1. Alien astronomers on a planet 8 light years distant, view the orbit of Earth around the Sun (which has a mass of $M = 2.0 \times 10^{30}$ kg).

(a) Show that, if the orbit (which can be considered circular with constant speed) is viewed edge-on by the aliens that the Earth exhibits simple harmonic motion (SHM). Show all steps. [6]

(b) By equating the above acceleration with that due to gravity, $a = -\frac{GM}{r^2}$, from the period of the Earth’s orbit calculate the distance at which the Earth orbits the Sun (gravitational constant is $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$). Note that the radius of the orbit is equal to the maximum SHM displacement and so this should be used in the acceleration of the SHM). [8]

(c) Using this radius, calculate the maximum speed of the Earth apparent to the aliens as it orbits the Sun. [4]

(d) [1231 only] Which size of telescope observing electromagnetic radiation with a frequency of $1 \times 10^{15}$ Hz would be required by the aliens in order to resolve this orbit (1 light year is $9.46 \times 10^{15}$ m)? [4]

(e) [1231 only] In which electromagnetic band are the aliens observing? If this is also the band of their vision, speculate about the surface temperature of their sun. [2]

[Total 24]

2. A sinusoidal wave travels along a string of length 5.0 m with increasing $x$. The wave has a frequency of 80 Hz and an amplitude of 1 cm. The string has a mass of 50 g and is under a tension of 4.0 N.

(a) Find:
   i. The speed of the wave in the string. [2]
   ii. The wavelength. [2]
   iii. The angular frequency. [2]

(b) Write an equation describing the above travelling wave which has zero displacement at $x = 0$ and $t = 0$. [4]

(c) Use this expression to give the speed of an oscillating element in the string as a function of time and determine the maximum velocity of such an element. [4]

(d) You now wish to set up a standing wave on the same length of string.
   i. Write the equation of a second wave that would interfere with the wave in part b. to give such a standing wave. [2]
   ii. Give the equation of the resulting standing wave [1231 to show derivation of the equation used]. [4]

[Total 20]

3. (a) Two speakers emit sounds of different frequencies and unrelated phases. Speaker A has a power output of 5 W and speaker B a power output of 2 W. Given the geometry illustrated below, determine the resulting sound level (in dB with respect to 1 pW m$^{-2}$) at point C:
   i. If only speaker A emits sound. [3]
   ii. If only speaker B emits sound. [3]
   iii. Both speakers emit sound. [4]
iv. [1231 only] Justify your reasoning in part iii. [2]

(b) The speed of sound is 343 m s$^{-1}$ and the density of air is 1.22 kg m$^{-3}$. If speaker A alone which emits at a wavelength of 20 cm, what is the resulting pressure amplitude at C? [4]

(c) If an observer walks away from speaker A in the direction A→C, at a speed of 6 km h$^{-1}$, at what frequency do they hear the speaker? [2]

(d) [1231 only] If instead the observer walks away in the x-direction, what is the frequency heard as he passes through point C? [4]

[Total 22]

4. A light beam enters a medium and is internally reflected at the critical angle, as shown below.

(a) Determine the angle of incidence, $\theta$, of the light ray [6].
(b) If the light has a wavelength of 550 nm and a speed of $3 \times 10^5$ km s$^{-1}$ before entering, calculate
   i. Its wavelength within the medium. [2]
   ii. Its velocity in the medium. [2]

(c) What happens to the light ray at point P? [4]

(d) [1231 only] Verify that energy is conserved in part b (consider the light as a stream of photons). [2]

[Total 16]

5. (a) Show that a single light source passing through a narrow slit of width $a$ ($a \lesssim \lambda$) produces dark fringes on a screen according to $\sin \theta = \frac{m\lambda}{a}$, where $m$ is an integer. [6]

(b) What is the principle which explains why a single light source passing though an aperture produces such an interference pattern? [2]

(c) Sketch the intensity distribution of this pattern (label the axes). [4]

(d) An x-ray beam of frequency $3 \times 10^{18}$ Hz illuminates certain planes in a crystal. The first bright interference pattern occurs at $\theta = 33^\circ$.
   i. Determine the distance between the planes. [3]
   ii. Are there any more maxima at larger angles. [3]

[Total 18]