PHYS1169 Light notes. Joe Wolfe, UNSW

Physical optics \(\rightarrow\) interference and diffraction

Electromagnetic radiation

**Speed** \( c = 3.00 \times 10^8 \text{ ms}^{-1} \)

\[2.997923458 \times 10^{-8} \text{ ms}^{-1}\] (This is definition of metre)

\(\lambda:\) for visible light, \(400 \text{ nm} < \lambda < 800 \text{ nm}\)

Typical 
\[
\frac{c}{\lambda} \sim \frac{3 \times 10^8 \text{ ms}^{-1}}{5 \times 10^{-7} \text{ m}} = 6 \times 10^{14} \text{ Hz} = 600 \text{ THz}
\]


Geometrical optics only works if size \(>>\lambda\). Why?

**Huygen's principle:** each point on a wave front \(~\) a new source

Is light

a) a particle (Newton) \(\rightarrow\) geometric optics

b) a wave (Young) \(\rightarrow\) physical optics

c) both

d) none of the above \(\rightarrow\) quantum physics

Hence: **Young's experiment**

Light through one slit (gives *coherent source*) then *two slits* gives interference pattern on screen.

**Constructive interference** if

\(d \sin \theta = m \lambda\)

**Destructive interference** if

\(d \sin \theta = \left( m + \frac{1}{2} \right) \lambda\)

At an angle \(\theta\), the phase difference \(\phi\) is

\[
\frac{\phi}{2\pi} = \frac{\Delta \text{ pathlength}}{\lambda} = \frac{d \sin \theta}{\lambda}
\]

\[
\therefore \quad \phi = \frac{2\pi}{\lambda} d \sin \theta \quad \text{how to add two sin waves? phasor diagrams}\]
\[ a_{\text{tot}} = 2a \cos \beta \]
\[ \beta = \phi/2 = \frac{\pi}{\lambda} \cdot d \sin \theta \]

Intensity \( \propto \) amplitude^2 \quad \therefore \quad I \propto 4a^2 \cos^2 \beta 

\[ I_{\text{max}} \text{ if } \beta = \phi/2 = 0 \]
\[ I = I_{\text{max}} \cos^2 \beta \quad \text{where } \beta = \frac{\pi}{\lambda} \cdot d \sin \theta \]

Numerical example:

**Example** 3 narrow slits radiate uniformly and in phase. Interference pattern on screen. (a) Show phase diagrams for (i) central maximum (ii) 1st order maximum (iii) minima between i and ii.

(b) Sketch \( I(\phi) \)

i) \[ \phi = 0, \text{ central max} \]

ii) \[ \phi = 2\pi, \text{ 1st order max} \]

iii) \[ \phi = 2\pi/3 \quad \phi = 4\pi/3 \quad \text{both} \to 0 \text{ resultant} \]

b) if \( \phi = \pi, \) amplitude = 1/3 central max 
\[ \therefore \text{ intensity } = I_{\text{max}}/9 \]
**Diffraction grating** has very many slits. Used
to measure $\lambda$ very accurately.
If there are $N$ slits per unit length, $d = 1/N$.
The first minimum is *very* close (small $\phi$ to close
polygon), ie very narrow maxima

For constructive interference
\[ d \sin \theta = m\lambda \]

$m = 1 \rightarrow$ 1st order spectrum
$m = 2 \rightarrow$ 2nd order spectrum
\[ \theta_{\text{red}} = \sin^{-1} \frac{m\lambda_{\text{red}}}{d} \]
\[ \theta_{\text{blue}} = \sin^{-1} \frac{m\lambda_{\text{blue}}}{d} \]

**Diffraction from a slit** (Later from a circle)

Use Huygen's construction: beam of finite width, interference if $d \sim \lambda$

**Circular aperture:**

First minimum at
\[ \sin \theta = 1.22 \frac{\lambda}{d} \]
d is diameter

can resolve $\sim \theta$ with lens diam $d$. Rayleigh's criterion
Coherence length
Wave trains have finite length - coherence length

Only interfere if $\Delta$ pathlength $< l$

Examples
Radio transmitter $E = E_m \sin (kx - \omega t)$ $E_m$, $\omega$, $k$ vary slowly
Hot body (e.g. lamp) $l \sim 1$ m, but different regions have different, random phase. To get interference, use a pin hole and keep $\Delta$ path $<< 1$ m
LASER (Light Amplification by Stimulated Emission of Radiation) $l >>$ km

Interference in thin films
Light in a medium travels at $c/n$.
$$\lambda_{\text{medium}} = \lambda/n$$
$$\Delta \phi = 2\pi \frac{\Delta \text{ path}}{\lambda_{\text{med}}} = 2\pi n \frac{\Delta \text{ path}}{\lambda}$$

Define Optical path length $\equiv n \cdot \text{pathlength}$
$$\Delta \phi = 2\pi \frac{\Delta \text{ optical pathlength}}{\lambda}$$

