Waves are moving pattern of displacements. May transmit energy and signals.

Travelling waves, superposition and interference, velocity, reflection and transmission, harmonic waves, spherical and plane waves.

**Sound.**

Doppler effect, standing waves in strings and air columns, beats, decibel scale

**Light.**

- **Lab** Ray approximation & geometric optics:
- **Lab** Reflection and refraction, Huygen's principle, total internal reflection, mirrors, images, lenses, magnifier, compound microscope, telescope

**Interference and Diffraction**

- Conditions for interference, Young's experiment, and interference pattern, phasor addition, reflection, thin films, diffraction

**Mechanical waves**

<table>
<thead>
<tr>
<th>example</th>
<th>type</th>
<th>restoring force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave in string</td>
<td>transverse</td>
<td>tension in string</td>
</tr>
<tr>
<td>Water wave</td>
<td>transverse</td>
<td>gravity</td>
</tr>
<tr>
<td>Sound wave</td>
<td>longitudinal</td>
<td>air pressure</td>
</tr>
</tbody>
</table>

**Only pattern travels, not medium.**

Displacement motion \( \frac{\partial y}{\partial t} \) is usually slower than wave speed

\[ f(x - vt) \text{ is a wave travelling at } v \text{ in } +x \text{ dir:} \]

\[ y(x - vt) \text{ is the equation of a wave travelling to the right.} \]

\[ y(x + vt) \text{ is a wave travelling to the left} \]

\[ e.g. \ y = y_m \sin \left( \frac{2\pi}{\lambda} (x - vt) \right) \]
At one point (e.g. $x = 0$)

One cycle of SHM takes $T$, wave travels $\lambda$, \(\therefore\)

\[ v = \frac{\lambda}{T} = f\lambda \]

**define** $\omega = 2\pi f$, wave number $k = \frac{2\pi}{\lambda}$

**y = $y_m \sin (kx - \omega t)$**

**Speed of wave**

\[ v_{\text{wave}} = \sqrt{\frac{\text{springy const}}{\text{inertial const}}} \quad v_{\text{string}} = \sqrt{\frac{T}{\mu}} \]

\[ v_{\text{sound}} = \sqrt{\frac{\gamma P}{\rho}} \quad (c = \sqrt{\frac{k_{\text{elec}}}{k_{\text{mag}}}} = \sqrt{\frac{1}{\mu\epsilon_0}}) \]

\[ \gamma = \frac{cP}{c_v} \] is ratio of specific heats

**Example** A wave has $y = y_m \sin (kx - \omega t)$,

- $y_m = 10 \text{ nm}$, $k = 18.5 \text{ m}^{-1}$, $\omega = 6300 \text{ rad.s}^{-1}$
- i) what is the speed of the wave?
- ii) What is (max) average speed of particles?

\[ v_{\text{wave}} = f\lambda = \frac{\omega}{2\pi} \cdot \frac{2\pi}{k} = \frac{\omega}{k} = \ldots = 340 \text{ ms}^{-1} \]

**Reflection:**

Going from less dense to more dense, waves are reflected with a phase change of $\pi$.
- e.g. reflection at a 'fixed' end
  - thin string to thick string,
  - air to water

From more dense to less dense, no phase change
- e.g. reflection at 'free' end, etc

**Superposition**

In a linear medium, waves superpose linearly, i.e. their displacements simply add.
Most media linear for small amplitude waves.

Superpose incident & reflected waves $\rightarrow$ standing waves
Standing waves

\[ y_1 = y_m \sin(kx - \omega t) \]

\[ y_2 = y_m \sin(kx + \omega t) \]

\[ y_T = \ldots (algebra) \ldots = 2y_m \sin kx \cos \omega t \]

\[ f_1 = \frac{v}{\lambda} = \frac{v}{2L} \]

\[ f_2 = \frac{2v}{\lambda} = 2f_1 \]

\[ (2^{nd} \text{ harmonic}) \]

\[ f_3 = \frac{3v}{\lambda} = 3f_1 \]

\[ (3^{rd} \text{ harmonic}) \]


