Example. At atmospheric pressure, water boils at 100 °C or 212 °F, and freezes at 0 °C or 32 °F. At what temperature do the two scales have the same value? When is the Farenheit temperature twice the Centigrade temperature?

Centigrade scale (symbol θ) and the Farenheit scale (symbol ϕ) are linearly related:

\[ ϕ = aθ + b \]

How do you know this?

Write down givens using this:

\[ 212 \ °F = a \ 100 \ °C + b \]

\[ 32 \ °F = 0 + b \]

Solve:

\[ b = 32 \ °F, \ a = 1.8 \ \frac{°F}{°C} \]

If \( ϕ = θ \):

\[ θ = aθ + b, \]

so \( ϕ = θ = -\frac{b}{a} = -40 °C = -40 °F \)

If \( ϕ = 2θ \):

\[ 2θ = aθ + b \]

so \( θ = \frac{b}{2 - a} = 160 °C = 320 °F. \)

Thermal Physics

**Thermodynamics:** laws relating macroscopic variables (P, V, T etc.).

**Statistical Mechanics:** molecular explanation.

**Difference between heat and temperature**

*Intensive or extensive properties?*

*Which relates to sense of hotness?*

**Define temperature:**

**Thermal equilibrium:**

Thermal properties do not change with time.

**Definition of Temperature (T):**

T is equal in any 2 bodies at thermal equilibrium.

**Zeroth Law of Thermodynamics:**

if \( T_A = T_B \) and \( T_B = T_C \), then \( T_A = T_C \).

What is temperature? How to measure it?

**Thermometers:** Hg in glass, thermocouple, thermistor, liquid crystal layer, constant volume gas thermometer.

**Scales.** Obvious definition of temperature θ: choose a property X and make X proportional to or linear with θ. This can only be done once for any temp scale θ.

**Reference temperature**

Melting or freezing? Depends on the pressure.
Thermal Expansion

\[ T + \Delta T \]

\[ L \]

Usually, \( \frac{\Delta L}{L} \approx \Delta T \) for small \( \Delta T \)

\[ \therefore \text{ Define } \frac{\Delta L}{L} = \alpha \Delta T \]

\( \alpha \) is coefficient of linear expansion

e.g. steel \( \alpha_{\text{st}} = 1.1 \times 10^{-5} \text{ K}^{-1} \)

Al \( \alpha_{\text{Al}} = 2.3 \times 10^{-5} \text{ K}^{-1} \)

Example Bridge span is 1 km long.

Mid-winter, \( T = -5^\circ \text{C} \) summer, \( T = 45^\circ \text{C} \)

What is \( \Delta L \)?

\[ \Delta L = \alpha \Delta T L = ... = 55 \text{ cm} \]

Volume Increase

define \( \frac{\Delta V}{V} = \beta \Delta T \)

\( \beta \) = coefficient of volume expansion

\[ \Delta V = (L + \Delta L)^3 - L^3 \]

\[ = L^3 \left( 1 + \frac{\Delta L}{L} \right)^3 - L^3 \]

\[ = L^3 \left( 1 + 3 \frac{\Delta L}{L} + ... \right) - L^3 \]

\[ = \Delta V \left( 3 \alpha \Delta T + ... \right) \]

\[ \therefore \beta = 3 \alpha \]

Note: Water is unusual: \( 0^\circ - 4^\circ \text{C}, \beta < 0 \)

Example What is change in \( \rho \) for steel between \( 0^\circ \text{C} \) and \( 100^\circ \text{C} \)?

\[ \rho = \frac{M}{V} \therefore d\rho = -\frac{M}{V^2} dV \]

\[ \therefore \Delta \rho = -\frac{M}{V^2} \Delta V = -\rho \frac{\Delta V}{V} \]

\[ \therefore \frac{\Delta \rho}{\rho} = -\frac{\Delta V}{V} = -\beta \Delta T = ... = -0.33\% \]
**Example:** Bimetallic Strip, 10 cm long, made of 1 mm Al and 1 mm steel. Straight at 0 °C, what angle at 50 °C?

