Notes on Heat for PHYS1221-1231. Joe Wolfe, UNSW

Thermal Physics
Thermodynamics: laws relating macroscopic variables (P, V, T etc.).
Statistical Mechanics: molecular explanation.

Difference between heat and temperature
Extensive property $\propto$ amount of material
Intensive property doesn't depend on amount

Which of heat and temperature is intensive?
Which relates to sense of hotness?

Define temperature:

Thermal equilibrium:
Thermal properties do not change with time

Definition of Temperature (T):
T is equal in any 2 bodies at thermal equilibrium.

Zero\textsuperscript{th} Law of Thermodynamics:
if $T_A = T_B$ and $T_B = T_C$, then $T_A = T_C$.

What is temperature? How to measure it?

Thermometers: Hg in glass, thermocouple, thermistor, liquid crystal layer, constant volume gas thermometer

Scales. Obvious definition of temperature $\theta$: choose a property $X$ and make $X$ proportional to or linear with $\theta$. This can only be done once for any temp scale $\theta$.

Reference temperature
Melting or freezing? Depends on the pressure.

Thermal Expansion

\[ \frac{\Delta L}{L} = \alpha \Delta T \]

Usually, $\frac{\Delta L}{L} \propto \Delta T$ for small $\Delta T$

Define $\frac{\Delta L}{L} = \alpha \Delta T$

$\alpha$ is coefficient of linear expansion

e.g. steel $\alpha_{st} = 1.1 \times 10^{-5}$ K$^{-1}$
Al $\alpha_{Al} = 2.3 \times 10^{-5}$ K$^{-1}$
Brass $\alpha_{br} = 1.9 \times 10^{-5}$ K$^{-1}$

Example Bridge span is 1 km long.
Mid-winter, $T = -5^\circ$C summer, $T = 45^\circ$C
What is $\Delta l$?

$\Delta L = \alpha \Delta T L = \ldots = 55 \text{ cm}$
**Example:** Bimetallic Strip, 10 cm long, made of 1 mm brass and 1 mm steel. Straight at 0 °C, what angle at 100 °C?

\[
L + \Delta L_{st} = 2\pi R \cdot \frac{\theta}{360°}
\]

\[
L + \Delta L_{br} = 2\pi (R + t) \cdot \frac{\theta}{360°}
\]

subtract →

\[
2\pi t \cdot \frac{\theta}{360°} = \Delta L_{br} - \Delta L_{st} = L\Delta T(\alpha_{br} - \alpha_{st})
\]

\[
\theta = 14°
\]

\[D L_{br} - D L_{st} = \frac{L}{D T(a_{br} - a_{st})} = 14°\]

E.g. oven switch:

**Volume Increase**

Define

\[
\frac{\Delta V}{V} = \beta \Delta T
\]

\[\beta = \text{coefficient of volume expansion}\]

\[
\Delta V = (L + \Delta L)^3 - L^3
\]

\[= L^3(1 + \frac{3\Delta L}{L})^3 - L^3\]

\[= L^3 \left(1 + \frac{3\Delta L}{L} + \cdots - 1\right)\]

\[= V \cdot 3\alpha \Delta T\]

\[\therefore \beta = 3\alpha\]

**Note:** Water is unusual: 0° - 4°C, \(\beta < 0\)

**Example** What is change in \(\rho\) for steel between 0° C and 100° C?

\[\rho = \frac{M}{V} \quad \therefore d\rho = -\frac{M}{V^2} dV\]

\[\therefore \Delta \rho = -\frac{M}{V^2} \Delta V = -\rho \frac{\Delta V}{V}\]

\[\therefore \frac{\Delta \rho}{\rho} = -\frac{\Delta V}{V} = -\beta \Delta T = \cdots = -0.33\%\]
**Example:** thermometer, $V_{\text{cyylinder}} \ll V_{\text{sphere}}$

What is its calibration slope $\frac{\partial x}{\partial T}$?