Sound is a compression wave - longitudinal

Radiation

Intensity \[ I = \frac{\text{power}}{\text{area}} \]

Example: What is the intensity of solar radiation? \( P_{\text{sun}} = 3.9 \times 10^{26} \text{ W.} \)

Earth is 150 million km from sun.

\[ I = \frac{P}{4\pi r^2} = \ldots = 1.38 \text{ kWm}^{-2} \]

above atmosphere, radiation

Isotropic radiation:

Intensity \( \propto \text{Power} \propto \text{amplitude}^2 \propto \text{pressure}^2 \)
Sound intensity level:

\[ L_I = 10 \log_{10} \frac{I}{I_o} \]  

where \( I_o = 10^{-12} \text{ W.m}^{-2} \)  

(L_I in decibels)

\[ L_2 - L_1 = 10 \left( \log_{10} \frac{I_2}{I_o} - \log_{10} \frac{I_1}{I_o} \right) = 10 \log_{10} \frac{I_2}{I_1} \]

\[ \frac{p_2}{p_1} \quad \Delta L_p \quad \frac{I_2}{I_1} \quad \Delta L_I \]

\[ \sqrt{2} \quad 3 \text{ dB} \quad 2 \quad 3 \text{ dB} \]

\[ \sqrt{10} \quad 10 \text{ dB} \quad 10 \quad 10 \text{ dB} \]

http://www.phys.unsw.edu.au/music/dB.html

www.phys.unsw.edu.au/PHYSICS_/SPEECH_HELIUM/speech.html

**Example.** If sound level \( L_I = 3 \text{ dB} \) at 10 cm from a source radiating uniformly, what is the acoustic power of the source?

\[ 3 \text{ dB} = L_I = 10 \log \frac{I}{I_o} \]

\[ 0.3 = \log \frac{I}{I_o} \]

\[ I/I_o = \text{antilog} 0.3 = 10^{0.3} = 2 \]

\[ I = 2 I_o = 2 \times 10^{-12} \text{ W.m}^{-2} \]

\[ I = \frac{P}{A} = \frac{P}{4\pi r^2} \]

\[ P = \ldots = 0.25 \text{ pW} \]

**Doppler effect.**

\[ f' = f \left( \frac{v + v_o}{v - v_s} \right) \]

\( v_o \) and \( v_s \) are positive for approaching

measure all velocities with respect to medium

**Example.** You walk towards a wall, blowing a whistle at \( f = 500 \text{ Hz} \). You hear beats at 5 Hz between your whistle and the reflected sound. How fast are you walking?

You hear your own whistle at frequency \( f \).

The wall receives

\[ f' = f \frac{v + v_o}{v - v_s} = f \frac{0}{v - v_w} \]

This is the source of the reflection. You hear

\[ f'' = f' \frac{v + v_o}{v - v_s} = f' \frac{v + v_w}{v - 0} \]

\[ f'' = f' \frac{v + v_w}{v - v_w} \]

\[ f''(v - v_w) = f (v + v_w) \]

\[ (f'' - f)v = (f'' + f)v_w \]

\[ v_w = \frac{f'' - f}{f'' + f} = v \frac{5}{1005} = 1.7 \text{ ms}^{-1} \]
Shock wave
Crests combine to form a shock wave
Cone has $\sin \theta = \frac{v}{v_s}$
$\frac{v_s}{v} \equiv \text{Mach number}$

Beats

\[
y_1 = A \cos 2\pi f_1 t \\
y_2 = A \cos 2\pi f_2 t \\
y_1 + y_2 = 2A \cos 2\pi \frac{f_1 + f_2}{2} t \cos 2\pi \frac{f_2 - f_1}{2} t
\]

\[
\begin{array}{c}
\text{average frequency} \\
\text{Beat frequency is } f_2 - f_1
\end{array}
\]

One cycle (diagram) of $\cos 2\pi \frac{f_2 - f_1}{2} t$ has two beats
Young's experiment  Coherent source \(\rightarrow\) two slits gives interference pattern on screen.

Constructive interference (max) if \(d \sin \theta = m \lambda\).

