

Thermal Physics

Thermodynamics:

laws relating macroscopic variables (P, V, T etc.).

Statistical Mechanics: molecular explanation.

Difference between heat and temperature

Extensive property \propto amount of material

Intensive property doesn't depend on amount

Which of heat and temperature is intensive?

Which relates to sense of hotness?

Define temperature:

Thermal equilibrium:

Thermal properties do not change with time

Definition of Temperature (T):

T is equal in any 2 bodies at thermal equilibrium.

Zeroth Law of Thermodynamics:

if $T_A = T_B$ and $T_B = T_C$, then $T_A = T_C$.

What is temperature? How to measure it?

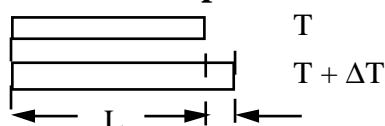
Thermometers: Hg in glass, thermocouple, thermistor, liquid crystal layer, constant volume gas thermometer

Scales. Obvious definition of temperature θ : choose a property X and make X proportional to or linear with θ . This can only be done once for any temp scale θ .

Reference temperature

Melting or freezing? Depends on the pressure.

Thermal Expansion



Usually, $\frac{\Delta L}{L} \propto \Delta T$ for small ΔT

\therefore Define $\frac{\Delta L}{L} = \alpha \Delta T$

α is **coefficient of linear expansion**

e.g. steel $\alpha_{st} = 1.1 \times 10^{-5} \text{ K}^{-1}$

Al $\alpha_{Al} = 2.3 \times 10^{-5} \text{ K}^{-1}$

Brass $\alpha_{br} = 1.9 \times 10^{-5} \text{ K}^{-1}$

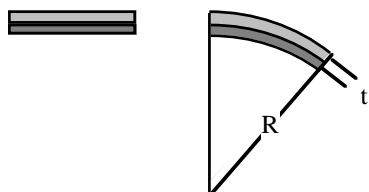
Example Bridge span is 1 km long.

Mid-winter, $T = -5^\circ\text{C}$ summer, $T = 45^\circ\text{C}$

What is Δl ?

$$\Delta L = \alpha \Delta T \cdot L = \dots = 55 \text{ cm}$$

Example: Bimetallic Strip, 30 cm long, made of 1 mm brass and 1 mm steel. Straight at 0 °C, what angle at 100 °C?

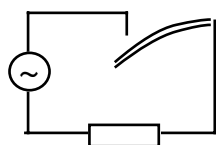


$$L + \Delta L_{st} = 2\pi R \cdot \frac{\theta}{360^\circ}$$

$$L + \Delta L_{br} = 2\pi(R + t) \frac{\theta}{360^\circ} \quad \text{subtract} \rightarrow$$

$$2\pi t \frac{\theta}{360^\circ} = \Delta L_{br} - \Delta L_{st} = L\Delta T(\alpha_{br} - \alpha_{st})$$

$$\theta = 14^\circ$$

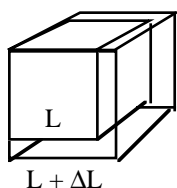


e.g. oven switch:

Volume Increase

define $\frac{\Delta V}{V} = \beta \Delta T$

β = coefficient of volume expansion



$$\begin{aligned} \Delta V &= (L + \Delta L)^3 - L^3 \\ &= L^3 \left(1 + \frac{\Delta L}{L}\right)^3 - L^3 \end{aligned}$$

$$= L^3 \left(1 + \frac{3\Delta L}{L} + \dots - 1\right)$$

$$\approx V 3\alpha \Delta T$$

$$\therefore \beta \approx 3\alpha$$

Note: Water is unusual: 0° - 4°C, $\beta < 0$

Example What is change in ρ for steel between 0° C and 100° C?