\[
L + \Delta L_{st} = 2\pi R \cdot \frac{\theta}{360°}
\]

\[
L + \Delta L_{A1} = 2\pi (R + t) \cdot \frac{\theta}{360°}
\]

subtract →

\[
2\pi t \cdot \frac{\theta}{360°} = \Delta L_{A1} - \Delta L_{st} = L \Delta T (\alpha_{A1} - \alpha_{st})
\]

\[
\theta = \frac{\Delta L_{A1} - \Delta L_{st}}{L \Delta T (\alpha_{A1} - \alpha_{st})} = 3.4 °C
\]

e.g. oven switch:

**Example:** thermometer, \(V_{ol_{cylinder}} << V_{ol_{sphere}}\)

What is its calibration slope \(\frac{\partial x}{\partial T}\) ?

\[
\frac{\Delta V}{V} = \beta \Delta T
\]

\[
a \delta x = \Delta V_{fl} - \Delta V_{sphere}
\]

\[
= \beta_{fl} V \Delta T - \beta_{g1} V \Delta T
\]

\[
\frac{\partial x}{\partial T} = \frac{V}{a} (\beta_{fl} - \beta_{g1})
\]
Ideal gas temperature scale

Uses reference temp:

Triple point - co-existence of ice, water, steam

\[ \text{call it } \theta_{\text{tr}}. \]

**defines** \( P \propto \theta \) for constant volume of gas

But gases are not (quite) ideal

e.g. consider boiling temp \( \theta_s \) at some \( P \):

\[ \frac{\theta_s}{\theta_{\text{tr}}} \]

is different for different gases and at different densities.

At very low density or pressure,

all gases \( \rightarrow \) ideal, \( \therefore \)

\[ \frac{\theta_s}{\theta_{\text{tr}}} \rightarrow \text{same limit} \]

![Diagram of fixed volume of gas with P and T scales]

\[ T = T_{\text{tr}} \lim_{\rho_{\text{tr}} \to 0} \left( \frac{P}{P_{\text{tr}}} \right)_V \]

where \( T_{\text{tr}} = 273.16 \text{ K} \)

Why 273.16? This defines the Kelvin so that

\[ \Delta T = 1 \text{ K} \quad \iff \quad \Delta T = 1 \text{ °C} \]

(Working defn. is more complicated)

**Celsius Scale:** \( T_C = T - 273.15^\circ \)

\[ T_C = 0 \text{ °C water freezes} \]
\[ T_C = 100 \text{ °C water boils} \quad \{ \text{at } P_A \} \]

**Fahrenheit Scale**

**Heat**

**Definition:** that which is transferred between a system and its surroundings as result of \( \Delta T \) only.

**Joule** showed:

mechanical energy \( \rightarrow \) heat \( \quad (\text{by friction etc.}) \)

\( \left( \text{Carnot showed} \right) \)

heat at high \( T \) \( \rightarrow \) heat at low \( T \) + work

\( \therefore \) measure heat as energy; i.e. S.I.unit. Joule (J)
Heat Capacity: (for a body)  \( C = \frac{\Delta Q}{\Delta T} \) extensive quantity

Specific Heat: (of a substance)  \( c = \frac{\Delta Q}{M\Delta T} \) intensive quantity

\( c_{H_2O} = 4.2 \text{ kJ.kg}^{-1}\text{K}^{-1} \),  \( c_{\text{Al}} = 900 \text{ J.kg}^{-1}\text{K}^{-1} \)

Latent Heat: heat required for change of phase (at constant \( T \)).

Example. A 240 V kettle has a working resistance of 50 \( \Omega \). Put in 500 ml of water at 20 °C and turn on. How long before it boils dry? (Specific heat of \( c_w = \text{water} = 4.2 \text{ J.kg.K}^{-1} \), Latent heat of vaporisation \( L_{\text{vap}} = 2.3 \text{ MJ.kg}^{-1} \).)