\[
\Delta V \frac{V}{V'} \equiv \beta \Delta T
\]

\[
a \delta x \approx \Delta V_{\text{fl}} - \Delta V_{\text{sphere}} = \beta_{\text{fl}} V_{\Delta T} - \beta_{\text{gl}} V_{\Delta T}
\]

\[
\frac{\partial x}{\partial T} = \frac{V}{a} (\beta_{\text{fl}} - \beta_{\text{gl}})
\]

Other temperature coefficients:

**Resistivity $\rho$:**

\[
\rho = \rho(T=T_0)(1 + \alpha_1(T - T_0) + \alpha_2(T - T_0)^2 + \ldots)
\]

Warning: $\rho$ is $> 0$ for metals, but $< 0$ for semiconductors

\[
\sigma = \frac{1}{\rho} = \ldots \equiv \sigma(T=T_0)(1 - \alpha_1(T - T_0))
\]

<table>
<thead>
<tr>
<th>Thermal runaway possible</th>
<th>Thermally stable</th>
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<tr>
<td><img src="image1" alt="Thermal runaway" /></td>
<td><img src="image2" alt="Thermally stable" /></td>
</tr>
</tbody>
</table>
Ideal gas temperature scale
Uses reference temp:
Triple point - co-existence of ice, water, steam

\[
\text{call it } \theta_{tr}.
\]

defines \( P \propto \theta \) for constant volume of gas \( (P \text{ is pressure } \equiv \text{Force per unit area}) \)
But gases are not (quite) ideal

e.g. consider boiling temp \( \theta_s \) at some \( P \):

\[
\frac{\theta_s}{\theta_{tr}} \text{ is different for different gases and at different densities.}
\]

At very low density or pressure,

all gases \( \rightarrow \) ideal, \( : \frac{\theta_s}{\theta_{tr}} \rightarrow \text{ same limit} \)

\[
T = T_{tr} \cdot \lim_{P_{tr} \rightarrow 0} \left( \frac{P}{P_{tr}} \right)_V
\]

where \( T_{tr} = 273.16 \text{ K} \)

Why 273.16? This defines the Kelvin so that

\[
\Delta T = 1 \text{ K} \iff \Delta T = 1 ^\circ \text{C}
\]

(Working definition is more complicated)

Celsius Scale: \( T_C = T - 273.15^\circ \)

\[
T_C = 0 ^\circ \text{C water freezes} \\
T_C = 100 ^\circ \text{C water boils} \quad \text{at } P_A
\]

Fahrenheit Scale
**Heat**

**Definition:** that which is transferred between a system and its surroundings as result of $\Delta T$ only.

**Joule** showed:

mechanical energy $\rightarrow$ heat \textit{(by friction etc.)}.

\begin{align*}
\text{Carnot showed} & \hspace{1cm} \text{heat at high } T \rightarrow \text{heat at low } T + \text{work} \\
\therefore \text{ measure heat as energy; i.e. S.I. unit. Joule (J)}
\end{align*}

**Heat Capacity:** (for a body) $C = \frac{\Delta Q}{\Delta T}$ \textit{extensive quantity}

**Specific Heat:** (of a substance) $c = \frac{\Delta Q}{M \Delta T}$ \textit{intensive quantity}

\begin{align*}
e.g. \hspace{0.5cm} c_{\text{H}_2\text{O}} &= 4.2 \text{ kJ.kg}^{-1}\text{K}^{-1}, \quad c_{\text{Al}} = 900 \text{ J.kg}^{-1}\text{K}^{-1}
\end{align*}

**Latent Heat:** heat required for change of phase (at constant $T$).

**Example.** A 240 V kettle has a working resistance of 50 $\Omega$. Put in 500 ml of water at 20 °C and turn on. How long before it boils dry? (Specific heat of $c_w = \text{water} = 4.2 \text{ J.kg.K}^{-1}$, Latent heat of vaporisation $L_{\text{vap}} = 2.3 \text{ MJ.kg}^{-1}$.)

