Notes on Thermal Physics for PHYS1121-1131. Joe Wolfe, UNSW

Thermal Physics
Thermodynamics:
   laws relating macroscopic variables (P, V, T etc.).
Statistical Mechanics: molecular explanation.
Difference between heat and temperature
   Extensive property  ∝ amount of material
   Intensive property  doesn't depend on amount
   Which of heat and temperature is intensive?
   Which relates to sense of hotness?

Define temperature:
Thermal equilibrium:
   Thermal properties do not change with time

Definition of Temperature (T):
   T is equal in any 2 bodies at thermal equilibrium.

Zeroth Law of Thermodynamics:
   if T_A = T_B and T_B = T_C, then T_A = T_C.

What is temperature? How to measure it?

Thermometers: Hg in glass, thermocouple, thermistor, liquid crystal layer, constant volume gas thermometer
Scales. Obvious definition of temperature θ: choose a property X and make X proportional to or linear with θ. This can only be done once for any temp scale θ.

Reference temperature
   Melting or freezing? Depends on the pressure.

Thermal Expansion

\[ \frac{\Delta L}{L} \approx \Delta T \text{ for small } \Delta T \]

\[ \therefore \text{ Define } \frac{\Delta L}{L} = \alpha \Delta T \]

\( \alpha \) is coefficient of linear expansion

e.g. steel \( \alpha_{st} = 1.1 \times 10^{-5} \text{ K}^{-1} \)

Al \( \alpha_{Al} = 2.3 \times 10^{-5} \text{ K}^{-1} \)

Brass \( \alpha_{br} = 1.9 \times 10^{-5} \text{ K}^{-1} \)

Example Bridge span is 1 km long.
Mid-winter, T = -5°C  summer,  T = 45°C
What is \( \Delta L \)?

\[ \Delta L = \alpha \Delta T \cdot L = \ldots \ = 55 \text{ cm} \]
Example: Bimetallic Strip, 30 cm long, made of 1 mm brass and 1 mm steel. Straight at 0 °C, what angle at 100 °C?

\[
L + \Delta L_{\text{st}} = 2\pi R \cdot \frac{\theta}{360}\]

\[
L + \Delta L_{\text{br}} = 2\pi (R + t) \cdot \frac{\theta}{360}\]

subtract →

\[
2\pi \frac{\theta}{360} = \Delta L_{\text{br}} - \Delta L_{\text{st}} = L \Delta T (\alpha_{\text{br}} - \alpha_{\text{st}})
\]

\[
\theta = \frac{\Delta L_{\text{br}} - \Delta L_{\text{st}}}{L \Delta T (\alpha_{\text{br}} - \alpha_{\text{st}})} = 14°
\]

e.g. oven switch:

Volume Increase

Define \( \frac{\Delta V}{V} = \beta \Delta T \)

\( \beta = \text{coefficient of volume expansion} \)

\[
\Delta V = (L + \Delta L)^3 - L^3 = L^3 \left(1 + \frac{3\Delta L}{L} + \ldots -1\right)
\]

\( = V 3\alpha \Delta T \)

\( \therefore \beta \approx 3 \alpha \)

Note: Water is unusual: 0° - 4°C, \( \beta < 0 \)

Example: What is change in \( \rho \) for steel between 0° C and 100° C?

\( \rho = \frac{M}{V} \)

\( \therefore \Delta \rho = - \frac{M}{V^2} dV \)

\( \therefore \Delta \rho \approx - \frac{M}{V^2} \Delta V = - \rho \frac{\Delta V}{V} \)

\( \therefore \frac{\Delta \rho}{\rho} = - \frac{\Delta V}{V} = - \beta \Delta T = \ldots = -0.33\% \)

Example: thermometer, \( \text{Vol}_{\text{cylinder}} \ll \text{Vol}_{\text{sphere}} \)
What is its calibration slope \( \frac{\partial x}{\partial T} \)?

\[ \frac{\Delta V}{V} = \beta \Delta T \]
\[ a \Delta x = \Delta V_{fl} - \Delta V_{sphere} \]
\[ = \beta_{fl} V \Delta T - \beta_{g1} V \Delta T \]
\[ \frac{\partial x}{\partial T} = \frac{V}{a} (\beta_{fl} - \beta_{g1}) \]

Other temperature coefficients:
Resisitivity \( \rho \):
\[ \rho = \rho_{(T=T_0)}(1 + \alpha_1(T - T_0) + \alpha_2(T - T_0)^2 + ..... ) \]
Warning: \( \alpha \) is \( > 0 \) for metals, but \( < 0 \) for semiconductors
\[ \sigma \equiv \frac{1}{\rho} = ..... = \sigma_{(T=T_0)}(1 - \alpha_1(T - T_0)) \]

Thermal runaway
Possible

Thermally stable
Ideal gas temperature scale

Uses reference temp:
Triple point - co-existence of ice, water, steam
call it $\theta_{tr}$.