Destructive interference (min) if \(d \sin \theta = \left(m + \frac{1}{2}\right) \lambda\).

\[
\frac{\phi}{2\pi} = \frac{\Delta \text{path}}{\lambda} = \frac{d \sin \theta}{\lambda} \implies \phi = \frac{2\pi}{\lambda} d \sin \theta
\]

\(a_{\text{tot}} = 2a \cos \beta\)

\(\beta = \phi/2 = \frac{\pi}{\lambda} d \sin \theta\)

Intensity \(\propto\) amplitude\(^2\) \(\implies I \propto 4a^2 \cos^2 \beta\)

Diffraction grating has very many slits.
Used to measure \(\lambda\) very accurately.
If there are \(N\) slits per unit length, \(d = 1/N\).
The first minimum is very close (small \(\phi\) to close polygon), i.e. very narrow maxima

For constructive interference
\(d \sin \theta = m\lambda\)

\(m = 1 \rightarrow 1\text{st order spectrum}\)

\(m = 2 \rightarrow 2\text{nd order spectrum}\)

\(\theta_{\text{red}} = \sin^{-1} \frac{m\lambda_{\text{red}}}{d}\)

\(\theta_{\text{blue}} = \sin^{-1} \frac{m\lambda_{\text{blue}}}{d}\)

Thin film interference

Optical path length \(\text{OPL} \equiv n \cdot \text{pathlength}\)

\(\Delta \phi = 2\pi \frac{\Delta \text{optical pathlength}}{\lambda}\)
e.g. Newton's rings & non reflective coating
For destructive
ΔOPL = λ/2
2nd = λ/2
\[ d \sim \frac{\lambda}{4n} \]

Air wedge

Denstuctive interference if
\[ 2d = m\lambda \]

Constructive interference if
\[ 2d = \left( m + \frac{1}{2} \right)\lambda \]

Diffraction from a slit
\[ I = I_{\text{max}} \left( \frac{\sin \alpha}{\alpha} \right)^2 \quad \text{where} \quad \alpha = \frac{\pi a \sin \theta}{\lambda} \]

Resolution from circular aperture:
First minimum at \[ \sin \theta = 1.22 \frac{\lambda}{d} \]
can resolve \( \theta \) with lens diam \( d \). Rayleigh's criterion

Polarisation.

EM waves are transverse waves: \( \mathbf{E} \perp \mathbf{v} \) \( \therefore \) can be polarised. Usually light has waves with \( \mathbf{E} \) in all directions

Polaroid materials
allow \( \mathbf{E} \) in only one dir\( \uparrow \)
\( \mathbf{E}_{\text{transmitted}} = \mathbf{E} \cos \theta \)

Malus’ Law:
\[ I_{\text{trans}} = I_{\text{in}} \cos^2 \theta \]

Average of \( \cos^2 \theta \) over all angles is 1/2 \( \therefore \)

Polarisation by reflection
When refracted ray \( \perp \) reflected \( \rightarrow \) plane polarised reflected wave (Brewster’s angle \( \theta_B \)). If \( \theta_B + \theta_r = 90^\circ \)

\[
\sin \theta_r = \cos \theta_B \quad \Rightarrow \quad n = \frac{\sin \theta_B}{\cos \theta_B} \quad \Rightarrow \quad \theta_B = \tan^{-1} n
\]

**Example.** An oil slick (\( n = 1.20 \)) floats on water. What are the thicknesses for which red light (\( \lambda = 700 \text{ nm} \)) is reflected weakly? What does the slick look like at its thinnest point?

\( n_{\text{water}} > n_{\text{oil}} \)

**Constructive interference** if

\[ \Delta \text{OPL} = m \lambda \]

**Destructive interference** if

\[ \Delta \text{OPL} = \left( m + \frac{1}{2} \right) \lambda \]

i) If red has destructive interference,

\[
\Delta \text{OPL} = 2nd = \left( m + \frac{1}{2} \right) \lambda_{\text{red}}
\]

\[
d = \frac{\lambda_{\text{red}}}{2n} \left( m + \frac{1}{2} \right)
\]

\[ m = 0, \quad m = 1, \quad m = 2 \quad \ldots \]

\[ = 150 \text{ nm}, 440 \text{ nm}, 730 \text{ nm} \quad \text{etc} \]

ii) If \( d \ll \lambda \), \( \pi \) phase difference on both paths so constructive interference for all \( \lambda \), so it looks bright and ‘white’.

