glueball wave function as
\[ |\Psi\rangle = F(U) e^{R(U)} |0\rangle \]  

\[ F(U) = F_1 + F_2 + \cdots \]  

The eigenvalue equation for $F$ is

\[ \sum_i [(E_i, [E_i, F])] + 2[E_i, F][E_i, R] = \Delta w F \]  

where $\Delta w^2/2a$ is the glueball mass. The truncated eigenvalue equation at order $n$ is

\[ \sum_i \left[ E_i, \sum_{i=1}^n F_i \right] + \sum_{i+j \leq m} [E_i, F_i][E_i, R_j] \]

\[ = \Delta w \sum_{i=1}^n F_i \]  

The results for $\Delta w$ are plotted in Fig. 2. We see that both the third- and fourth-order curves show good scaling behavior extending to the deep weak coupling region. The mass gap is

\[ \frac{\Delta m}{e^2} \approx \begin{cases} 1.59 \text{ (third order)} \, \, , \\ 1.84 \text{ (fourth order)} \, \, . \end{cases} \]  

At fourth order, the mass gap remains the same whether or not the four graphs in Eq. (8) are included.

In contrast with the rapid rising of the mass gap in the weak coupling region as in Ref. [4], we obtain good scaling behavior for the mass gap in both third-order and fourth-order calculations. It seems paradoxical that we obtain scaling results even though we have truncated the wave function at low orders. The clue lies in the treatment of the term $\sum_i [E_i, R][E_i, R]$. We carefully keep intact all graphs generated by each term $\sum_j [E_i, gr(i)][E_i, gr(j)]$. The continuum limit of this term is then preserved. Our truncation scheme effectively softens the cutoff, which leads to better scaling compared with other truncation schemes.

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