**defines**  $P \propto \theta$ for constant volume of gas  \quad (P is pressure \equiv Force per unit area)

But gases are not (quite) ideal

e.g. consider boiling temp $\theta_s$ at some $P$:

$\frac{\theta_s}{\theta_{tr}}$ is different for different gases and at different densities.

At very low density or pressure,

all gases $\rightarrow$ ideal, $\therefore \frac{\theta_s}{\theta_{tr}} \rightarrow$ same limit

\[ T = T_{tr} \lim_{\rho_{tr} \to 0} \left( \frac{P}{P_{tr}} \right) \]

where $T_{tr} = 273.16 \text{ K}$

Why 273.16? This defines the Kelvin so that

$\Delta T = 1 \text{ K} \iff \Delta T = 1 \degree \text{ C}$

(Working definition is more complicated)

**Celsius Scale:**  $T_C = T - 273.15^\circ$

\[
\begin{align*}
T_C = 0 \degree \text{C water freezes} \\
T_C = 100 \degree \text{C water boils}
\end{align*}
\]

\} \quad \text{at } P_A

**Fahrenheit Scale**
Heat symbol Q

**Definition:** that which is transferred between a system and its surroundings as result of \( \Delta T \) only.

**Joule** showed:
mechanical energy \( \rightarrow \) heat \( (by \text{ friction etc.}) \).

\( \text{(Carnot showed)} \)
\( \text{heat at high } T \rightarrow \text{heat at low } T + \text{work} \)
\( \therefore \) measure heat as energy; i.e. S.I. unit. Joule (J)

**Heat Capacity:** (for a body) \( C = \frac{\Delta Q}{\Delta T} \) extensive quantity

**Specific Heat:** (of a substance) \( c = \frac{\Delta Q}{M \Delta T} \) intensive quantity

\[ e.g. \quad c_{\text{H}_2\text{O}} = 4.2 \text{ kJ.kg}^{-1}\text{K}^{-1} \quad c_{\text{Al}} = 900 \text{ J.kg}^{-1}\text{K}^{-1} \]

**Latent Heat:** heat required for change of phase (at constant \( T \)).

**Example.** A 240 V kettle has a working resistance of 50 \( \Omega \). Put in 500 ml of water at 20 °C and turn on. How long before it boils dry?
(Specific heat of \( c_w = \text{water} = 4.2 \text{ J.kg.K}^{-1}, \quad \text{Latent heat of}\) vaporisation \( L_{\text{vap}} = 2.3 \text{ MJ.kg}^{-1}. \)

Energy in = power.t = \( \frac{V^2}{R} \) t = (1.15 kW).t
= Q to raise T of water + Q to evaporate water
= \( m_w c_w (T_f - T_i) \) + \( m_w L_{\text{vap}} \)
= \( (0.5 \text{ kg}) (4.2 \times 10^3 \text{ J.kg}^{-1}\text{K}^{-1}) (100 - 20)\text{°C} \)
\[ + \quad (0.5 \text{ kg}) (2.3 \times 10^6 \text{ J.kg}^{-1}) \]
= 168 kJ + 1.15 MJ = 1.32 MJ
\( \therefore \quad t = \frac{1.32 \text{ MJ}}{1.15 \text{ kW}} = \ldots = 1150 \text{ s} = 19 \text{ minutes} \)

(it boils after \( \frac{168 \text{ kJ}}{1.15 \text{ kW}} = 2.5 \text{ mins} \))
Example A mass $m_w$ of water at temperature $T_1$ is added to a mass $m_i$ of ice at temperature $T_2$. (No heat is lost to the environment.) What is the final temperature at thermal equilibrium?