Energy in = power.t = \( \frac{V^2}{R} \) t = (1.15 kW).t
= Q to raise T of water + Q to evaporate water
= \( m_w c_w (T_f - T_i) \) + \( m_w L_{\text{vap}} \)
= (0.5 kg) (4.2 \( \times 10^3 \) J.kg\(^{-1}\)K\(^{-1}\)) (100 - 20)°C
+ (0.5 kg) (2.3 \( \times 10^6 \) J.kg\(^{-1}\))
= 168 kJ + 1.15 MJ = 1.32 MJ
\( \therefore \) t = \( \frac{1.32 \text{ MJ}}{1.15 \text{ kW}} \) = ... = 1150 s = 19 minutes
(it boils after \( \frac{168 \text{ kJ}}{1.15 \text{ kW}} = 2.5 \text{ mins} \))

Work: energy transmitted from one system to another without \( \Delta T \) or transfer of Q.

\( \text{e.g. work done by force } F \)
\( dW = F ds \)
\( \text{e.g. work done against pressure } P \)

\[ dW = F ds = PA ds = PdV \]

Internal Energy
Heat dQ added to a system increases its internal energy U.
Work dW done by the system lowers its internal energy.

1st Law  \( dU = dQ - dW \) where U is a state function
Digression  Equation of state of condensed phase

\[ V = V_o \left( 1 - \frac{P}{B} + \beta (T - T_o) \right) \]

Volumetric coefficient of thermal expansion \( \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P \)

Bulk modulus \( \frac{1}{B} = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T \)

Tut prob 3:

\[ \text{strain} = \frac{\Delta L}{L_o} = \frac{\text{stress}}{\text{modulus}} = \frac{\text{force/area}}{Y} \]

Phase diagram in one dimension:

\[ L = L_o \left( 1 + \frac{\Delta L_{\text{therm}}}{L_o} + \frac{\Delta L_{\text{stress}}}{L_o} \right) \]

\[ = L_o \left( 1 + \alpha (T - T_o) + \frac{m g}{A Y} \right) \]

Heat conduction

Reservoirs at \( T_H \) and at \( T_C \). \( H \) is rate of heat transfer through a material in steady state.

\[ H = k A \frac{T_H - T_C}{l} \]

defines the thermal conductivity \( k \)

H in W, so \( k \) in W.m\(^{-1}\).K\(^{-1}\).

<table>
<thead>
<tr>
<th>Material</th>
<th>Value (W.m(^{-1}).K(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>401</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>14</td>
</tr>
<tr>
<td>Glass</td>
<td>1</td>
</tr>
<tr>
<td>Water</td>
<td>0.5</td>
</tr>
<tr>
<td>Pine (wood)</td>
<td>0.11</td>
</tr>
<tr>
<td>Dry air</td>
<td>0.026</td>
</tr>
</tbody>
</table>
Thermal resistance or R-value sometimes used for building materials

\[ R \equiv \frac{l}{k} \quad \text{so} \quad H = A \frac{\Delta T}{R} \]

(High conductivity, low R value and *vice versa*.

**Example.** What is the R value of 1 cm pine? When his AustStudy is cut off, a student lives in a pine packing crate, area 8 m², thickness 1.0 cm. If the shivering student produces 300 W, which is lost by conduction through the crate, how much warmer is it inside the crate?

\[ R \equiv \frac{l}{k} = 0.09 \text{ K.m}^2\text{W}^{-1} \]

\[ H \equiv kA \frac{T_H - T_C}{l} \quad \text{or} \quad A \frac{\Delta T}{R} \]

\[ T_H - T_C = \frac{lH}{kA} \]

\[ = \frac{0.010 \text{ m} \times 300 \text{ W}}{0.11 \text{ W.m}^{-1}.\text{K}^{-1} \times 8 \text{ m}^2} \]

\[ = 3 \text{ K.} \quad \text{Other benefits: wind, rain, radiation} \]

What if he has a friend? 2 students at 300 W → 600 W → 7 K.

**Example.** To reduce thermal noise, a low temperature circuit is immersed in liquid nitrogen (77 K, \( L = 199 \text{kJ.kg}^{-1} \)). It is connected to the outside circuitry by 3 well-insulated copper wires, length \( l = 100 \text{ mm} \), diameter 0.3 mm. What is the rate of N₂ evaporation due to the heat conducted down the wires?

\[ \text{Power to evaporate N}_2 = \text{heat transfer} \]

\[ L \frac{dm}{dt} = H \equiv kA \frac{T_H - T_C}{l} \]

\[ \frac{dm}{dt} = kA \frac{T_H - T_C}{L} \]

\[ = \ldots \]

\[ = 90 \mu\text{gs}^{-1} = 340 \text{ mg.hr}^{-1} \]
Example. A flask of coffee, initially at 90 °C, cools to 81 °C in one hour in 20 °C atmosphere. How long will it take to cool to 60 °C?