Reflections remember reflections in strings
From less dense to more dense
$$\Delta \phi = \pi$$
From more dense to less dense
$$\Delta \phi = 0$$

transmitted wave has no phase change

Example: Non reflective coating useful in camera lens etc

Coating has $1 < n < n_{\text{glass}}$

How thick should it be to give minimum reflection?
Both reflections have $\pi$ phase change. As $d \to 0$, $\to$ constructive interference.
For destructive, we want $\Delta \text{ optical pathlength} = \lambda/2$
$$2nd = \lambda/2$$
$$d \sim \frac{\lambda}{4n} = \frac{550 \text{ nm}}{4 \times 1.2} \sim 100 \text{ nm}$$
Air wedge

\[ d \]

\[ \pi \]

**Destructive interference if**

\[ 2d = m \lambda \]

**Constructive interference if**

\[ 2d = \left( m + \frac{1}{2} \right) \lambda \]

At L.H. end, destructive (dark), then count fringes to get thickness.  
(see corridor display)

Also: Newton's rings, oil slicks

**Polarisation.**

EM waves are **transverse** waves, \( \therefore \) can be polarised. Usually light has waves with \( E \) in all directions

Unpolarised  Plane polarised

**Polaroid materials** allow \( E \) in only one dirn

\[ E_x \]

\[ \theta \]

\[ \text{polarisation direction} \]

\[ E_{\text{transmitted}} = E \cos \theta \]

**Malus' Law:**

\[ I_{\text{trans}} = I_{\text{in}} \cos^2 \theta \]

Average of \( \cos^2 \theta \) over all angles is \( 1/2 \)  \( \therefore \)

Unpolarised  plane polarised  plane polarised
Example. Two crossed polarisers (polar\(\alpha\) dir\(\alpha\) are at right angles) have a third polariser between them. What is the transmitted intensity? What angle for the middle polariser gives maximum transmission?

\[
I = \frac{I_0}{2} \cos^2 \theta \sin^2 \theta = \frac{I_0}{8} \sin^2 2\theta
\]
**Polarisation by reflection**

For wave in medium, $\mathbf{E}$ of light causes oscillation $\parallel \mathbf{E}$. This oscillation can produce (only) transverse waves, hence polarisation of reflected wave.

When refracted ray $\perp$ reflected $\rightarrow$ plane polarised reflected wave (Brewster's angle $\theta_B$).

$\rightarrow$ polaroid sunglasses (*see your optics kit*)

**Scattering** of light also polarises

More effective for short $\lambda$, $\rightarrow$ blue sky

**Example.** What is Brewster's angle for a medium with $n = 1.40$?

Refraction: $n = \frac{\sin \theta_B}{\sin \theta_r}$

If refracted and reflected are at $90^\circ$,

$\theta_B + \theta_r = 90^\circ$

so $\quad \sin \theta_r = \cos \theta_B$

$\Rightarrow \quad n = \frac{\sin \theta_B}{\cos \theta_B} = \tan \theta_B$

$\theta_B = \tan^{-1} 1.4 = 54^\circ$
**Example.** An oil slick \((n = 1.20)\) floats on water. What are the thicknesses for which red light \((\lambda \approx 700 \text{ nm})\) is reflected weakly? What does the slick look like at its thinnest point?

![Diagram of oil slick on water]

\(n_{\text{water}} > n_{\text{oil}}\)

**Constructive interference** if

\[\Delta \text{OPL} = m \lambda\]

**Destructive interference** if

\[\Delta \text{OPL} = \left( m + \frac{1}{2} \right)\lambda\]

i) If red has destructive interference,

\[\Delta \text{OPL} = 2\text{nd} = \left( m + \frac{1}{2} \right)\lambda_{\text{red}}\]

\[d = \frac{\lambda_{\text{red}}}{2n} \left( m + \frac{1}{2} \right)\]

\[m = 0, \quad m = 1, \quad m = 2 \quad \ldots\]

\[= 150 \text{ nm}, 440 \text{ nm}, 730 \text{ nm} \quad \text{etc}\]

ii) If \(d << \lambda\), \(\pi\) phase difference on both paths so constructive interference for all \(\lambda\), so it looks bright and 'white'.

**Example.** Same problem, but for scuba diver!

![Diagram of oil slick on water]

**Destructive interference** if \(\Delta \text{OPL} = m \lambda\)

**Constructive interference** if \(\Delta \text{OPL} = \left( m + \frac{1}{2} \right)\lambda\)

**Example.** A binary star has an orbital radius of 100 light seconds, and is 10 light years from us. How big must an optical telescope be to resolve them? (take \(\lambda = 550 \text{ nm}\)). A radio telescope (take \(\lambda = 21 \text{ cm}\)).

\[100 \text{ s} < 10 \text{ yr} \quad \rightarrow \quad \theta \text{ is small} \]

Rayleigh criterion: \(\sin \theta_R = 1.22 \frac{\lambda}{a}\)

\[\sin \theta_R \equiv \theta_R \equiv \frac{100 \text{ light seconds}}{10 \text{ light years}}\]

\[1.22 \frac{\lambda}{a} = \frac{c \times 100 \text{ seconds}}{c \times 10 \text{ years}}\]

\[\lambda = 550 \text{ nm} \rightarrow \quad a = 2 \text{ m} \quad \lambda = 21 \text{ cm} \rightarrow \quad a = 800 \text{ km}\]