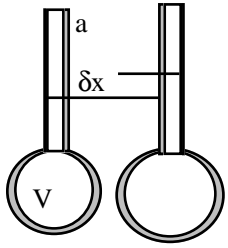
$$\rho = \frac{M}{V} \quad \therefore d\rho = -\frac{M}{V^2} dV$$

$$\therefore \Delta\rho \approx -\frac{M}{V^2} \Delta V = -\rho \frac{\Delta V}{V}$$

$$\therefore \frac{\Delta\rho}{\rho} = -\frac{\Delta V}{V} = -\beta \Delta T = \dots = -0.33\%$$

Example: thermometer, $Vol_{cylinder} \ll Vol_{sphere}$

What is its calibration slope $\frac{\partial x}{\partial T}$?



$$\frac{\Delta V}{V} \equiv \beta \Delta T$$

$$a \delta x \approx \Delta V_{fl} - \Delta V_{sphere}$$

$$= \beta_{fl} V \Delta T - \beta_{g1} V \Delta T$$

$$\frac{\partial x}{\partial T} = \frac{V}{a} (\beta_{fl} - \beta_{g1})$$

Other temperature coefficients:

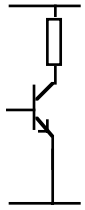
Resistivity ρ :

$$\rho = \rho(T=T_0)(1 + \alpha_1(T - T_0) + \alpha_2(T - T_0)^2 + \dots)$$

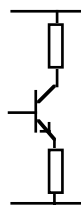
Warning: α is > 0 for metals, but < 0 for semiconductors

$$\sigma \equiv \frac{1}{\rho} = \dots \equiv \sigma(T=T_0)(1 - \alpha_1(T - T_0))$$

Thermal runaway possible



Thermally stable



Ideal gas temperature scale

Uses reference temp:

Triple point - co-existence of ice, water, steam
call it θ_{tr} .

defines $P \propto \theta$ for constant volume of gas (P is pressure \equiv Force per unit area)

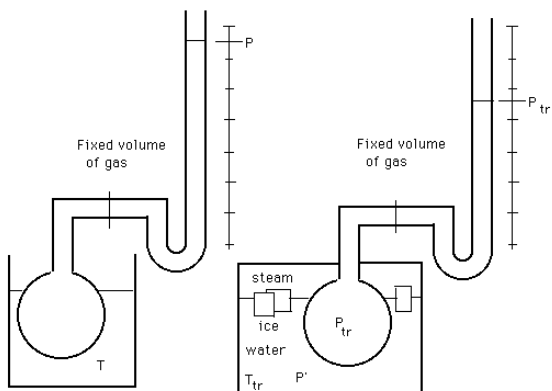
But gases are not (quite) ideal

e.g. consider boiling temp θ_s at some P:

$\frac{\theta_s}{\theta_{tr}}$ is different for different gases and at different densities.

At very low density or pressure,

all gases \rightarrow ideal, $\therefore \frac{\theta_s}{\theta_{tr}} \rightarrow$ same limit



$$T = T_{tr} \cdot \lim_{P_{tr} \rightarrow 0} \left(\frac{P}{P_{tr}} \right)_V$$

where $T_{tr} = 273.16 \text{ K}$

Why 273.16? This defines the Kelvin so that

$$\Delta T = 1 \text{ K} \quad \Leftrightarrow \quad \Delta T = 1 \text{ }^\circ\text{C}$$

(Working definition is more complicated)

Celsius Scale: $T_C = T - 273.15^\circ$

$$\left. \begin{array}{l} T_C = 0 \text{ }^\circ\text{C} \text{ water freezes} \\ T_C = 100 \text{ }^\circ\text{C} \text{ water boils} \end{array} \right\} \text{ at } P_A$$

Fahrenheit Scale

Heat

symbol Q

Definition: that which is transferred between a system and its surroundings as result of ΔT only.

Joule showed:

mechanical energy \rightarrow heat (by friction etc.).

(**C a r n o t** showed
heat at high T \rightarrow heat at low T + work)

\therefore measure heat as energy; i.e. S.I.unit. Joule (J)

Heat Capacity: (for a body) $C = \frac{\Delta Q}{\Delta T}$ extensive quantity

Specific Heat: (of a substance) $c = \frac{\Delta Q}{M\Delta T}$ intensive quantity

e.g. $c_{H_2O} = 4.2 \text{ kJ.kg}^{-1}\text{K}^{-1}$, $c_{Al} = 900 \text{ J.kg}^{-1}\text{K}^{-1}$

Latent Heat: heat required for change of phase (at constant T).