---

**Temperature (T):**

\( T \) is equal in any 2 bodies at thermal equilibrium

**Thermal Expansion**

\[ \frac{\Delta L}{L} = \alpha \Delta T \quad \frac{\Delta V}{V} = \beta \Delta T \]

\( \alpha \) is coefficient of linear expansion

\( \beta \) is coefficient of volume expansion

\[
\Delta V = (L + \Delta L)^3 - L^3
\]

\[ = \ldots \]

\[ = V.3\alpha \Delta T \]

\[ \therefore \quad \beta = 3 \alpha \]

**Heat Capacity:** (for a body) \( C = \frac{\Delta Q}{\Delta T} \) extensive

**Specific Heat:** (of substance) \( c = \frac{\Delta Q}{M \Delta T} \) intensive

**Latent Heat:** heat required per unit mass for change of phase (at constant \( T \)).

**Work** done against pressure \( P \)
\[ dW = F \cdot ds = PA \, ds = PdV \]

**Work** done against pressure \( P \)

**1st Law** \( dU = dQ - dW \)

where \( U \) is a state function

**Isobaric** \( P \) const

**Adiabatic Process:** \( \Delta Q = 0 \) (fast or insulated)

**Calculate work:**

Isobaric: \( W = \int P \, dV = P \Delta V \)

Isothermal: \( W = \int P \, dV = \int \frac{nRT}{V} \, dV = nRT \, ln \frac{V_f}{V_i} \)

Adiabatic: \( PV^{\gamma} = \) constant

**Kinetic Theory**

\( \Delta \) momentum = \( 2mv_x \)

\( |F| = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L} \)

\( F_{all} = PA = \frac{Nmv_x^2}{L} \)

\( v^2 = v_x^2 + v_y^2 + v_z^2 \); random motion \( \Rightarrow v_{x,\text{rms}}^2 = \frac{1}{3} \frac{v^2}{v_x^2} \), so:

\[ \frac{1}{2} m \, v^2 = \bar{\varepsilon} \quad \text{and} \quad PV = NkT \]

\[ \therefore \bar{\varepsilon} = \frac{1}{2} m \, v^2 = \frac{3}{2} \frac{PV}{N} = \frac{3}{2} kT \]

\( v_{f.m.s.} \equiv \sqrt{v^2} = \sqrt{\frac{3kT}{m}} \)

**Heat conduction**

\[ H = kA \frac{T_H - T_C}{l} \]

\( k \) is thermal conductivity

**Thermal resistance or R-value** sometimes used for building materials

\[ R = \frac{l}{k} \quad \text{so} \quad H = A \frac{\Delta T}{R} \]

(High conductivity, low R value and vice versa.)
Mechanics and forces

**Electric force**

\[ F_{\text{elec}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2} \hat{r} \]

**van der Waals force** (electrodynamic force)

\[ F_{\text{vdw}} \propto \frac{1}{r^6} \text{ always attractive} \]

**Properties of condensed phases**

Inter-atomic & intermolecular forces and energies

\[ U(r) \]

\[ \rightarrow \text{ Linear elasticity (parabolic minimum in } U(r)) \]

Linear approximation to inter-molecular forces

**Stress**

\[ \sigma \equiv \frac{F}{A} \]

**Strain**

\[ \varepsilon \equiv \text{dimensionless change, e.g. } \frac{\delta L}{L} \]

Hooke’s Law: \[ \frac{\sigma}{\varepsilon} = \text{elastic modulus} \]

**Longitudinal stress:**

\[ \sigma_{\text{long}} \]

**Young’s modulus**

\[ Y = \frac{F}{A} \frac{L}{\delta L} = \frac{FL}{A\delta L} \]

