Marking scheme for preliminary test

Question 1

This explanation not required: just the graphs, and the deceleration phase is not required.

Runner b starts at \((t,s) = (0,0)\). She accelerates from rest, so her graph is a parabola, initially with slope zero, and finally with slop \(v\). She then continues with constant \(v\): a straight line on the \(s(t)\) graph. In this phase, her position is the same as that of runner a, so their \(s(t)\) are identical for a straight line segment (during which the baton is passed).

(Runner a then decelerates (a parabola with negative acceleration) until her velocity is zero \((s(t)\) horizontal).

4 marks for a clear diagram

iii) Runner a has displacement \(s_a = vt - d\)

v) Runner b has displacement \(s_b = \frac{1}{2}at^2\) during acceleration phase

vi) Runner b accelerates at \(a\) from rest to final speed \(v\) in time \(T\) where \(v = aT\), so \(T = \frac{v}{a} = 4\) s.

vii) at \(t = T\), \(s_b = s_a\)

\[
\frac{1}{2}aT^2 = vT - d
\]

\[
d = vT - \frac{1}{2}aT^2
\]

\[
vT - \frac{1}{2}aT^2 = \frac{1}{2}v^2 - \frac{1}{2}v^2/a = \frac{1}{2}v^2/a
\]

\[
d = 16\ m.
\]
Question 2 (14 marks)

i) No acceleration so $\Sigma$forces = 0. Resolve forces in the directions normal to and in the plane.

In the normal: $N = W \cos\theta + F \sin\theta$

In the plane: $F \cos\theta = W \sin\theta + F_f$

$F \cos\theta = W \sin\theta + \mu_k N$

Eliminate $N$:

$F \cos\theta = W \sin\theta + \mu_k (W \cos\theta + F \sin\theta)$

$F = mg \frac{\sin\theta + \mu_k \cos\theta}{\cos\theta - \mu_k \sin\theta}$

ii) Let $ds$ be the displacement up the plane:

Power $= \frac{d\text{(work)}}{dt} = \frac{F \cdot ds}{dt}$

$= \frac{F \cos\theta \cdot ds}{dt} = Fv \cos\theta$

iii) If there is no friction force in the plane of the diagram, then the horizontal acceleration $a$ satisfies

$ma = N \sin\theta$

There is no vertical acceleration so

$W = N \cos\theta$

and the centripetal acceleration required is $v^2/R$, so

$m \frac{v^2}{R} = N \sin\theta = \frac{W}{\cos\theta} \sin\theta = mg \tan\theta$

$\theta = \tan^{-1} \left( \frac{v^2}{Rg} \right)$