Example. A 240 V kettle has a working resistance of 50 Ω . Put in 500 ml of water at 20 $^{\circ}\text{C}$ and turn on. How long before it boils dry? (Specific heat of $c_w = \text{water} = 4.2 \text{ J.kg.K}^{-1}$, Latent heat of vaporisation $L_{\text{vap}} = 2.3 \text{ MJ.kg}^{-1}$.)

$$\text{Energy in} = \text{power.t} = \frac{V^2}{R} t = (1.15 \text{ kW}).t$$

= Q to raise T of water + Q to evaporate water

$$= m_w c_w (T_f - T_i) + m_w l_{\text{vap}}$$

$$= (0.5 \text{ kg}) (4.2 \cdot 10^3 \text{ J.kg}^{-1}\text{K}^{-1}) (100 - 20)^{\circ}\text{C}$$

$$+ (0.5 \text{ kg}) (2.3 \cdot 10^6 \text{ J.kg}^{-1})$$

$$= 168 \text{ kJ} + 1.15 \text{ MJ} = 1.32 \text{ MJ}$$

$$\therefore t = \frac{1.32 \text{ MJ}}{1.15 \text{ kW}} = \dots = 1150 \text{ s} = 19 \text{ minutes}$$

$$\text{(it boils after } \frac{168 \text{ kJ}}{1.15 \text{ kW}} = 2.5 \text{ mins)}$$

Example A mass m_w of water at temperature T_1 is added to a mass m_i of ice at temperature T_2 . (No heat is lost to the environment.) What is the final temperature at thermal equilibrium?

\exists 4 possible outcomes:

- i) $T > 0^\circ\text{C}$ (all ice melts)
- ii) $T = 0^\circ\text{C}$ & some ice melts
- iii) $T = 0^\circ\text{C}$ & some water freezes
- iv) $T < 0^\circ\text{C}$ (all water freezes)

To warm ice to 0° requires $Q_i = m_i c_i (0^\circ - T_i)$
 To cool water to 0° loses $Q_w = m_w c_w (T_w - 0^\circ)$
 To melt all ice requires $Q_m = m_i L$
 To freeze all water loses $Q_f = m_w L$

If $Q_w > Q_i + Q_m$, then all ice melts and final $T > 0$

Q lost by water = Q gained by ice

$$m_w c_w (T_w - T) = m_i c_i (0^\circ - T_i) + m_i L + m_i (T - 0^\circ)$$

heat ice melt it warm melted ice

$$T(m_i + m_w c_w) = m_i c_i T_i + m_w c_w T_w - m_i L$$

$$T = \dots\dots$$

If $Q_i + Q_m > Q_w > Q_i$, then some ice melts but $T = 0^\circ$

Q lost by water = Q gained by ice

$$m_w c_w (T_w - 0^\circ) = m_i c_i (0^\circ - T_i) + m_m c_w L$$

heat all ice melt some ice $m_m < m_i$

$$m_m = \dots\dots$$

etc for $Q_i > Q_w + Q_f$ and $Q_w + Q_f > Q_i > Q_w$
all water freezes some water freezes

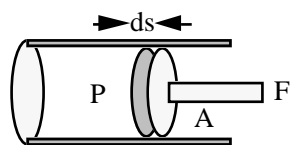
Work: energy transmitted from one system to another without ΔT or transfer of Q.

e.g. work done by force F

$$dW = \mathbf{F} \cdot d\mathbf{s}$$

e.g. work done against pressure P

$$\left(P \equiv \frac{\text{force}}{\text{area}} \right)$$



$$dW = \mathbf{F} \cdot d\mathbf{s} = PA ds = PdV$$

Example 1 kg water, initially at 20°C , is boiled away at P_A . How much of the input energy goes as work?

water volume = 10^{-3} m^3 Steam: $V_f \gg V_i$

$$W \approx PV_f = n RT = \frac{\text{mass}}{\text{mol. mass}} RT = \dots = 172 \text{ kJ}$$

cf. Heat to raise to 100°C

$$Q_1 = c m \Delta T = \dots = 336 \text{ kJ}$$

$$\text{Heat to transition } Q_2 = mL = \dots = 2.26 \text{ MJ}$$

Internal Energy

Heat dQ added to a system increases its internal energy U .