4 possible outcomes:

i) $T > 0 \, ^\circ\text{C}$ (all ice melts)

ii) $T = 0 \, ^\circ\text{C}$ & some ice melts

iii) $T = 0 \, ^\circ\text{C}$ & some water freezes

iv) $T < 0 \, ^\circ\text{C}$ (all water freezes)

To warm ice to 0° requires $Q_i = m_i c_i (0° - T_i)$

To cool water to 0° loses $Q_w = m_w c_w (T_w - 0°)$

To melt all ice requires $Q_m = m_i L$

To freeze all water loses $Q_f = m_w L$

If $Q_w > Q_i + Q_m$, then all ice melts and final $T > 0$

$Q$ lost by water = $Q$ gained by ice

$m_w c_w (T_w - T) = m_i c_i (0° - T_i) + m_i (T - 0°)$

$T(m_i + m_w c_w) = m_i c_i T_i + m_w c_w T_w - m_i L$

$T = ....$

If $Q_i + Q_m > Q_w > Q_i$, then some ice melts but $T = 0°$

$Q$ lost by water = $Q$ gained by ice

$m_w c_w (T_w - 0°) = m_i c_i (0° - T_i) + m_m c_w L$

$m_m = ....$

etc for $Q_i > Q_w + Q_f$ and $Q_w + Q_f > Q_i > Q_w$

all water freezes

some water freezes

Work: energy transmitted from one system to another without ΔT or transfer of $Q$.

e.g. work done by force $F$

$\text{d}W = F \cdot \text{ds}$

e.g. work done against pressure $P$

($P = \frac{\text{force}}{\text{area}}$)

$\text{d}W = F \cdot \text{ds} = PA \, \text{ds} = PdV$

Example 1 kg water, initially at 20°C, is boiled away at $P_A$. How much of the input energy goes as work?

water volume = 10^{-3} \, \text{m}^3

Steam: $V_f >> V_i$

$W = PV_f = n \, RT = \frac{\text{mass}}{\text{mol. mass}} \, RT = ... = 172 \, \text{kJ}$

cf. Heat to raise to 100°C

$Q_1 = c \, m \, \Delta T = ... = 336 \, \text{kJ}$

Heat to transition $Q_2 = mL = ... = 2.26 \, \text{MJ}$
Internal Energy
Heat dQ added to a system increases its internal energy U.
Work dW done by the system lowers its internal energy.

$$\text{1st Law of Thermodynamics} \quad dU = dQ - dW$$
where U is a state function

---

Kinetic Theory of Gases

Ideal gas equation of state

(E\(\equiv\) limit for all gases at low \(p\):)

\[
PV = nRT = NkT
\]

\(n\) = no. of moles \(N\) = no. of molecules \(R\) = gas constant \(k\) = Boltzmann's Constant

\[
= 8.31 \text{ JK}^{-1} \quad = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ JK}^{-1}
\]

Example. Spherical balloon. Skin (total) has mass \(\sigma = 10\text{g.m}^{-2}\).
How big does it need to be to lift 200 kg load if (i) it contains hot air at 100 \(^\circ\)C? (ii) Helium at STP?

Archimedes: \(W_{\text{displaced}} = W_{\text{balloon}}\)

\[
\frac{4}{3} \pi r^3 \rho_{\text{air}} g = \frac{4}{3} \pi r^3 \rho_{\text{gas}} g + 4\pi r^2 \sigma g + mg
\]

\[
r^3 (\rho_{\text{air}} - \rho_{\text{gas}}) - 3\sigma r^2 = \frac{3}{4\pi} m \quad (\text{neglect quadratic term or solve cubic})
\]

\[
r \equiv \sqrt[3]{\frac{3m}{4\pi \rho_{\text{air}} (1 - \rho_{\text{gas}}/\rho_{\text{air}})}}
\]

He: \(\rho_{\text{gas}}/\rho_{\text{air}} = 4/30\) \(\rightarrow r \equiv 3.6\) m

Hot air: \(\rho = \frac{Nm}{V} = \frac{Pm}{kT}\)

\[
\therefore \frac{\rho_{\text{hot}}}{\rho_{\text{cold}}} = \frac{T_{\text{cold}}}{T_{\text{hot}}} = \frac{273 \text{ K}}{373 \text{ K}} \quad \rightarrow r \equiv 5.3\ \text{m} \quad \text{check approx}^n\]
Kinetic theory: Ideal Gas Postulates

i) gas made of (identical) molecules

ii) these obey Newton's laws, with random motion

iii) no. of molecules is large \((\sim \text{Avagadro's number})\)

iv) total volume molecules is negligible fraction \((\sim 10^{-3})\)

v) no interaction except during collision \((\text{average } U_{\text{interaction}} < 10^{-4}\text{ K.E.})\)

vi) collisions elastic, negligible duration. \((\sim 10^{-3}\text{ of time})\)

Parallel plates, area \(A\). Volume \(V = AL\).