**Bulk modulus**

\[ \kappa = -\frac{P}{\delta V/V} \]

**Poisson’s ratio**

\[ \nu = -\frac{\varepsilon_{\text{lat.}}}{\varepsilon_{\text{long.}}} = -\frac{\delta d/d}{\delta L/L} \]
Rigidity modulus (shear modulus)

\[
G = \frac{F/A}{\delta L/L} = \frac{\tau}{\gamma}
\]

Example

F applied to the ends. P applied to other 4 sides. What is P so that cross section A is unchanged?

\[
\varepsilon_y = -v\sigma_y/Y + \sigma_y/Y - v\sigma_y/Y \\
0 = -vF/YA - P/Y + vP/Y \\
vF/A = P(1 - v) \\
P = \frac{F}{A} \frac{v}{(1 - v)}
\]

Hysteresis

Other non-elasticity

Inter-atomic & intermolecular forces

Ionic solids: electrostatic force

Hydrogen bonds: H has +δ charge

van der Waals attraction: attraction between transient dipoles \( \propto r^{-6} \) at short range

Crystalline solids

Packing factor: fraction of space occupied by touching hard spheres

Example: Calculate packing factor and \( \rho \) of FCC

Face diagonal = 4r
Side of unit cube = 4r/sin 45°
\[= 2\sqrt{2}r\]
p.f. = \frac{\text{Volume of spheres in unit cube}}{\text{Volume of unit cube}}
= \frac{8 \text{ corners} + 6 \text{ faces}}{(2\sqrt{2}r)^3}
= \frac{(8 \times \frac{1}{8} + 6 \times \frac{1}{2}) \frac{4}{3}\pi r^3}{(2\sqrt{2}r)^3}
= 74\%

\rho = \frac{(8 \times \frac{1}{8} + 6 \times \frac{1}{2}) \text{ atomic mass}}{a^3}
= \frac{4m}{a^3}

**Ionic crystals**
e.g. NaCl: ions of similar size, each ion has six neighbours of opposite charge (coordination number six)
(It's like two interlaced FCCs)

![Ionic crystals diagram](image)

**Covalent crystals**: share outer electrons. Depends on the angles of the electron orbitals

**Metallic crystal**: 'ions' in a sea of shared electrons. Often close packed in FCC or HCP. This, gives high \( \rho \), especially if atomic number is large (e.g. Au, Pt)

![Metallic crystal diagram](image)

**Intermediate bond types:**

\[ \text{covalent} \quad \text{ZnS} \quad \text{Si} \quad \text{TiC} \]
\[ \text{ionic} \quad \text{NaCl} \quad \text{AlN} \quad \text{Cu} \quad \text{metallic} \]

**Amorphous solids** (a.k.a. glass, vitreous phase)

**Metallic glass**: Cool the metal very quickly, e.g. small drops in liquid N\(_2\).

**Polymers**
Long chains repeating one unit e.g. poly(ethylene) PE
(what sort of hydrocarbon is this?)

poly(vinyl acetate) PVA
Long flexible chains: usually tangle rather than crystallise, especially if they have side groups. Attractive force is vdw (and tangling)

Amorphous (partly) Crystalline

Crystalline polymers are only partly crystallised. Need uniform cross section, e.g. poly(vinylidene chloride)

Cross linking. Chemical bonds rigidify 3D structure. e.g. resins, vulcanisation in rubber (S bonds).

Point defect
vacancy, interstice, substitution

Line defects Edge and screw

Plane defects, espec. microcracks
### Surface tension and surface energy

A molecule in bulk is uniformly attracted in each direction. Molecule at surface has no attraction to atmosphere, so work done against the net force in order to make a surface.

Work to make new surface is done against the

**surface tension** \( \gamma \).

\[ \gamma = \text{force per unit length in the plane of the surface} \]

E.g., raise wire dy

\[ F = 2\gamma L \quad \text{(two sides)} \]

\[
\begin{align*}
\text{Work} & \quad dW = F \cdot dy = 2\gamma L \cdot dy \\
\text{dA} & = 2Ldy \\
\therefore \quad \gamma & = \frac{dW}{dA} \quad \text{(two sides)}
\end{align*}
\]

So surface tension = surface free energy per unit area.