Work dW done **by** the system lowers its internal energy.

$$\text{1st Law of Thermodynamics} \quad dU = dQ - dW$$

where U is a state function

Kinetic Theory of Gases

Ideal gas equation of state

(\cong limit for all gases at low ρ):

$$PV = nRT = NkT$$

n = no. of moles	N = no. of molecules
R = gas constant	k = Boltzmann's Constant
$= 8.31 \text{ JK}^{-1}$	$= \frac{R}{N_A} = 1.38 \cdot 10^{-23} \text{ JK}^{-1}$

Example. Spherical balloon. Skin (total) has mass $\sigma = 10 \text{ g.m}^{-2}$.

How big does it need to be to lift 200 kg load if (i) it contains hot air at 100 C? (ii) Helium at STP?

Archimedes: $W_{\text{displaced}} = W_{\text{balloon}}$

$$\frac{4}{3} \pi r^3 \rho_{\text{air}} g = \frac{4}{3} \pi r^3 \rho_{\text{gas}} g + 4\pi r^2 \sigma g + mg$$

$$r^3(\rho_{\text{air}} - \rho_{\text{gas}}) - 3\sigma r^2 = \frac{3}{4\pi} m \quad (\text{neglect quadratic term or solve cubic})$$

$$r \cong \sqrt[3]{\frac{3m}{4\pi\rho_{\text{air}}(1 - \rho_{\text{gas}}/\rho_{\text{air}})}}$$

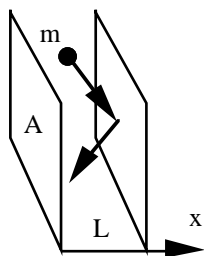
He: $\rho_{\text{gas}}/\rho_{\text{air}} = 4/30 \quad \rightarrow r \cong 3.6 \text{ m}$

Hot air: $\rho = \frac{Nm}{V} = \frac{Pm}{kT}$

$$\therefore \frac{\rho_{\text{hot}}}{\rho_{\text{cold}}} = \frac{T_{\text{cold}}}{T_{\text{hot}}} = \frac{273 \text{ K}}{373 \text{ K}} \quad \rightarrow r \cong 5.3 \text{ m} \quad \text{check approx}^n$$

Kinetic theory: Ideal Gas Postulates

- i) gas made of (identical) molecules
- ii) these obey Newton's laws, with random motion
- iii) no. of molecules is large (\sim Avagadro's number)
- iv) total volume molecules is negligible fraction ($\sim 10^{-3}$)
- v) no interaction except during collision (average $U_{interaction} < 10^{-4}$ K.E.)
- vi) collisions elastic, negligible duration. ($\sim 10^{-3}$ of time)



parallel plates, area A. Volume $V = AL$.
 N molecules (mass m) of an ideal gas.
 Each collision \rightarrow
 Δ momentum = $2mv_x$

time between collisions $t = 2L/v_x$.

$$\bar{F} = \frac{\Delta \text{ momentum}}{\Delta \text{ time}} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L} \quad \therefore \quad F \text{ on all } N \text{ molecules is}$$

$$F_{\text{all molecules}} = \frac{Nm \overline{v_x^2}}{L} = PA$$

$$v^2 = v_x^2 + v_y^2 + v_z^2; \quad \text{random motion} \Rightarrow \overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2} \Rightarrow \overline{v_x^2} = \frac{1}{3} \overline{v^2}, \text{ so:}$$

$$PAL = PV = Nm \overline{v_x^2} = \frac{N}{3} m \overline{v^2} \quad P = \frac{Nm}{3V} \overline{v^2} = \frac{1}{3} \rho \overline{v^2}$$

Molecular speeds:

v_{rms} root mean square velocity

$$v_{r.m.s.} \equiv \sqrt{\overline{v^2}}$$

c) What is v_{rms} in atmosphere? (approximate it as an ideal gas at P_A , with $\rho_A = 1.3 \text{ kg.m}^{-3}$)

$$\rightarrow v_{rms} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 10^5}{1.3}} = 480 \text{ ms}^{-1}$$

Meaning of temperature:

We had $PV = \frac{N}{3} m \overline{v^2}$

$$\frac{1}{2} m \overline{v^2} = \bar{\epsilon} \equiv \text{average K.E. per molecule}$$

But T defined by (1 and 5): $PV = NkT$

$$\therefore \bar{\epsilon} = \frac{1}{2} m \overline{v^2} = \frac{3}{2} \frac{PV}{N} = \frac{3}{2} kT \quad (7)$$

For ideal gas **all energy E is kinetic** so:

$$E = N \bar{\epsilon} = \frac{3}{2} NkT \quad (8)$$

$T \propto$ average K.E. of molecules in an ideal gas.

