1. A planet has a circular orbit around a star 10 light-years from Earth with a period of 100 Earth days. If the orbit is edge-on as viewed from Earth (i.e. the planet appears to oscillate back and forth on a straight line), the mass of the star is 1.5 times that of the Sun (1 solar mass = $2.0 \times 10^{30}$ kg) and is 5 times as luminous:

(a) Show that the orbit exhibits simple harmonic motion (SHM) as viewed from Earth. Show all steps in your derivation. [6]

(b) By equating the above acceleration with that due to gravity, $a = -\frac{GM}{r^2}$, calculate the distance at which the planet orbits the star (gravitational constant is $G = 6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$ s$^{-2}$). [6]

(c) Using this radius, calculate the apparent maximum speed of the planet as viewed from Earth. [4]

(d) If astronomers on Earth where observing this system at a frequency of $1.420 \times 10^9$ Hz, how large would the aperture/dish of the telescope have to be in order to resolve this orbit (1 light year is $9.46 \times 10^{15}$ m)? [3]

(e) How could such an aperture be constructed in practice? [1]

(f) [1231 only] Given that the Earth orbits the Sun at an distance of 149,600,000 kilometres, calculate how the intensity of the radiation from the star incident on the planet compares with that from the Sun on the Earth. [4]

[Total 24]

2. A sinusoidal wave travels along a string of length 8.0 m in the direction of decreasing $x$. The wave has a frequency of 60 Hz and an amplitude of 2 cm. The string has a mass of 40 g and is under a tension of 4.5 N.

(a) Find:
   i. The speed of the wave in the string. [2]
   ii. The wavelength. [2]
   iii. The angular frequency. [2]

(b) Write an equation describing the above travelling wave which has zero displacement at $x = 0.5$ metres and $t = 0$. [4]

(c) Given that the transverse velocity of an infinitesimal element on the string is given by $u = \frac{dy}{dt}$:
   i. Derive an expression for the average rate of kinetic energy in the wave. [6]
   ii. Given that the potential energy has the same value as the kinetic energy, what is the power carried by the wave? [2]
   iii. [1231 only] Show that $v = f \lambda$ for a travelling wave. [4]

[Total 22]

3. A foghorn of power 10 W emits a sound of frequency of 200 Hz. The speed of sound in air is 343 m s$^{-1}$ and the density of air is 1.22 kg m$^{-3}$.

(a) Calculate the maximum distance at which the foghorn would be theoretically audible to the human ear, using $10^{-12}$ W m$^{-2}$ as the threshold of hearing. [4]

(b) What is the maximum transverse displacement of an air particle (i.e. the amplitude) at this distance? [2]
(c) How much louder does the foghorn sound (i.e. how does the sound level compare) to the pilot of small aircraft at 1 km from the horn?
Assume low altitude, i.e. the speed of sound is 343 m s\(^{-1}\). Also assume the intensity of the sound suffers no attenuation due to the aircraft canopy or the pilot’s headphones. [4]

(d) If the pilot is flying away from the foghorn in a westerly direction at 180 km/hour, at what frequency does he hear the foghorn? [4]
[1231 only] The aircraft now experiences a 90 km/hour wind from the north. At what frequency would the pilot hear the foghorn in this case? [4]

[Total 18]

4. A light beam in air \((n = 1.00)\) enters a prism in the shape of an equilateral triangle and of refractive index 1.5 at an angle \(\theta\).

(a) Determine the smallest value of \(\theta\) for which the light ray can emerge from the other side [6].

(b) What happens at angles of incidence smaller than this? Give an example of a useful application of this phenomenon. [2]

(c) If the light has a wavelength of 650 nm and a speed of \(3 \times 10^8\) m s\(^{-1}\) before entering, calculate
i. Its wavelength within the medium. [2]
ii. Its velocity in the medium. [2]

(d) [1231 only] If the light beam emerges into water \((n = 1.3)\) [to the right of the prism in the diagram], calculate the angle at which the ray emerges if \(\theta = 60^\circ\). Check that it does emerge. [4]

[Total 16]

5. (a) Show that a single light source passing through a narrow slit of width \(a\) \((a \ll \lambda)\) produces dark fringes on a screen according to \(\sin \theta = m\frac{\lambda}{a}\), where \(m\) is an integer. [6]
   i. Sketch the intensity distribution of this pattern (label the axes). [4]
   ii. How does the intensity on the screen at a phase offset of 30° compare with that at the axis? Note that this is the single slit term of the equation given in the formula sheet. [4]

(b) i. Explain the difference between polarised and unpolarised (natural) light. [2]
   ii. How does linearly polarised light differ from circularly polarised light? [2]
   iii. Briefly describe one way of polarising natural light. [2]

[Total 20]