\(N\) molecules (mass \(m\)) of an ideal gas.

Each collision →
\(\Delta\) momentum = \(2mv_x\)

Time between collisions \(t = \frac{2L}{v_x}\).

\[
\frac{F}{\Delta \text{time}} = \frac{2mv_x^2}{2L_v_x} = \frac{mv_x^2}{L} \quad \therefore \quad F \text{ on all } N \text{ molecules is}
\]

\[
F_{\text{all molecules}} = \frac{Nm v_x^2}{L} = PA
\]

\[
v_x^2 = v_y^2 + v_z^2; \quad \text{random motion} \Rightarrow v_x^2 = v_y^2 = v_z^2 \quad \Rightarrow \quad v_x^2 = \frac{1}{3} \bar{v}^2 , \text{ so:}
\]

\[
P A L = PV = Nm \bar{v}_x^2 = \frac{N}{3} m \bar{v}^2 \quad P = \frac{Nm}{3V} \bar{v}^2 = \frac{1}{3} \rho \bar{v}^2
\]

Molecular speeds:

\(v_{\text{rms}}\) root mean square velocity

\[
v_{\text{rms}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 10^5}{1.3}} = 480 \text{ ms}^{-1}
\]

Meaning of temperature:

We had \(PV = \frac{N}{3} m \bar{v}^2\)

\[
\frac{1}{2} m \bar{v}^2 = \bar{\epsilon} = \text{average K.E. per molecule}
\]

But \(T\) defined by (1 and 5): \(PV = NkT\)

\[
\therefore \quad \bar{\epsilon} = \frac{1}{2} m \bar{v}^2 = \frac{3}{2} \frac{PV}{N} = \frac{3}{2} kT \quad (7)
\]

For ideal gas all energy \(E\) is kinetic so:

\[
E = N \bar{\epsilon} = \frac{3}{2} NkT \quad (8)
\]

\(T \propto \text{average K.E. of molecules in an ideal gas.}\)

3 degrees of motional freedom (x, y, z)
i.e. \( \frac{1}{2} kT \) per degree of freedom  
(At ordinary temperatures, \( kT \approx 4 \times 10^{-21} \text{ J} \))

### Molecular Speeds Again:

\[
\frac{1}{2} m \overline{v^2} = \frac{3}{2} kT
\]

\( v_{r.m.s.} = \sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{m}} \)

**Example:** What is the \( v_{rms} \) of \( O_2 \), \( N_2 \) and \( H_2 \) at \( T = 293 \text{ K} \)?

\[
(7) \quad v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT}{N_A \text{ mol wt}}}
\]

For \( O_2 \):

\[
= \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 293 \times 6.02 \times 10^{23}}{0.032}}
\]

\( = 478 \text{ ms}^{-1} \)

For \( N_2 \rightarrow 511 \text{ ms}^{-1} \)  
For \( H_2 \rightarrow 1.91 \text{ kms}^{-1} \)  
C.F. \( v_{escape} = 11 \text{ kms}^{-1} \)

Note that for air, \( v_{rms} > v_{sound} \)

But recall from waves:

\[
V_s = \sqrt{\frac{K_{ad}}{\rho}} = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma kT}{m}}
\]

So \( \frac{v_{rms}}{v_s} = \sqrt{\frac{3}{\gamma}} \approx 1.5 \)  
\( \gamma \) discussed later.

**Example** What is the \( v_{rms} \) due to thermal motion (Brownian motion) of: pollen grain (\( m \sim 10^{-15} \text{ kg} \)) and apple (\( m \sim 0.2 \text{ kg} \))

\[
v_{rms} = \sqrt{\frac{3kT}{m}} \quad \text{(Brownian motion 1st analysed by Einstein, 1904)}
\]

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r \approx \frac{3m}{4\pi \rho_{air}(1 - \rho_{gas}/\rho_{air})}
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\]

*Check approx
Special Cases

1. **Isobaric Process** - $P$ constant
   \[ W = \int P \, dV = P(V_f - V_i) \]
   \[ \therefore \Delta U = \Delta Q - P \Delta V \]
   Example
   \[ \Delta Q = M L \quad (= \text{mass \cdot latent heat}) \]
   \[ \Delta U = ML - P \Delta V \]
   Also, things done at atmospheric pressure