3 degrees of motional freedom (x, y, z)

i.e. $\frac{1}{2} kT$ per **degree of freedom** (At ordinary temperatures, $kT \cong 4 \cdot 10^{-21} \text{ J}$)

molecular speeds again:

$$\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT$$

$$v_{\text{r.m.s.}} \equiv \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}}$$

Example: What is v_{rms} of O_2 , N_2 , and H_2 at $T = 293\text{K}$?

$$(7) \quad v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT N_A}{\text{mol wt}}}$$

$$\text{for } \text{O}_2: \quad = \sqrt{\frac{3 \times 1.38 \cdot 10^{-23} \times 293 \times 6.02 \cdot 10^{23}}{0.032}}$$

$$= 478 \text{ ms}^{-1}$$

$$\text{for } \text{N}_2 \rightarrow 511 \text{ ms}^{-1} \quad \text{for } \text{H}_2 \quad 1.91 \text{ kms}^{-1} \quad \text{c.f. } v_{\text{escape}} = 11 \text{ kms}^{-1}$$

note that for air $v_{\text{rms}} > v_{\text{sound}}$

but recall from waves:

$$v_s = \sqrt{\frac{K_{\text{ad}}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma kT}{m}}$$

$$\text{so } \frac{v_{\text{rms}}}{v_s} = \sqrt{\frac{3}{\gamma}} \sim 1.5 \quad \gamma \text{ discussed later.}$$

Example What is the v_{rms} due to thermal motion (Brownian motion) of: pollen grain ($m \sim 10^{-15} \text{ kg}$) and apple ($m \sim 0.2 \text{ kg}$)

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad \text{pollen} \Rightarrow 2 \text{ mm s}^{-1} \quad (\text{Brownian motion 1st analysed by Einstein, 1904})$$

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check approx¹

Special Cases

1. **Isobaric Process** - P constant

$$W = \int P dV = P(V_f - V_i)$$

$$\therefore \Delta U = \Delta Q - P\Delta V$$

example $\Delta Q = ML$ (= mass . latent heat)

$$\Delta U = ML - P\Delta V$$

Also, things done at atmospheric pressure

2. **Isochoric** – volume constant

$$\therefore W = 0, \quad \therefore \Delta Q = \Delta U$$

3. **Adiabatic Process:** no heat flow, $\Delta Q = 0$

either fast or insulated

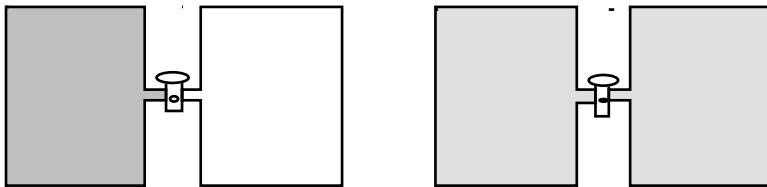
e.g. compression stroke in engine

sound wave compression

most muscle contractions

Free expansion:

open tap



No work done, Experimentally, find $\Delta Q = 0$

$$\therefore \Delta U = 0$$

$\therefore U$ of ideal gas doesn't depend on ρ ,

i.e. $U = U(T)$

Special cases of the First Law

$$\Delta U = Q - W$$

Process	Q	W	ΔU
Adiabatic	$Q = 0$	$\therefore \Delta U = -W$	$\int PdV$
Constant volume	$W = \int PdV = 0$	$\therefore \Delta U = Q$	Q
Closed cycle	$\Delta U = 0$	$\therefore Q = W$	Q
Free expansion	$Q = W = 0$	$\therefore \Delta U = 0$	0

For a gas, specific heat at constant pressure (c_p) is greater than specific heat at constant volume (c_v).

Why?