### Contact angles and menisci

The surface energy of solid-vapour is \( S_S \),

Solid-liquid is \( S_{LS} \)

Surface tension of liquid is \( \gamma \)

Contact angle \( \theta \)

Suppose that the meniscus rises as shown. Work done by tension

\[ dW = -\gamma Ldx \cos \theta = -\gamma dA \cos \theta \]

\[ = \text{new solid-liquid energy} - \text{solid-vapour energy} \]

\[ = S_{LS} dA - S_S dA \]

\[ \gamma \cos \theta = S_S - S_{LS} \]

### Some surface free energies

<table>
<thead>
<tr>
<th>Class</th>
<th>Material</th>
<th>( S ) mJ.m(^{-2} )</th>
<th>( \frac{\gamma}{\text{mN.m}(^{-1})} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Liquid</td>
<td>water</td>
<td>0.073</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hg</td>
<td>0.051</td>
<td></td>
</tr>
<tr>
<td>Glass</td>
<td>SiO(_2)</td>
<td>4.4</td>
<td></td>
</tr>
<tr>
<td>Ionic</td>
<td>NaCl</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>solid</td>
<td>KCl</td>
<td>0.11</td>
<td></td>
</tr>
<tr>
<td>Mica</td>
<td>in air</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td></td>
<td>in vacuum</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Covalent</td>
<td>Al(_2)O(_3) (sapphire)</td>
<td>6-32</td>
<td></td>
</tr>
<tr>
<td>solid</td>
<td>C (diamond)</td>
<td>5.24</td>
<td></td>
</tr>
<tr>
<td>Metal</td>
<td>Zn</td>
<td>0.105</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pb</td>
<td>0.763</td>
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<tr>
<td></td>
<td>Si</td>
<td>1.24</td>
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<tr>
<td>Polycrystalline</td>
<td>SiC</td>
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<tr>
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<td>Graphite</td>
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<tr>
<td></td>
<td>Granite</td>
<td>200</td>
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</tr>
<tr>
<td></td>
<td>Cast Iron</td>
<td>1,520</td>
<td></td>
</tr>
</tbody>
</table>
Pressure excess inside balanced by surface tension
\[ \Delta P \pi r^2 = \gamma \cdot 2\pi r \]
\[ \Delta P = \frac{\gamma}{r} \]

Young-Laplace Equation

Surface energy of solids

Complicated by mechanical strength of materials. Work to cleave
\[ = U_{\text{surf}} + U_{\text{bend}} \]

Put in expression for deflection of cantilever spring:
\[ F = \frac{3Yd^3y^2}{8x^4} \]

Diverse comments about various types of materials and behaviour

Be careful talking of 'strength of materials'
- Sometimes you want high E (\( \therefore \) small \( e \))
- Other times you want high \( \sigma_{\text{max}} \)
- Yet other times you want low E (\( \because \) large \( e \))

Macroscopic \( \sigma_{\text{max}} \) usually \( \ll \) microscopic \( \sigma_{\text{max}} \)
- Composite materials aim to minimise the difference by limiting propagation of dislocations
- In composites, \( \sigma_{\text{micro}} \neq \sigma_{\text{macro}} \)

Ductility refers to the ease of plastic deformation without rupture.
- Sometimes 'good', sometimes 'bad'
- Work hardening: 'dislocation tangle'. Dislocation stops at slip plane,
- fewer moveable dislocations \( \therefore \) less ductility
- Ductile rupture: break bonds, eventually form macroscopic rupture
For some materials, mechanical properties may depend on time scale (rheology).

For some materials, mechanical properties depend on temperature. Plastic deformation is easier at high T, brittle fracture is more likely at low T.

Material properties may also be changed by

- Chemical reactions
- Radiation (UV or even light from some polymers, $\gamma$ and X for other materials)
- Hydration (for hydrophilic, fibrous materials)