Energy in = power.$t = \frac{V^2}{R}$.$t = (1.15 \text{ kW}).t$ = Q to raise T of water + Q to evaporate water

= $m_w c_w (T_f - T_i) + m_w L_{\text{vap}}$

= (0.5 kg) (4.2 $10^3$ J.kg$^{-1}$K$^{-1}$) (100 - 20)$^\circ$C + (0.5 kg) (2.3 $10^6$ J.kg$^{-1}$)

= 168 kJ + 1.15 MJ = 1.32 MJ

$\therefore \hspace{0.5cm} t = \frac{1.32 \text{ MJ}}{1.15 \text{ kW}} = \ldots = 1150 \text{ s} = 19 \text{ minutes}$

(it boils after $\frac{168 \text{ kJ}}{1.15 \text{ kW}} = 2.5 \text{ mins}$)
Example A mass $m_w$ of water at temperature $T_1$ is added to a mass $m_i$ of ice at temperature $T_2$. (No heat is lost to the environment.) What is the final temperature at thermal equilibrium?

$\exists$ 4 possible outcomes:

i) $T > 0 \, ^\circ C$ (all ice melts)

ii) $T = 0 \, ^\circ C$ & some ice melts

iii) $T = 0 \, ^\circ C$ & some water freezes

iv) $T < 0 \, ^\circ C$ (all water freezes)

To warm ice to $0^\circ$ requires $Q_i = m_i c_i (0^\circ - T_i)$

To cool water to $0^\circ$ loses $Q_w = m_w c_w (T_w - 0^\circ)$

To melt all ice requires $Q_m = m_i L$

To freeze all water loses $Q_f = m_w L$

If $Q_w > Q_i + Q_m$, then all ice melts and final $T > 0$

Q lost by water = Q gained by ice

$m_w c_w (T_w - T) = m_i c_i (0^\circ - T_i) + m_i L + m_i (T - 0^\circ)$

heat ice melt it warm melted ice

$T (m_i + m_w c_w) = m_i c_i T_i + m_w c_w T_w - m_i L$

$T = ....$

If $Q_i + Q_m > Q_w > Q_i$, then all ice melts but $T = 0^\circ$

Q lost by water = Q gained by ice

$m_w c_w (T_w - 0^\circ) = m_i c_i (0^\circ - T_i) + m_m L$

heat all ice melt some ice $m_m < m_i$

$m_m = ....$

etc for $Q_i > Q_w + Q_f$ and $Q_w + Q_f > Q_i > Q_w$

all water freezes some water freezes

Work: energy transmitted from one system to another without $\Delta T$
or transfer of Q.

e.g. work done by force $F$

$dW = F \cdot ds$

e.g. work done against pressure $P$

\[ P = \frac{\text{force}}{\text{area}} \]

\[
\begin{array}{c}
\text{P} \\
\hline
\text{F}
\end{array}
\]

$dW = F \cdot ds = PA \cdot ds = PdV$

Example 1 kg water, initially at 20˚C, is boiled away at $P_A$. How much of the input energy goes as work?

water volume = $10^{-3}$ m$^3$

Steam: $V_f >> V_i$

$W = PV_f = n RT = \frac{\text{mass}}{\text{mol. mass}} RT = \ldots = 172 \, \text{kJ}$

cf. Heat to raise to 100˚C

$Q_1 = C m \Delta T = \ldots = 336 \, \text{kJ}$

Heat to transition $Q_2 = mL = \ldots = 2.26 \, \text{MJ}$
Internal Energy

Heat dQ added to a system increases its internal energy U. Work dW done by the system lowers its internal energy.

1st Law of Thermodynamics
\[ dU = dQ - dW \]
where U is a state function

---

Kinetic Theory of Gases

Ideal gas equation of state
(\( \equiv \) limit for all gases at low \( \rho \)):
\[ PV = nRT = NkT \]
- no. of moles
- no. of molecules
- gas constant
- Boltzmann's Constant
\[ = 8.31 \text{ JK}^{-1} \quad = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ JK}^{-1} \]

Example. Spherical balloon. Skin (total) has mass \( \sigma = 10 \text{ g.m}^{-2} \).
How big does it need to be to lift 200 kg load if (i) it contains hot air at 100°C? (ii) Helium at STP?

