Particle dynamics Physics 1A, UNSW

Newton's laws: S & J: Ch 5.1 – 5.9, 6.1

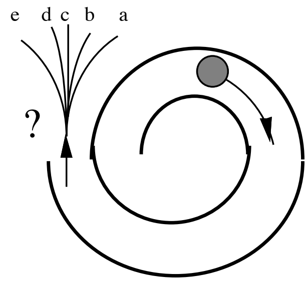
force, mass, acceleration also weight Physclips Chapter 5

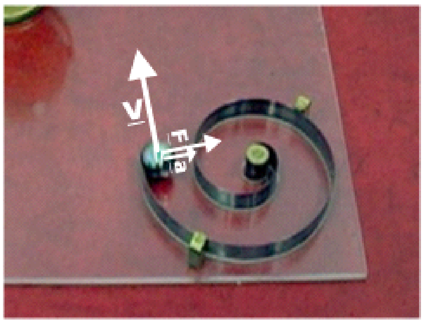
Friction - coefficients of friction Physclips Chapter 6

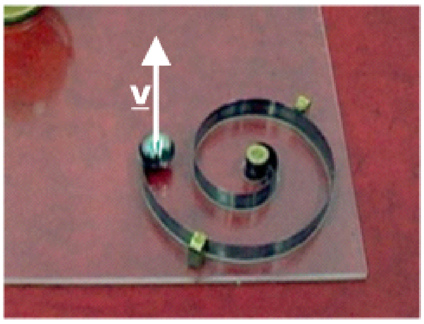
Hooke's Law

Dynamics of circular motion

Question. Top view of ball. What is its trajectory after it leaves the race?







then what?

Aristotle: $\underline{v} = 0$ *is "natural" state* *(not in syllabus)*

Galileo & Newton: $\underline{a} = 0$ *is "natural" state*

Galileo: what if we remove the side of the bowl?

Newton's Laws

First law "zero (total) force ⇒ zero acceleration"

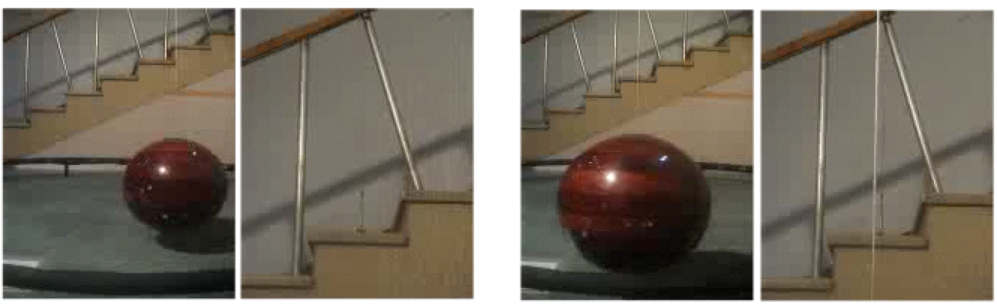
(It's actually a bit more subtle. More formally, we should say:

If $\Sigma \underline{\mathbf{F}} = 0$, there exist reference frames in which $\underline{\mathbf{a}} = 0$

called **Inertial frames**

What is an inertial frame? One in which Newton's laws are true.

- observation: w.r.t. these frames, distant stars don't accelerate



Experiment in the foyer

In inertial frames:

Second law $\Sigma\,\mathbf{\underline{F}} = m\,\mathbf{\underline{a}}$ *Σ is important: it is the **total force** that determines acceleration*

$\Sigma F_x = ma_x$ $\Sigma F_y = ma_y$ $\Sigma F_z = ma_z$ *3D –> 3 equations*

1st law is special case of 2nd What does the 2nd law mean?

$\Sigma\,\mathbf{\underline{F}} = m_i\,\mathbf{\underline{a}}$ and $\mathbf{\underline{W}} = m_g\,\mathbf{\underline{g}}$

are m_i and m_g necessarily the same?

called inertial and gravitational masses

$\mathbf{\underline{F}} = m\,\mathbf{\underline{a}}$

$\mathbf{\underline{a}}$ is already defined, but this leaves us with a puzzle:

i) Does this equation define m?

ii) Does this law define $\mathbf{\underline{F}}$?

iii) Is it a physical law?

iv) All of the above?

v) How?

