Phys 3030
Electromagnetism

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Room 59A OMB
Course information

Literature:

- **Textbook:** *Introduction to Electrodynamics*, D.J. Griffiths
- **Additional:** Landau & Lifshits vol.2 (!)
- **Lecture notes on the Web**
  www.phys.unsw.edu.au
  Courtesy of
  – Victor Flambaum
  – M.Kuchiev (some notes, hopefully)
\[ \partial \mathbf{F} = \mathbf{j} \]

\[ \partial \mathbf{\tilde{F}} = 0 \]
Assessment:

- Four assignments 15%
- Mid-Session test 1hr 15%
- Final Exam 2 hr 70%
Week 1

• Rounding up electrostatic and magnetostatic
• Introduction to Electrodynamic
Electrostatics

If there is a charge $q$, then it creates

Potential $V$ and electric field $E$

\[ E = -\nabla V \]

\[ V = \frac{1}{4\pi \varepsilon_0} \frac{q}{r} , \quad \frac{1}{4\pi \varepsilon_0} = 9 \cdot 10^9 \]

\[ E = \frac{1}{4\pi \varepsilon_0} \frac{qr}{r^3} \]
Maxwell equations for electrostatics

\[ \mathbf{E} = -\nabla V \quad \iff \quad \nabla \times \mathbf{E} = 0 \]

\[ \mathbf{E} = \frac{1}{4\pi\varepsilon_0} \frac{q\mathbf{r}}{r^3} \quad \iff \quad \nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \rho \quad \text{(Gauss law)} \]

Gauss (Ostrogradsky) law

\[ \oint \mathbf{E} \cdot d\mathbf{s} = \frac{1}{\varepsilon_0} \int \rho \, dV = \frac{1}{\varepsilon_0} \sum q_i = \frac{Q}{\varepsilon_0} \]
Magnetostatics: steady current creates magnetic field $\mathbf{B}$

Vector potential $\mathbf{A}$

$\mathbf{B} = \nabla \times \mathbf{A} \quad \Leftrightarrow \quad \nabla \cdot \mathbf{B} = 0$

$\nabla \times \mathbf{B} = \mu_0 \mathbf{j}$ (Ampere's law)

$\mu_0 = \frac{10^{-6}}{4\pi}, \quad \frac{1}{\varepsilon_0 \mu_0} = 9 \cdot 10^{16} = c^2,$

$c = 3 \cdot 10^8 \text{ m/s}$

$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$
Maxwell equations for fields

\[ \nabla \cdot \mathbf{E} = \frac{1}{\varepsilon_0} \rho \quad \nabla \cdot \mathbf{B} = 0 \]

\[ \nabla \times \mathbf{E} = 0 \quad \nabla \times \mathbf{B} = \mu_0 \mathbf{J} \]

\[ \frac{\partial \mathbf{F}}{\partial t} = \mathbf{j} \]

\[ \frac{\partial \mathbf{F}}{\partial t} = 0 \]
Equations for potentials

\[ \mathbf{E} = -\nabla V \quad \mathbf{B} = \nabla \times \mathbf{A} \]

\[ \Delta V = -\frac{1}{\varepsilon_0} \rho \]

\[ \Delta \mathbf{A} = -\mu_0 \mathbf{J} \]
Solution of the equations

\[
\Delta V = -\frac{1}{\varepsilon_0} \rho
\]

\[
\Delta A = -\mu_0 \mathbf{J}
\]

\[
V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r')}{|r-r'|} \, d^3 r'
\]

\[
A(r) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(r')}{|r-r'|} \, d^3 r'
\]

\[
(r = |r-r'|, \quad d^3 r' = d\tau')
\]
Example 1

Consider homogeneous distribution of a charge along a straight line

\[ E = \frac{1}{2\pi\varepsilon_0} \frac{\sigma n}{r}, \]

\( \sigma \) is the density of charge,
\( r \) is a distance from the line,
\( n \) is a unit vector from the line towards the observation point.
Example 2

Consider the steady current $I$ along a straight line

$$ B = \frac{\mu_0}{2\pi} \frac{I \times n}{r} $$
Combine together examples 1,2

- Assume that there is the homogeneous distribution of charge along a line
- Consider it in the reference frame, which moves with velocity $-v$ along this line
- Then in this frame there is the charge distribution \textit{plus} the current
- There are present both the electric and magnetic fields
Important: geometry in the new reference frame does not differ from the initial one:

- Electric field remains perpendicular to the line (time invariance)
- Magnetic field curls around the line
\textbf{B perpendicular to }\mathbf{E}

\[ \mathbf{E} = \frac{1}{2\pi\varepsilon_0} \frac{\sigma' \mathbf{n}}{r} \]