Archimedes: \( W_{\text{displaced}} = W_{\text{balloon}} \)
\[ \frac{4}{3} \pi r^3 \rho_{\text{air}} = \frac{4}{3} \pi r^3 \rho_{\text{gas}} + 4 \pi r^2 \sigma + mg \]
\[ r^3 (\rho_{\text{air}} - \rho_{\text{gas}}) - 3 \pi r^2 = \frac{3}{4} m \quad \text{(or solve cubic)} \]

\[ r \approx \sqrt[3]{3m} \quad \frac{4 \pi \rho_{\text{air}} (1 - \rho_{\text{gas}}/\rho_{\text{air}})}{4 \pi \rho_{\text{air}} (1 - \rho_{\text{gas}}/\rho_{\text{air}})} \]

He: \( \rho_{\text{gas}}/\rho_{\text{air}} = 4/30 \)
\[ \rightarrow \quad r \approx 3.6 \quad \text{m} \]

Hot air: \( \rho = \frac{Nm}{V} = \frac{Pm}{kT} \)
\[ \therefore \quad \frac{\rho_{\text{hot}}}{\rho_{\text{cold}}} = \frac{T_{\text{cold}}}{T_{\text{hot}}} = \frac{273}{373} \quad \text{K} \quad \rightarrow \quad r \approx 5.3 \quad \text{m} \quad \text{check approx^n} \]
Kinetic theory: Ideal Gas Postulates

i) gas made of (identical) molecules

ii) these obey Newton's laws, with random motion

iii) no. of molecules is large  \( (\sim \text{Avagadro's number}) \)

iv) total volume molecules is negligible fraction  \( (\sim 10^{-3}) \)

v) no interaction except during collision  \( (\text{average } U_{\text{interaction}} < 10^{-4} \text{ K.E.}) \)

vi) collisions elastic, negligible duration.  \( (\sim 10^{-3} \text{ of time}) \)

parallel plates, area A. Volume V = AL.

N molecules (mass m) of an ideal gas.

Each collision →

\[ \Delta \text{momentum} = 2mv_x \]

\[ \text{time between collisions} \quad t = 2L/v_x. \]

\[ \vec{F} = \frac{\Delta \text{momentum}}{\Delta \text{time}} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L} \]

\[ \therefore \quad F \text{ on all } N \text{ molecules is} \]

\[ F_{\text{all molecules}} = \frac{Nmv_x^2}{L} = PA \]

\[ v^2 = v_x^2 + v_y^2 + v_z^2; \quad \text{random motion} \implies \bar{v_x^2} = \bar{v_y^2} = \bar{v_z^2} \implies \bar{v_x^2} = \frac{1}{3} \bar{v^2}, \text{ so:} \]

\[ P \cdot A \cdot L = N \cdot \bar{v_x^2} = \frac{N}{3} \cdot m \cdot \bar{v^2} \]

\[ \rho = \frac{N \cdot m}{3\cdot V} \cdot \bar{v^2} = \frac{1}{3} \rho \cdot \bar{v^2} \]

Molecular speeds:

\[ v_{\text{rms}} \quad \text{root mean square velocity} \]

\[ v_{\text{r.m.s.}} \equiv \sqrt{\bar{v^2}} \]

c) What is \( v_{\text{rms}} \) in atmosphere? (approximate it as an ideal gas at \( P_A \), with \( \rho_A = 1.3 \text{ kg.m}^{-3} \))

\[ \rightarrow v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 10^5}{1.3}} = 480 \text{ ms}^{-1} \]

Meaning of temperature:

We had \[ PV = \frac{N}{3} \cdot m \cdot \bar{v^2} \]

\[ \frac{1}{2} m \cdot \bar{v^2} = \bar{\varepsilon} \equiv \text{average K.E. per molecule} \]

But T defined by (1 and 5): \[ PV = NkT \]

\[ \therefore \quad \bar{\varepsilon} = \frac{1}{2} m \cdot \bar{v^2} = \frac{3}{2} \frac{PV}{N} = \frac{3}{2} kT \quad (7) \]