\[ H = kA \frac{T - T_C}{L} \]

\[ H = -\frac{dQ}{dt} = -\frac{mcdT}{dt} \]

\[ \frac{dT}{dt} = -kA \frac{T - T_C}{mcL} \]

Characteristic time: \( \tau = \frac{mcL}{kA} \) Solve DE →

\[ T = T_C + (T_0 - T_C) e^{-t/\tau} \]

81 °C = 20 °C + (70 °C)e\(^{-1 \text{ h}}/\tau\)

\( \therefore \tau = 7.3\ h \quad \rightarrow \ 4.1\ h \) to cool to 60 °C

**Kinetic Theory of Gases**

The Ideal Gas - postulates

- equation of state
- r.m.s. velocity
- Temperature
- Internal energy
- specific heats

**Ideal gas equation of state**

(\( \approx \) limit for all gases at low \( \rho \)):

\[ PV = nRT = NkT \]

- no. of moles
- no. of molecules
- gas constant
- Boltzmann's Constant

\[ R = 8.31\ JK^{-1}, \quad k = \frac{R}{N_A} = 1.38 \times 10^{-23}\ JK^{-1} \]

*NB not the same \( k \) and \( R \) as in heat conduction!*

**Kinetic theory: Ideal Gas Postulates**

i) gas made of (identical) molecules

ii) these obey Newton's laws, with random motion

iii) no. of molecules is large \( (\sim \text{Avagadro's number}) \)

iv) total volume molecules is negligible fraction \( (\sim 10^{-3}) \)

v) no interaction except during collision \( (\text{average } U_{\text{interaction}} < 10^{-4}\ K.E.) \)

vi) collisions elastic, negligible duration. \( (\sim 10^{-3} \text{ of time}) \)
parallel plates, area $A$. Volume $V = AL$.

N molecules (mass $m$) of an ideal gas.

Each collision $\rightarrow$

$\Delta$ momentum $= 2mv_x$

time between collisions $t = \frac{2L}{v_x}$.

$F = \frac{\Delta$ momentum}{\Delta$ time} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$

F on all N molecules is

$F_{\text{all molecules}} = \frac{Nm\bar{v}_x^2}{L} = PA$

$v^2 = v_x^2 + v_y^2 + v_z^2$;

random motion $\Rightarrow \bar{v}_x^2 = \bar{v}_y^2 = \bar{v}_z^2$ $\Rightarrow \bar{v}_x^2 = \frac{1}{3} \bar{v}^2$, so:

$P \bar{A}L = PV = \frac{Nm\bar{v}_x^2}{3} = \frac{N}{3} m \bar{v}^2$

$P = \frac{N m \bar{v}^2}{3V} = \frac{1}{3} \rho \bar{v}^2$

**Molecular speeds:**

$v_{\text{rms}}$ root mean square velocity

$v_{\text{r.m.s.}} = \sqrt{\bar{v}^2}$

(c) What is $v_{\text{rms}}$ in atmosphere? (approximate it as an ideal gas at $P_A$, with $\rho_A = 1.3 \text{ kg.m}^{-3}$)

$\Rightarrow v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 10^5}{1.3}} = 480 \text{ ms}^{-1}$

**Meaning of temperature:**

We had $PV = \frac{N}{3} m \bar{v}^2$

*both sides are familiar*

$\frac{1}{2} m \bar{v}^2 = \bar{v} \equiv \text{average K.E. per molecule}$

But $T$ defined by (1 and 5): $PV = NkT$

$\therefore \bar{v} = \frac{1}{2} m \bar{v}^2 = \frac{3 PV}{2N} = \frac{3}{2} kT$ (7)

For ideal gas all energy $E$ is kinetic so:

$E = N \bar{v} \bar{v} = \frac{3}{2} NkT$ (8)

