1. \[ g(T) = g_1 + \frac{g_2 - g_1}{T_2 - T_1} (T - T_1) \approx 31.2 \]

2. Out of the oven, the rod of length \( L \) reads \( R \) on the ruler. In the oven, the rod has expanded by \( \Delta L \) and reads \( R' > R \), because it has expanded proportionally more than has the ruler. For the rod, write

\[ \Delta L = L \alpha_r \Delta T \]

(subscripts \( r \) for rod and \( s \) for steel).

The diagram shows that \( \Delta L \) is the sum of two lengths:

\( \Delta R \) is the expansion of the length of the ruler up to the reading \( R \), which is

\[ \Delta R = L \alpha_s \Delta T. \]

Add to this the length \( \Delta R \) of the increased reading \( R' - R \).

Strictly, the length \( \Delta R \) is very slightly greater than what \( R' - R \) reads on the ruler, but that difference is well below the precision of the problem. So

\[ \Delta L = L \alpha_r \Delta T = L \alpha_s \Delta T + (R' - R) \]

so

\[ \alpha_r = \frac{L \alpha_s \Delta T + (R' - R)}{L \Delta T} = ... = 23 \text{ ppm.K}^{-1}. \]

3. \[ \frac{\delta L}{L} = ? \quad L = L_1(1 + \alpha_1 \Delta T) + L_2(1 + \alpha_2 \Delta T) \]
\[
\Delta T = L\left(1 + \frac{L_1\alpha_1 + L_2\alpha_2}{L}\right)\Delta T
\]

\[
d_1 = 11.10^{-6} \quad L = 52.4 \text{ cm}
\]
\[
\alpha_2 = 19.10^{-6} \quad d = 13.10^{-6} \text{ K}^{-1}
\]

\[
\frac{L_1\alpha_1 + L_2\alpha_2}{L} = \alpha
\]

\[
L_1 + L_2 = L
\]

\[
L_1 = L - L_2
\]

\[
L_1\alpha_1 + L_2\alpha_2 = L\alpha (L - L_2)\alpha_1 + L_2\alpha_2 = L\alpha
\]

\[
L_2 = \frac{L(\alpha - \alpha_1)}{\alpha_2 - \alpha_1} \approx 13.1 \text{ cm}
\]

\[
L_1 = \frac{L(\alpha - \alpha_2)}{\alpha_1 - \alpha_2} \approx 39.3 \text{ cm}
\]

4. Kinetic theory and the ideal gas

\[
P_1 = 1.00 \text{ atm} = 76.0 \text{ cm Hg} (= 1013 \times 10^2 \text{ Pa} = 1.013 \times 10^5 \text{ Pa})
\]

\[
T_1 = 22.0^\circ C
\]

\[
V_1 = 3.47 \text{ m}^3
\]

\[
P_2 = 36.0 \text{ cm Hg}
\]

\[
T_2 = -48.0^\circ C
\]

\[
V_2 - ?
\]

\[
\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}
\]

\[
T(\text{k}) = T(\text{o C}) + 273
\]

\[
V_2 = \frac{P_1}{P_2} \cdot \frac{T_2}{T_1} \cdot V_1 \approx 5.59 \text{ m}^3
\]

N.B. Because only the ratio of pressures appears, we may use the peculiar but common units: cm Hg, which means cm of mercury in a manometer.

5.

\[
\Delta p = 2mv \cos \theta
\]

\[
F = \frac{\Delta p}{\Delta t}
\]

\[
\text{pressure} = \frac{\sum F}{A} = \frac{\Delta p}{\Delta t} \cdot \frac{v}{A} = \frac{2mv \cos \theta v}{A} \approx 0.189 \frac{N}{\text{cm}^2}
\]

\[
m = 3.3 \times 10^{-27} \text{ kg}
\]

\[
A = 2.0 \times 10^{-4} \text{ m}^2
\]

\[
Q = 55^\circ
\]

\[
v = 1.0 \times 10^3 \text{ m/s}
\]
m = 0.315 kg
NH₃ \mu = (14+3) \times 10^{-3} \text{ kg/mole}
P = 1.35 \times 10^6 \text{ Pa}
P V = \frac{m}{\mu} R T
T = 273 + 77 = 350 \text{ K}
V = \frac{m R T}{\mu \rho} = 0.0399 \text{ m}^3
R = 8.31 \text{ J.K}^{-1}\text{/mole}
p_1 = 8.68 \times 10^5 \text{ Pa} \quad T_1 = 295 \text{ K}
m_1 = \frac{\mu p V}{RT_1} = 0.240 \text{ kg} \quad \Rightarrow \quad \Delta m = m - m_1 = 0.075 \text{ kg} = 75 \text{ g}

7.

\rho_0 = 1.22 \text{ kg/m}^3
V = 2180 \text{ m}^3
M = 249 \text{ kg}
m_1 = 272 \text{ kg}
\rho V = \frac{m}{\mu} R T; \quad p = \text{ const}
\frac{\mu}{RT} = \rho \equiv \frac{m}{V}
\rho T = \frac{\mu}{R}
\rho_0 T_0 = \rho_1 T_1

Mass of the air displaced by the balloon = \rho_0 V
Archimedes' principle: weight of floating object = weight of fluid displaced. So,
M + m + \rho_1 V = \rho_0 V
\frac{\rho_0 T_0}{T_1} V = \rho_0 V - M - m_1

T_0 = \frac{\rho_0 T_0 V}{\rho_0 V - M - m_1}
\approx 361 \text{ K} = 89^\circ \text{C}

\rho_0 = 1.22 \text{ kg/m}^3
V = 2180 \text{ m}^3
M = 249 \text{ kg}
m_1 = 272 \text{ kg}
\rho V = \frac{m}{\mu} R T; \quad p = \text{ const}
\frac{\mu}{RT} = \rho \equiv \frac{m}{V}
\rho T = \frac{\mu}{R}
\rho_0 T_0 = \rho_1 T_1

Past exam question

a) In time t, the mass of water added is $m = F \rho t$

In steady state, energy added = heat to raise $T$ + heat to boil water

$P t = m c \Delta T + m L = F \rho t (c \Delta T + L)$

$P = F \rho (c \Delta T + L)$

$= 0.020 \text{ l/s} \times 1 \text{ kg/l} \times (80 \text{ K} \times 4.2 \text{ kJ.kg}^{-1}\text{K}^{-1} + 2.3 \text{ MJ.kg}^{-1}) = 880 \text{ W}$
b) i) \[ D = L_A - L_B = (L_o + D_o)(1 + \alpha_A(T - T_o)) - L_o(1 + \alpha_B(T - T_o)) \]
   = (various simplifications possible, but not asked for)

ii) \[ D = (L_o + D_o)\alpha_A T - L_o \alpha_B T + \text{constant terms} \]
   if independent of \( T \), \( (L_o + D_o)\alpha_A = L_o \alpha_B \)

\[ L_o(\alpha_B - \alpha_A) = D_o \alpha_A \]

\[ D_o/L_o = (\alpha_B - \alpha_A)/\alpha_A \]