For ideal gas all energy \( E \) is kinetic so:

\[ E = N \bar{\varepsilon} = \frac{3}{2} NkT \quad (8) \]

\( T \sim \text{average K.E. of molecules in an ideal gas.} \)

3 degrees of motional freedom (x, y, z)
i.e. \( \frac{1}{2} kT \) per degree of freedom \hspace{1em} (At ordinary temperatures, \( kT \approx 4 \times 10^{-21} \text{ J} \))

molecular speeds again:

\[
\frac{1}{2} m v^2 = \frac{3}{2} kT
\]

\[v_{\text{r.m.s.}} = \sqrt{\frac{v^2}{m}} = \sqrt{\frac{3kT}{m}}\]

**Example:** What is \( v_{\text{rms}} \) of O\(_2\), N\(_2\), and H\(_2\) at \( T = 293 \text{ K} \)?

\[
(7) \quad v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{N_A \text{ mol wt}}}
\]

for O\(_2\):

\[
= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 293 \times 6.02 \times 10^{23}}{0.032}}
\]

\[= 478 \text{ ms}^{-1}\]

for N\(_2\) \( \rightarrow 511 \text{ ms}^{-1} \) \hspace{1em} for H\(_2\) \( 1.91 \text{ kms}^{-1} \) \hspace{1em} c.f. \( v_{\text{escape}} = 11 \text{ kms}^{-1} \)

Note that for air \( v_{\text{rms}} > v_{\text{sound}} \)

But recall from waves:

\[v_s = \sqrt{\frac{K_{\text{ad}}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma kT}{m}}\]

so \( \frac{v_{\text{rms}}}{v_s} = \sqrt{\frac{\gamma}{3}} \approx 1.5 \) \hspace{1em} \( \gamma \) discussed later.

**Example** What is the \( v_{\text{rms}} \) due to thermal motion (Brownian motion) of: pollen grain (m \( \sim 10^{-15} \text{ kg} \)) and apple (m \( \sim 0.2 \text{ kg} \))

\[v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \text{ pollen} \Rightarrow 2 \text{ mm s}^{-1} \quad (\text{Brownian motion 1st analysed by Einstein, 1904})\]

**Example.** Spherical balloon. Skin (total) has mass \( \sigma = 10 \text{ g.m}^{-2} \). How big does it need to be to lift 200 kg load if (i) it contains hot air at 100 C? (ii) Helium at STP?

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\[r \approx \frac{3m}{4\pi \rho_{\text{air}}(1 - \rho_{\text{gas}}/\rho_{\text{air}})}\]

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\[\therefore \quad \frac{\rho_{\text{hot}}}{\rho_{\text{cold}}} = \frac{T_{\text{cold}}}{T_{\text{hot}}} = \frac{273 \text{ K}}{373 \text{ K}} \quad \Rightarrow r \approx 5.3 \text{ m} \]

*check approx
Special Cases

1. **Isobaric Process** - $p$ constant
   
   $W = \int P \, dV = P(V_f - V_i)$
   
   $\therefore \Delta U = \Delta Q - P\Delta V$

   example $\Delta Q = M L$ (mass, latent heat)
   
   $\Delta U = ML - P\Delta V$

   Also, things done at atmospheric pressure

2. **Isochoric** – volume constant
   
   $\therefore W = 0$, $\therefore \Delta Q = \Delta U$

3. **Adiabatic Process:** no heat flow, $\Delta Q = 0$
   
   either fast or insulated
   
   e.g. compression stroke in engine
   
   sound wave compression
   
   most muscle contractions

   **Free expansion:** open tap

   No work done, Experimentally, find $\Delta Q = 0$

   $\therefore \Delta U = 0$

   $\therefore U$ of ideal gas doesn't depend on $\rho$, i.e. $U = U(T)$

   **Special cases of the First Law**

   $\Delta U = Q - W$

   **Process**

   Adiabatic $Q = 0$ $\therefore \Delta U = -W$

   Constant volume $W = \int PdV = 0$ $\therefore \Delta U = Q$

   Closed cycle $\Delta U = 0$ $\therefore Q = W$

   Free expansion $Q = W = 0$ $\therefore \Delta U = 0$

For a gas, specific heat at constant pressure ($c_p$) is greater than specific heat at constant volume ($c_v$).