2. **Isochoric** - volume constant
   \[ \therefore W = 0, \quad \therefore \Delta Q = \Delta U \]

3. **Adiabatic Process**: no heat flow, $\Delta Q = 0$
   either fast or insulated
   e.g. compression stroke in engine
   sound wave compression
   most muscle contractions

**Free expansion**: open tap

No work done, Experimentally, find $\Delta Q = 0$
\[ \therefore \Delta U = 0 \]
\[ \therefore U \text{ of ideal gas doesn't depend on } p, \]
i.e. $U = U(T)$

**Special cases of the First Law**
\[ \Delta U = Q - W \]

<table>
<thead>
<tr>
<th>Process</th>
<th>$Q$</th>
<th>$W$</th>
<th>$\Delta U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adiabatic</td>
<td>$Q = 0$</td>
<td>$\therefore \Delta U = -W$</td>
<td>$0$</td>
</tr>
<tr>
<td>Constant volume</td>
<td>$W = \int P dV = 0$</td>
<td>$\therefore \Delta U = Q$</td>
<td>$Q$</td>
</tr>
<tr>
<td>Closed cycle</td>
<td>$\Delta U = 0$</td>
<td>$\therefore Q = W$</td>
<td>$Q$</td>
</tr>
<tr>
<td>Free expansion</td>
<td>$Q = W = 0$</td>
<td>$\therefore \Delta U = 0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

For a gas, specific heat at constant pressure ($c_p$) is greater than specific heat at constant volume ($c_v$).
Why?
The ratio $c_p/c_v$ is called $\gamma$.
\[ PV^\gamma = \text{constant for adiabatic process} \]

*Note: pressure varies more strongly with change in V for adiabatic than for isothermal, where $PV^\gamma = \text{constant.}$*
Example
Compressed gas \((P_1, T_1)\) escapes from a cylinder into the atmosphere \((P_A, T)\). Assume rapid, adiabatic expansion, no initial mixing. How cold does it get?

Adiabatic: \(PV^\gamma = \text{constant}\)

\[
\therefore \quad \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/\gamma}
\]

\[
\therefore \quad \frac{T_2}{T_1} = \frac{P_2V_2}{P_1V_1} = \frac{P_2}{P_1} \left(\frac{V_2}{V_1}\right)^{1/\gamma} = \left(\frac{P_2}{P_1}\right)^{1 - 1/\gamma}
\]

Example. Same problem but for car tyre:
"\(p\)" (gauge pressure) = 180 kPa. Take \(\gamma = 1.40\), \(T = 300\) K. Release some air into atmosphere \((P_2 = P_A = 101\) kPa). What is temperature of emerging air?

\(P_1 = \text{"p"} + P_A = 281\) kPa

\[
\therefore \quad T_f = \ldots\ldots\ldots = 224\text{ K} = -49^\circ\text{C}
\]

Example (a peculiar heat cycle)

\[\begin{align*}
\text{a) Isothermal expansion at } T_H \\
\text{d) heating at const. } V \\
\text{b) cooling at const. } V \\
\text{c) Isothermal compression}
\end{align*}\]

Isotherms: \(P = \frac{nRT}{V}\) \quad \therefore \quad W = \int P \, dV = \ldots = nRT \ln \frac{V_f}{V_i}

\[
\begin{align*}
\Delta U & = \Delta W & & \Delta Q \\
ad & = 0 & & nRT_H \ln \left(\frac{V_2}{V_1}\right) \\
b & = -nC_v(T_H - T_C) & & 0 \\
c & = 0 & & -nRT_C \ln(V_2/V_1) \\
d & = nC_v(T_H - T_C) & & 0 \\
\Sigma & = 0 & & nR(T_H - T_C) \ln \frac{V_2}{V_1}
\end{align*}
\]

efficiency = \frac{\text{work out}}{\text{heat in}} = \left(\frac{R(T_H - T_C) \ln \left(\frac{V_2}{V_1}\right)}{RT_H \ln(V_2/V_1) + C_v(T_H - T_C)}\right)

\{ \text{all terms } > 0 \}

\{ T_H - T_C \leq T_H \} \quad \therefore \text{efficiency } < 100\%

P-V diagrams and thermal cycles

![P-V diagram](image)

Simple cycles typical of those found in tut and exam questions

![Simple cycles](image)

Idealised cycles sometimes approximated

![Idealised cycles](image)

Heat conduction

Reservoirs at $T_H$ and at $T_C$.