The ratio c_p/c_v is called γ .

$$PV^\gamma = \text{constant for adiabatic process}$$

Note: pressure varies more strongly with change in V for adiabatic than for isothermal, where $PV^1 = \text{constant}$.

Example

Compressed gas (P_1, T_1) escapes from a cylinder into the atmosphere (P_A, T). Assume rapid, adiabatic expansion, no initial mixing. How cold does it get?

Adiabatic: $PV^\gamma = \text{constant}$

$$\therefore \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/\gamma}$$

$$\therefore \frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} = \frac{P_2}{P_1} \left(\frac{P_1}{P_2}\right)^{1/\gamma} = \left(\frac{P_2}{P_1}\right)^{1 - 1/\gamma}$$

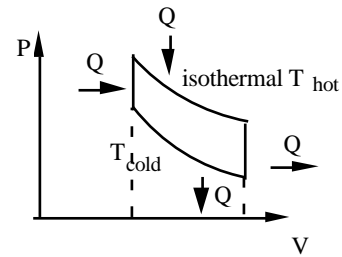
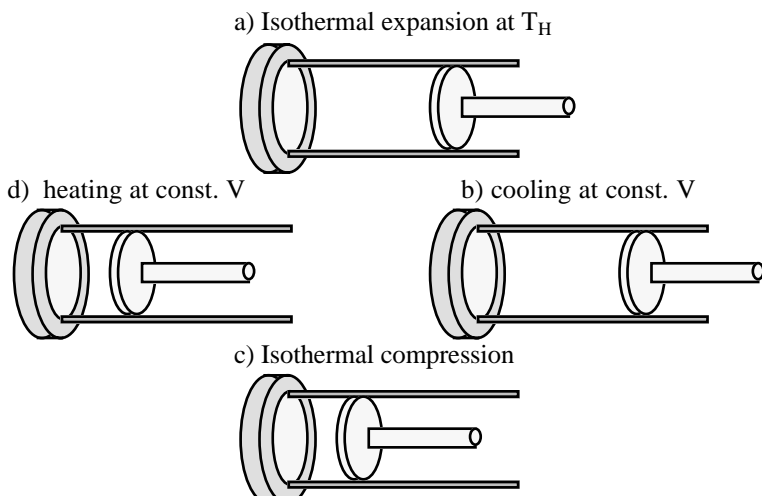
Example. Same problem but for car tyre:

"p" (gauge pressure) = 180 kPa. Take $\gamma = 1.40$, $T = 300$ K. Release some air into atmosphere ($P_2 = P_A = 101$ kPa). What is temperature of emerging air?

$P_1 = \text{"p"} + P_A = 281$ kPa

$\therefore T_f = \dots\dots\dots = 224$ K = -49°C

Example (a peculiar heat cycle)



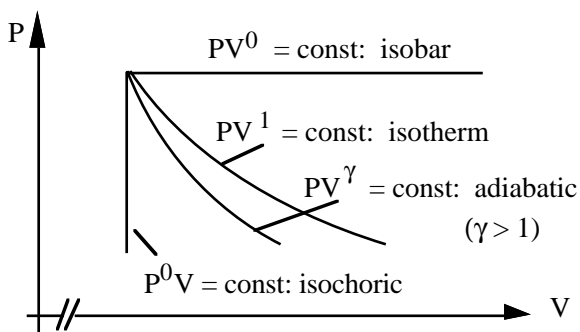
Isotherms: $P = \frac{nRT}{V} \quad \therefore W = \int P dV = \dots = nRT \ln \frac{V_f}{V_i}$

	ΔU	ΔW	ΔQ
a	0	$nRT_H \ln(V_2/V_1)$	$nRT_H \ln(V_2/V_1)$
b	$-nC_v(T_H - T_C)$	0	$-nC_v(T_H - T_C)$
c	0	$-nRT_C \ln(V_2/V_1)$	$-nRT_C \ln(V_2/V_1)$
d	$nC_v(T_H - T_C)$	0	$nC_v(T_H - T_C)$
Σ	0	$nR(T_H - T_C) \ln \frac{V_2}{V_1}$	$nR(T_H - T_C) \ln \frac{V_2}{V_1}$