Why?

The ratio $c_p/c_v$ is called $\gamma$.

$PV^\gamma = \text{constant for adiabatic process}$

*Note: pressure varies more strongly with change in $V$ for adiabatic than for isothermal, where $PV^I = \text{constant.}$*
Example
Compressed gas \((P_1, T_1)\) escapes from a cylinder into the atmosphere \((P_A, T)\).
Assume rapid, adiabatic expansion, no initial mixing. How cold does it get?

Adiabatic: \(PV^{\gamma} = \text{constant}\)

\[ \therefore \quad \frac{V_2}{V_1} = \left( \frac{P_1}{P_2} \right)^{1/\gamma} \]

\[ \therefore \quad \frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} = \left( \frac{P_2}{P_1} \right)^{1/\gamma} = \left( \frac{P_2}{P_1} \right)^{1 - 1/\gamma} \]

Example. Same problem but for car tyre:
"p" (gauge pressure) = 180 kPa. Take \(\gamma = 1.40\), \(T = 300\) K. Release some air into atmosphere \((P_2 = P_A = 101\) kPa). What is temperature of emerging air?

\(P_1 = "p" + P_A = 281\) kPa

\(\therefore \quad T_f = \ldots = 224\) K = -49°C

Example (a peculiar heat cycle)

\(a\) Isothermal expansion at \(T_H\)

\(b\) cooling at const. \(V\)

\(c\) Isothermal compression

\[ \Delta U \quad \Delta W \quad \Delta Q \]

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<thead>
<tr>
<th></th>
<th>(\Delta U)</th>
<th>(\Delta W)</th>
<th>(\Delta Q)</th>
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<tbody>
<tr>
<td>a</td>
<td>0</td>
<td>(-nC_v(T_H - T_C))</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>(-nC_v\ln(V_2/V_1))</td>
<td>(-nC_v(T_H - T_C))</td>
<td>0</td>
</tr>
<tr>
<td>c</td>
<td>0</td>
<td>(-nRT_C\ln(V_1/V_2))</td>
<td>(-nRT_C\ln(V_2/V_1))</td>
</tr>
<tr>
<td>d</td>
<td>(nC_v(T_H - T_C))</td>
<td>0</td>
<td>(nC_v(T_H - T_C))</td>
</tr>
</tbody>
</table>

\[ \Sigma \quad 0 \quad nR(T_H - T_C)\ln(V_2/V_1) \quad nR(T_H - T_C)\ln(V_2/V_1) \]

Efficiency = \(\frac{\text{work out}}{\text{heat in}} = \left( \frac{R(T_H - T_C)\ln(V_2/V_1)}{RT_H\ln(V_2/V_1) + C_v(T_H - T_C)} \right)\)

all terms > 0

\(T_H - T_C \leq T_H\) \(\therefore \text{efficiency < 100\%}\)
P-V diagrams and thermal cycles

Simple cycles typical of those found in tut and exam questions

Idealised cycles sometimes approximated

Heat conduction

Reservoirs at $T_H$ and at $T_C$. $H$ is rate of heat transfer through a material in steady state.

$$H \equiv kA \frac{T_H - T_C}{l}$$

defines the thermal conductivity $k$

$H$ in W, so $k$ in W.m$^{-1}$.K$^{-1}$.

Copper 401 W.m$^{-1}$.K$^{-1}$
Stainless Steel 14
Glass 1
Water 0.5
Pine (wood) 0.11
Dry air 0.026

Thermal resistance or R-value sometimes used for building materials

$$R \equiv \frac{l}{k}$$

so  $$H = A\frac{\Delta T}{R}$$

(High conductivity, low R value and vice versa.)
Example. What is the $R$ value of 1 cm pine?