$T \propto \text{average K.E. of molecules in an ideal gas.}$

3 degrees of motional freedom ($x, y, z$)

i.e. $\frac{1}{2} kT$ per degree of freedom

(At ordinary temperatures, $kT \equiv 4 \times 10^{-21} \text{ J}$)
molecular speeds again:

\[ \frac{1}{2} m v^2 = \frac{3}{2} kT \]

\[ v_{r.m.s.} \equiv \sqrt{\frac{v^2}{m}} = \sqrt{\frac{3kT}{m}} \]

**Example:** What is \( v_{rms} \) of O\(_2\), N\(_2\), and H\(_2\) at \( T = 293K \)?

(7) \[ v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{mol \text{ wt}}} \]

for O\(_2\) : \[ = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 293 \times 6.02 \times 10^{23}}{0.032}} \]

\[ = 478 \text{ ms}^{-1} \]

for N\(_2\) \( \rightarrow 511 \text{ ms}^{-1} \) for H\(_2\) 1.91 kms\(^{-1} \)

c.f. \( v_{escape} = 11 \text{ kms}^{-1} \)

So what?

note that for air \( v_{rms} > v_{sound} \)

but recall from waves:

\[ v_s = \sqrt{\frac{K_{ad}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma kT}{m}} \]

so \[ \frac{v_{rms}}{v_s} = \sqrt{\frac{3}{\gamma}} \sim 1.5 \]

**Example** What is the \( v_{rms} \) due to thermal motion (Brownian motion) of: pollen grain (m \( \sim 10^{-15} \) kg) and apple (m \( \sim 0.2 \) kg)

\[ v_{rms} = \sqrt{\frac{3kT}{m}} \]

pollen \( \Rightarrow 2 \text{ mm s}^{-1} \)

apple \( \Rightarrow 2.5 \times 10^{-10} \text{ ms}^{-1} \)

(Brownian motion 1st analysed by Einstein, 1904)

**Example:** What is the ratio of the speed of sound in He to that in air at the same temperature? How will this affect the pitch of a human voice when the lungs and vocal tract are (temporarily) filled with He?

\[ v_s = \sqrt{\frac{\gamma kT}{m}} \]

\[ \therefore \frac{v_{He}}{v_{air}} = \sqrt{\frac{\gamma_{He}m_{air}}{\gamma_{air}m_{He}}} \]

\[ \sqrt{\frac{\gamma_{He}}{\gamma_{air}}} = 1.1 \]

\[ \sqrt{\frac{m_{air}}{m_{He}}} = \sqrt{\frac{30}{4}} \approx 2.7 \]

Think carefully: does \( v_{sound} \) affect pitch?

\[ f_{air} = \frac{c}{\lambda} = \frac{c}{4L} \approx \frac{340 \text{ ms}^{-1}}{4 \times 0.17 \text{ m}} = 500 \text{ Hz} \]

\[ f_{He} \approx 1350 \text{ Hz} \]
Example. Spherical balloon. Skin (total) has mass $\sigma = 10 \text{g.m}^{-2}$.
How big does it need to be to lift 200 kg load if (i) it contains hot air at 100°C? (ii) Helium at STP?

Archimedes: $W_{\text{displaced}} = W_{\text{balloon}}$

$$\frac{4}{3} \pi r^3 \rho_{\text{air}} g = \frac{4}{3} \pi r^3 \rho_{\text{gas}} g + 4 \pi r^2 \sigma g + mg$$

$$r^3 (\rho_{\text{air}} - \rho_{\text{gas}}) - 3 \sigma r^2 = \frac{3}{4\pi} m \quad \text{(or solve cubic)}$$

$$r \equiv \frac{3m}{4\pi \rho_{\text{air}} (1 - \rho_{\text{gas}}/\rho_{\text{air}})}$$

He: $\rho_{\text{gas}}/\rho_{\text{air}} = 4/30 \quad \rightarrow \quad r \equiv 3.6 \text{ m}$

Hot air: $\rho = \frac{Nm}{V} = \frac{Pm}{kT}$

$$\therefore \quad \frac{\rho_{\text{hot}}}{\rho_{\text{cold}}} = \frac{T_{\text{cold}}}{T_{\text{hot}}} = \frac{273 \text{ K}}{373 \text{ K}} \quad \rightarrow \quad r \equiv 5.3 \text{ m}$$

(check approx.)