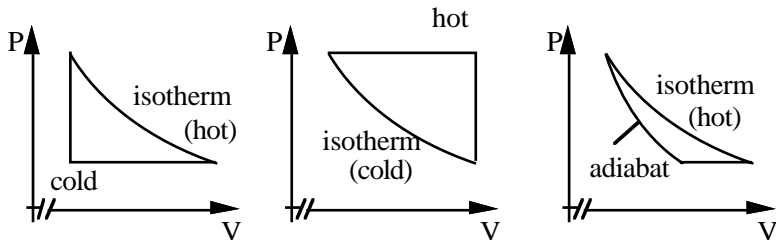
$$\text{efficiency} \equiv \frac{\text{work out}}{\text{heat in}} = \left(\frac{R(T_H - T_C) \ln(V_2/V_1)}{RT_H \ln(V_2/V_1) + C_v(T_H - T_C)} \right)$$

$\left. \begin{array}{l} \text{all terms} > 0 \\ T_H - T_C \leq T_H \end{array} \right\} \therefore \text{efficiency} < 100\%$

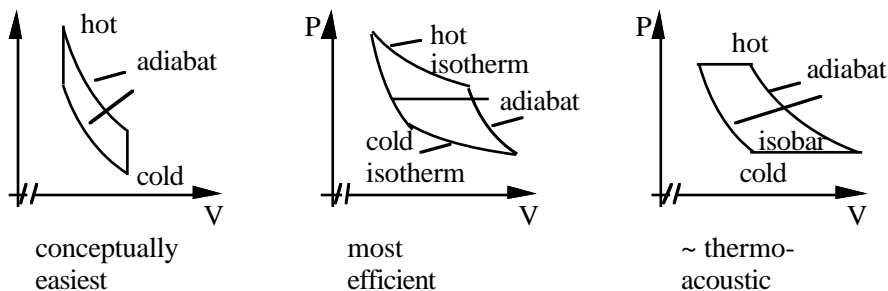
P-V diagrams and thermal cycles



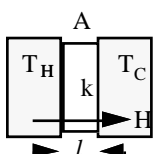
Simple cycles typical of those found in tut and exam questions



Idealised cycles sometimes approximated



Heat conduction



Reservoirs at T_H and at T_C .
 H is **rate of heat transfer** through a material in steady state.

$$H \equiv kA \frac{T_H - T_C}{l}$$

defines the **thermal conductivity k**

H in W, so k in $\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$.

Copper	401	$\text{W}\cdot\text{m}^{-1}\cdot\text{K}^{-1}$
Stainless Steel	14	
Glass	1	
Water	0.5	
Pine (wood)	0.11	
Dry air	0.026	

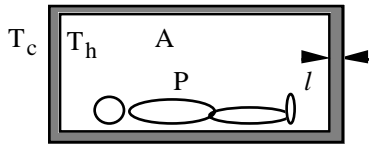
Thermal resistance or R-value sometimes used for building materials

$$R \equiv \frac{l}{k} \quad \text{so} \quad H = A \frac{\Delta T}{R}$$

(High conductivity, low R value and *vice versa*.)

Example. What is the R value of 1 cm pine?

When his Auststudy is cut off, a student lives in a pine packing crate, area 8 m^2 , thickness 1.0 cm . If the shivering student produces 300 W , which is lost by conduction through the crate, how much warmer is it inside the crate?



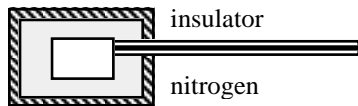
$$R \equiv \frac{l}{k} = 0.09 \text{ K.m}^2\text{W}^{-1}$$

$$H \equiv kA \frac{T_H - T_C}{l} \quad \text{or} = A \frac{\Delta T}{R}$$

$$T_H - T_C = \frac{lH}{kA}$$

$$= \frac{0.010 \text{ m} * 300 \text{ W}}{0.11 \text{ W.m}^{-1}.\text{K}^{-1} * 8 \text{ m}^2} = 3 \text{ K.} \quad \text{Other benefits: wind, rain, radiation}$$

Example. To reduce thermal noise, a low temperature circuit is immersed in liquid nitrogen (77 K , $L = 199 \text{ kJ.kg}^{-1}$). It is connected to the outside circuitry by 3 well-insulated copper wires, length $l = 100 \text{ mm}$, diameter 0.3 mm . What is the rate of N_2 evaporation due to the heat conducted down the wires?