When his Austudy is cut off, a student lives in a pine packing crate, area 8 m$^2$, thickness 1.0 cm. If the shivering student produces 300 W, which is lost by conduction through the crate, how much warmer is it inside the crate?

\[
R \equiv \frac{l}{k} = 0.09 \text{ K.m}^2\text{W}^{-1}
\]

\[
H \equiv kA \frac{T_H - T_C}{l} \quad \text{or} \quad A \frac{dT}{R}
\]

\[
T_H - T_C = \frac{lH}{kA}
\]

\[
= \frac{0.010 \text{ m} \times 300 \text{ W}}{0.11 \text{ W.m}^{-1}.\text{K}^{-1} \times 8 \text{ m}^2} = 3 \text{ K. Other benefits: wind, rain, radiation}
\]

Example. To reduce thermal noise, a low temperature circuit is immersed in liquid nitrogen (77 K, $L = 199 \text{ kJ.kg}^{-1}$). It is connected to the outside circuitry by 3 well-insulated copper wires, length $l = 100 \text{ mm}$, diameter 0.3 mm. What is the rate of $\text{N}_2$ evaporation due to the heat conducted down the wires?

\[
\text{Power to evaporate } \text{N}_2 = \text{ heat transfer}
\]

\[
L \frac{dm}{dt} = H \equiv kA \frac{T_H - T_C}{l}
\]

\[
\frac{dm}{dt} = kA \frac{T_H - T_C}{L} = \ldots = 90 \mu\text{g.s}^{-1} = 340 \mu\text{g.hr}^{-1}
\]

Example. A flask of coffee, initially at 90 °C, cools to 81 °C in one hour in 20 °C atmosphere. How long will it take to cool to 60 °C?

\[
H = kA \frac{T - T_C}{L}
\]

\[
H = - \frac{dQ}{dt} = - \frac{mc dT}{dt}
\]

\[
\frac{dT}{dt} = - \frac{kA}{mc} \frac{T - T_C}{L}
\]

Characteristic time: \[
\tau = \frac{mcL}{kA} \quad \text{Solve DE} \rightarrow \]

\[
T = T_C + (T_0 - T_C) e^{-t/\tau}
\]

81 °C = 20 °C + (70 °C)e^{-(1 h)/\tau}

\[
\therefore \quad \tau = 7.3 \text{ h} \quad \ldots \rightarrow 4.1 \text{ h to cool to 60 °C}
\]
Heat radiation

Rate of transfer by radiation

\[ H = e\sigma AT^4 \]

Stefan-Boltzmann constant

\[ \sigma = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4} \]
\[ e = \text{emissivity} \quad 0 < e < 1 \]

\( e \to 0 \) for shiny surfaces, \( e \to 1 \) for black surfaces

Note that a good emitter is also a good absorber.

Black body radiation

Black body has \( e \equiv 1 \).

'Black body' is in equilibrium with its radiation

In equilibrium with black body radiation, object comes to same temperature as the black body radiation.

Equilibrium: \( \text{radiation out} = \text{radiation in} \)

\[ \text{radiation out} = e\sigma AT_{\text{ob}}^4 = e\sigma AT_{\text{rad}}^4 \]
\[ \therefore \text{radiation in} = e\sigma AT_{\text{rad}}^4 \]

In black body radiation at temp \( T_{\text{rad}} \)

\[ H_{\text{nett}} = e\sigma A(T^4 - T_{\text{rad}}^4) \]

Example.

- A heat sink
- \( T = 50 \degree C \)
- Circuit

Dark-coloured heat sink (\( e \sim 1 \)) mounted on layer of (electrical) insulator with \( k = 1.2 \text{ Wm}^{-1}\text{K}^{-1} \). Area is 200 mm\(^2\), \( d = 0.1 \text{ mm} \).

Circuit must be kept at \( T \leq 50 \degree C \) in an environment at \( T = 30 \degree C \). What is max \( H \) produced by circuit?