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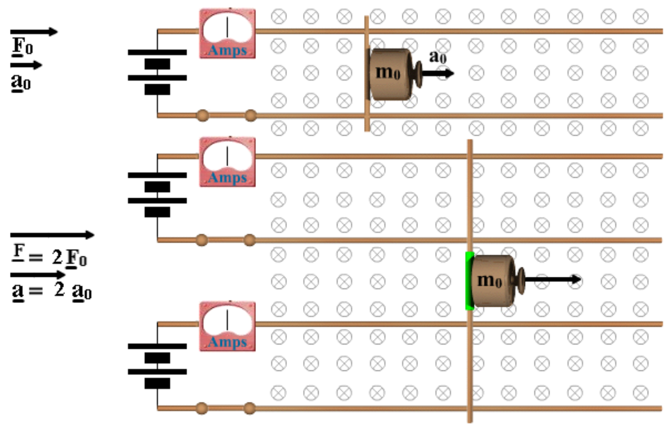
i) Given one mass, we could calibrate many forces by measuring the $\mathbf{\underline{a}}$ they produced.

ii) Similarly, for any one $\mathbf{\underline{F}}$, we could calibrate many m's by the accelerations produced

iii) The 2nd Law is the observation that the m's and F's thus defined are consistent. eg

Having used standard m to calibrate $\mathbf{\underline{F}}$, now produce 2$\mathbf{\underline{F}}$ (eg two identical F systems).

Is $\mathbf{\underline{a}}$ now doubled? Every such experiment is a test of Newton's second law.



Or, for those who want it logically:

***Ne*Newton 1**: "Every body persists in its state of rest or of uniform motion in a straight line unless it is compelled to change that state by forces impressed on it." *postulate*

An **inertial frame** of reference is one in which Newton's 1st law is true. *definition*

Such frames exist (and with respect to these frames, distant stars don't accelerate) *observation*:

$\therefore$ if $\Sigma\,\mathbf{\underline{F}} = 0$, $\mathbf{\underline{a}} = 0$ w.r.t. distant stars.

Force causes acceleration. $\mathbf{\underline{F}} \,//\, \mathbf{\underline{a}}$, $\mathbf{\underline{F}} \propto \mathbf{\underline{a}}$

Another way of writing **Newton 2**: To any body may be ascribed a (scalar) constant, mass, such that the acceleration produced in two bodies by a given force is inversely proportional to their masses,

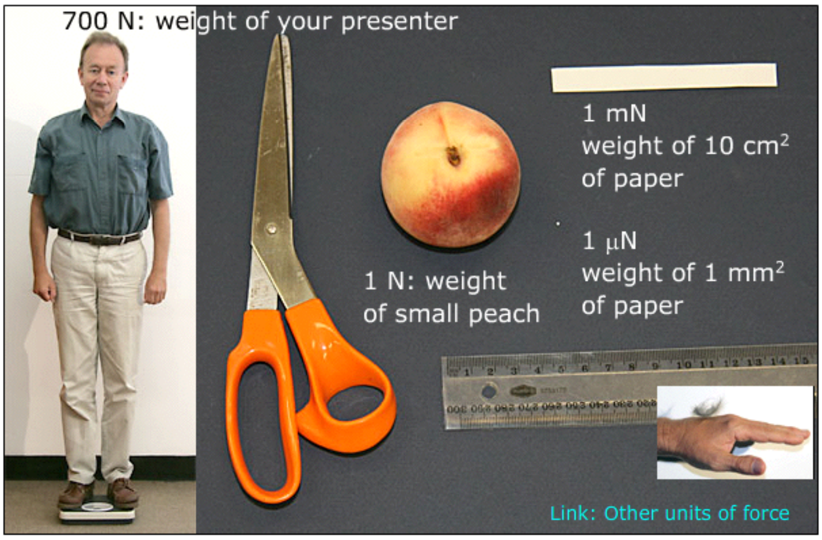
i.e. for same F, $\frac{m_2}{m_1} = \frac{a_1}{a_2}$

We already have metre, second, choose a standard body for kg, then choose units of F (Newtons) such that

$\Sigma \underline{\mathbf{F}} = m\,\underline{\mathbf{a}}$ **Newton's first and second laws**

(this eqn. is laws 1&2, definition of mass and units of force) So, how big are Newtons?