Power to evaporate $\text{N}_2 = \text{heat transfer}$

$$L \cdot \frac{dm}{dt} = H \equiv kA \frac{T_H - T_C}{l}$$

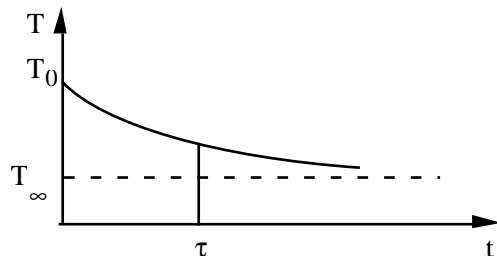
$$\frac{dm}{dt} = \frac{kA}{L} \frac{T_H - T_C}{l} = \dots = 90 \mu\text{gs}^{-1} = 340 \text{ mg.hr}^{-1}$$

Example. A flask of coffee, initially at 90°C , cools to 81°C in one hour in 20°C atmosphere. How long will it take to cool to 60°C ?

$$H = kA \frac{T - T_C}{L}$$

$$H = - \frac{dQ}{dt} \equiv - \frac{mcdT}{dt}$$

$$\frac{dT}{dt} = - \frac{kA}{mc} \frac{T - T_C}{L}$$



Characteristic time: $\tau = \frac{mL}{kA}$ Solve DE \rightarrow

$$T = T_C + (T_0 - T_C) e^{-t/\tau}$$

$$81^\circ\text{C} = 20^\circ\text{C} + (70^\circ\text{C})e^{-(1 \text{ h})/\tau}$$

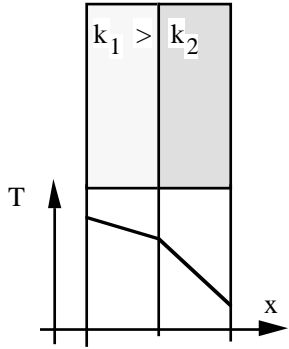
$$\therefore \tau = 7.3 \text{ h} \quad \dots \rightarrow 4.1 \text{ h to cool to } 60^\circ\text{C}$$

A note about steady state (T constant)

in one dimension, $H \equiv kA \frac{T_H - T_C}{l}$

becomes heat flux $\equiv \frac{H}{A} = k \frac{dT}{dx}$

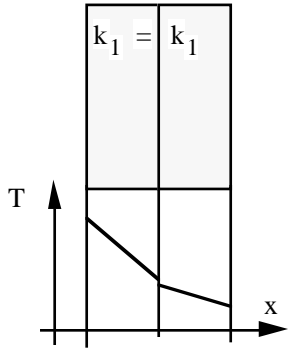
In steady state, rate of heat arriving = rate of heat departing



In steady state, heat flux is uniform. So

$$k_1 \left(\frac{dT}{dx} \right)_1 = k_2 \left(\frac{dT}{dx} \right)_2$$

Getting to steady state: *(details usually complicated)*

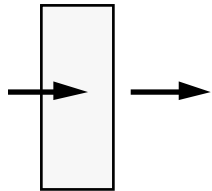


In this situation

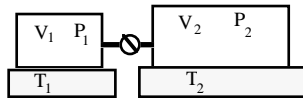
$$k_1 \left(\frac{dT}{dx} \right)_{\text{left}} > k_1 \left(\frac{dT}{dx} \right)_{\text{right}}$$

so the middle region receives more heat than it loses – so its temperature rises.

At boundary of vacuum:



Heat conduction to the surface
= heat radiation from the surface

Example

Two volumes of same gas. Open tap. What pressure?

Gas redistributes until P equal.

5 unknowns: N_1, N_2, N_1', N_2', P

Before $N_1 = \frac{P_1 V_1}{RT_1}$ $N_2 = \frac{P_2 V_2}{RT_2}$

After $N_1' = \frac{PV_1}{RT_1}$ $N_2' = \frac{PV_2}{RT_2}$

Conservation: $N_1 + N_2 = N_1' + N_2'$

Solve.....