Solve: find \( T \) of sink in steady state

Substitute to get \( H_{\text{max}} \)

In steady state, heat sink has

Conduction in = nett radiation out

\[ kA \frac{T_{\text{circ}} - T_{\text{sink}}}{d} = e\sigma A(T_{\text{sink}}^4 - T_{\text{env}}^4) \]
\[ \frac{k}{\sigma d} (T_{\text{circ}} - T_{\text{sink}}) = (T_{\text{sink}}^4 - T_{\text{env}}^4) \]

Hmm, which restricts heat more, radiation or conduction?
Linearise the messy term

\[ T_s^4 - T_e^4 = (T_s - T_e)(T_s^3 + T_s^2 T_e + T_s T_e^2 + T_e^3) \]
\[ \equiv (T_s - T_e)4T_e^3 \quad \text{if} \quad T_s - T_e \ll T_e. \]
\[ \frac{k}{4T_e^3 \sigma d} (T_c - T_s) \equiv T_s - T_e \]

numbers: \[ \frac{k}{4T_e^3 \sigma d} \equiv 2000, \]
\[ \therefore T_c \equiv T_s \]
\[ \therefore H_{\text{max}} \equiv \sigma A(T_s^4 - T_e^4) = 28 \text{ mW} \]

**Example.** Person gets out of hot shower with 1.5 m\(^2\) of skin (\(e = 0.7\)) at 35 °C. The walls, floor and ceiling are at 12 °C (treat them as black body). (bath mat is good insulator)
How fast do you lose energy?
\[ H = H_{\text{lost}} - H_{\text{gained}} = e \sigma A(T_{\text{skin}}^4 - T_{\text{rad}}^4) = .... = 143 \text{ W.} \]

**Example**

\[ \text{Two volumes of same gas. Open tap. What pressure?} \]
Gas redistributes until P equal.
5 unknowns: \(N_1, N_2, N_1', N_2', P\)

Before \[ N_1 = \frac{P_1 V_1}{RT_1} \quad N_2 = \frac{P_2 V_2}{RT_2} \]
After \[ N_1' = \frac{P V_1}{RT_1} \quad N_2' = \frac{P V_2}{RT_2} \]
Conservation: \[ N_1 + N_2 = N_1' + N_2' \quad \text{Solve......} \]

**Example**

\[ T_s \]

\[ \text{Circuit inside black case, } d = 2.5 \text{ mm, area } A = 0.02 \text{ m}^2, \text{ thermal conductivity } k = 0.02 \text{ W.K}^{-1}.\text{m}^{-1}. \text{ Assume internal temperature uniform. Surroundings and the case have emissivity of 1.0. In steady state:} \]
Outer surface of the case is at 35 °C, surroundings at 20°C. What is \(T_1\)?

Steady state: heat radiation loss = heat conduction
\[ A \sigma (e_o T_o^4 - e_s T_s^4) = \frac{kA}{d} (T_1 - T_o) \]
\[ T_i = \frac{d \sigma}{k} (T_o^4 - T_s^4) + T_o \]
\[ = \frac{(2.5 \cdot 10^{-3} \text{ m}) \times (5.67 \cdot 10^{-8} \text{ Wm}^{-2} \text{K}^{-4})}{0.02 \text{ W.K}^{-1}.\text{m}^{-1}} \left( (308 \text{ K})^4 - (293 \text{ K})^4 \right) + 35 \text{ °C} \]
\[ = 47 \text{ °C} \]
A note about steady state (T constant) in one dimension, \( H = kA \frac{TH - TC}{l} \) becomes heat flux \( \frac{H}{A} = k \frac{dT}{dx} \) In steady state, rate of heat arriving = rate of heat departing In steady state, heat flux is uniform. So \( k_1 \left( \frac{dT}{dx} \right)_1 = k_2 \left( \frac{dT}{dx} \right)_2 \)

Getting to steady state: (details usually complicated)

In this situation

\( k_1 \left( \frac{dT}{dx} \right)_{\text{left}} > k_1 \left( \frac{dT}{dx} \right)_{\text{right}} \)
so the middle region receives more heat than it loses – so its temperature rises.

At boundary of vacuum:

Heat conduction to the surface = heat radiation from the surface