$H$ is rate of heat transfer through a material in steady state.

$$H \equiv kA \frac{T_H - T_C}{l}$$

defines the thermal conductivity $k$

$H$ in W, so $k$ in W.m$^{-1}$.K$^{-1}$.

Copper \ 401 \ W.m$^{-1}$.K$^{-1}$
Stainless Steel \ 14
Glass \ 1
Water \ 0.5
Pine (wood) \ 0.11
Dry air \ 0.026

Thermal resistance or R-value sometimes used for building materials

$$R \equiv \frac{l}{k}$$

so $H = \frac{A \Delta T}{R}$

(High conductivity, low R value and vice versa.)
Example. What is the R value of 1 cm pine?

When his Auststudy is cut off, a student lives in a pine packing crate, area 8 m², thickness 1.0 cm. If the shivering student produces 300 W, which is lost by conduction through the crate, how much warmer is it inside the crate?

\[ R \equiv \frac{l}{k} = 0.09 \, \text{K.m}^2\text{W}^{-1} \]

\[ H \equiv kA \frac{T_H - T_C}{l} \quad \text{or} \quad A \frac{\Delta T}{R} \]

\[ T_H - T_C = \frac{lH}{kA} \]

\[ = \frac{0.010 \, \text{m} \times 300 \, \text{W}}{0.11 \, \text{W.m}^{-1}.\text{K}^{-1} \times 8 \, \text{m}^2} = 3 \, \text{K}. \quad \text{Other benefits: wind, rain, radiation} \]

Example. To reduce thermal noise, a low temperature circuit is immersed in liquid nitrogen (77 K, L = 199 kJ.kg⁻¹). It is connected to the outside circuitry by 3 well-insulated copper wires, length \( l = 100 \, \text{mm} \), diameter 0.3 mm. What is the rate of N₂ evaporation due to the heat conducted down the wires?

\[ \text{Power to evaporate N}_2 = \text{heat transfer} \]

\[ L \frac{dm}{dt} = H \equiv kA \frac{T_H - T_C}{l} \]

\[ \frac{dm}{dt} = \frac{kA T_H - T_C}{L} = ... = 90 \, \mu\text{gs}^{-1} = 340 \, \text{mg.hr}^{-1} \]

Example. A flask of coffee, initially at 90 °C, cools to 81 °C in one hour in 20 °C atmosphere. How long will it take to cool to 60 °C?

\[ H = kA \frac{T - T_C}{L} \]

\[ H = - \frac{dQ}{dt} = - mc \frac{dT}{dt} \]

\[ \frac{dT}{dt} = - \frac{kA T - T_C}{mc} \frac{1}{L} \]

\[ \tau = \frac{mcL}{kA} \quad \text{Solve DE} \rightarrow \]

\[ T = T_C + (T_0 - T_C) e^{-t/\tau} \]

\[ 81 \, ^\circ\text{C} = 20 \, ^\circ\text{C} + (70 \, ^\circ\text{C}) e^{-1 \, \text{h}/\tau} \]

\[ \therefore \quad \tau = 7.3 \, \text{h} \quad \ldots \rightarrow 4.1 \, \text{h to cool to 60 °C} \]
A note about steady state \((T\) constant\)
in one dimension, \(H \equiv kA \frac{T_H - T_C}{l}\)
becomes \heat flux \( \equiv \frac{H}{A} = k \frac{dT}{dx} \)
In steady state, \rate of heat arriving \( = \rate of heat departing\)

\[
\begin{array}{c|c}
\text{In steady state, heat flux is uniform. So} & \\
\text{So} & \\
\text{In this situation} & \\
\text{so the middle region receives more heat than it loses – so} & \\
\text{its temperature rises.} & \\
\text{At boundary of vacuum:} & \\
\text{Heat conduction to the surface} & \text{heat radiation from the surface}
\end{array}
\]
Two volumes of same gas. Open tap. What pressure?
Gas redistributes until P equal.
5 unknowns: $N_1$, $N_2$, $N_1'$, $N_2'$, $P$

Before

$$N_1 = \frac{P_1V_1}{RT_1} \quad N_2 = \frac{P_2V_2}{RT_2}$$

After

$$N_1' = \frac{PV_1}{RT_1} \quad N_2' = \frac{PV_2}{RT_2}$$

Conservation:

$$N_1 + N_2 = N_1' + N_2' \quad \text{Solve......}$$