(\(\sigma'\) is different from \(\sigma\))

\[ \mathbf{B} = \frac{\mu_0}{2\pi} \frac{\mathbf{I} \times \mathbf{n}}{r} \]

\[ \mathbf{I} = \sigma' \mathbf{v} \]

\[ \mathbf{B} = \frac{\mu_0}{\varepsilon_0} \mathbf{v} \times \mathbf{E} \]
“BvE theorem”

- If there is a reference frame \( K' \) in which \( B = 0 \), then in any other reference frame \( K \) there may exist both \( E \) and \( B \), which satisfy

\[
B = \frac{\mu_0}{\varepsilon_0} \mathbf{v} \times \mathbf{E}
\]

Here \( \mathbf{v} \) is the velocity of \( K' \) in \( K \)

- If there is a reference frame in which \( E = 0 \), then in any other reference frame

\[
E = -\frac{\varepsilon_0}{\mu_0} \mathbf{v} \times B
\]
“Intro” to Electrodynamics

• Suppose there is a current \( \mathbf{J}(\mathbf{r}, t) \).
• How to find the magnetic field?

Attempt:

\[
\mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{J}(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|} \ d^3r' \\
\]

fails
$$A(r, t) = \frac{\mu_0}{4\pi} \int \frac{J(r', t')}{|r-r'|} \, d^3r'$$

$$t - t' = \frac{1}{c} \frac{1}{|r-r'|}$$

$c = \text{const}$

Retardation must be accounted for!

It is “cool” and perfectly correct.
Magnetic field in ED

$$A(r, t) = \frac{\mu_0}{4\pi} \int \frac{J(r', t')}{|r-r'|} \, d^3r'$$

$$B = \nabla \times A$$
Charge, which moves with constant velocity

\[ B(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \frac{q \mathbf{v} \times \mathbf{R}}{|\mathbf{R}|^3} \frac{1 - v^2 / c^2}{(1 - \sin^2 \vartheta \ v^2 / c^2)^{3/2}} \]

\[ \mathbf{R} = \mathbf{r} - \mathbf{r}'(t) \quad \text{(here stands t, not t')} \]

\[ \cos \theta = \frac{\mathbf{v} \cdot \mathbf{R}}{|\mathbf{v}| |\mathbf{R}|} \]
Electromagnetic field of the charge, which moves with constant velocity

Remember “BvE theorem” $\mathbf{B} = \frac{\mu_0}{\varepsilon_0} \mathbf{v} \times \mathbf{E}$

$$\mathbf{E}(\mathbf{r}, t) = \frac{1}{4\pi\varepsilon_0} \frac{q \mathbf{R}}{|\mathbf{R}|^3} \frac{1 - v^2 / c^2}{(1 - \sin^2 \vartheta \ v^2 / c^2)^{3/2}}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \frac{q \mathbf{v} \times \mathbf{R}}{|\mathbf{R}|^3} \frac{1 - v^2 / c^2}{(1 - \sin^2 \vartheta \ v^2 / c^2)^{3/2}}$$
“Deformation” of the field

\[ E_{\parallel}(r, t) = \frac{1}{4\pi\varepsilon_0} \frac{q R_{\parallel}}{|R|^3} \frac{1}{\gamma^2} \quad \text{-- suppressed} \]

\[ E_{\perp}(r, t) = \frac{1}{4\pi\varepsilon_0} \frac{q R_{\perp}}{|R|^3} \gamma \quad \text{-- enhanced} \]

\[ \gamma = \frac{1}{\sqrt{1 - v^2 / c^2}} = \frac{\varepsilon}{mc^2} \geq 1 \quad \text{relativistic factor} \]
Relativistic factor

\[
\left( \frac{1 - v^2 / c^2}{1 - \sin^2 \Theta \cdot v^2 / c^2} \right)^{3/2}, \quad E = 0, \frac{1}{2}mc^2, mc^2, 4mc
\]
Something is wrong with $E$ and $V$?

We started from the magnetic field

If we try to repeat the trick with the electric field, it does not work, we get complete nonsense.

\[
A(r, t) = \frac{\mu_0}{4\pi} \int \frac{J(r', t')}{|r - r'|} d^3r'
\]

\[
B = \nabla \times A
\]

\[
V(r) = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r', t')}{|r - r'|} d^3r'
\]

\[
E = -\nabla V
\]
Something is missing in the relation $E$ versus $V$

$$E = -\nabla V \quad \Rightarrow \quad E_x = -\frac{\partial V}{\partial x}$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad \Rightarrow \quad B_x = \frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y}$$
Fix the problem with $\mathbf{E}$

\[ \mathbf{E} = -\nabla V - \dot{\mathbf{A}} \quad \Rightarrow \quad E_x = -\frac{\partial V}{\partial x} - \frac{\partial A_x}{\partial t} \]

\[ \mathbf{B} = \nabla \times \mathbf{A} \quad \Rightarrow \quad B_x = \frac{\partial A_y}{\partial z} - \frac{\partial A_z}{\partial y} \]