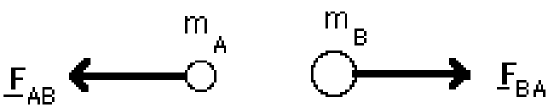
Mechanics > Newton's laws > 5.2 Second law



Newton 3: "To every action there is always opposed an equal reaction; or the mutual actions of two bodies upon each other are always equal and directed to contrary parts"

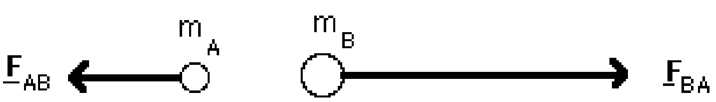
Or

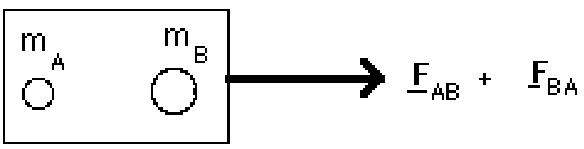
Forces always occur in pairs, $\underline{\mathbf{F}}$ and $-\underline{\mathbf{F}}$, one acting on each of a pair of interacting bodies.



Third law $\underline{\mathbf{F}}_{AB} = -\underline{\mathbf{F}}_{BA}$

Why so? What would it be like if internal forces *didn't* add to zero?

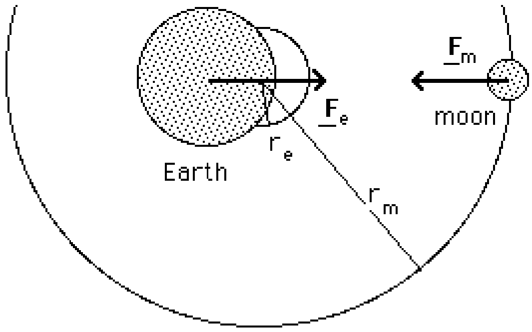




Important conclusion: internal forces in a system add to zero. So we can now write the 1st and 2nd laws:

$\Sigma \underline{\mathbf{F}}_{external} = m\,\underline{\mathbf{a}}$ *Total **external** force* = m $\underline{\mathbf{a}}$

Example Where is centre of earth-moon orbit?



$|F_e| = |F_m| = |F_g|$ equal & opposite *NB sign conventions*

each makes a circle about common centre of mass

$F_g = m_m a_m = m_m \omega^2 r_m$

$F_g = m_e a_e = m_e \omega^2 r_e$

$\therefore \quad \frac{r_m}{r_e} = \frac{m_e}{m_m} = \frac{5.98\ 10^{24}\ \text{kg}}{7.36\ 10^{22}\ \text{kg}} = 81.3 \qquad \text{(i)}$

earth-moon distance $r_e + r_m = 3.85\ 10^8$ m (ii) (two equations, two unknowns)

$r_e\ (1 + 81.3) = 3.85\ 10^8$ m gives $r_m = 3.80\ 10^8$ m, $r_e = 4.7\ 10^6$ m = 4700 km

$\therefore$ centre of both orbits is inside earth *(later we'll see that it is the centre of mass of the two)*

Using Newton's laws

How do we use them to solve problems?

Newton's 2nd $\Sigma \underline{\mathbf{F}} = m\underline{\mathbf{a}}$ the Σ is important: in principle, we have to consider them all.

Newton's 3rd (forces come in pairs, $\underline{\mathbf{F}}$ and $-\underline{\mathbf{F}}$), so:

- internal forces add to zero. They don't affect motion

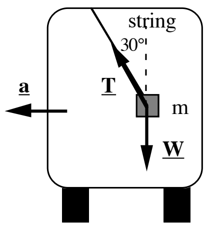
Therefore, applying Newton's 2nd, we use $\Sigma \underline{\mathbf{F}}_{\text{external}} = m\underline{\mathbf{a}}$

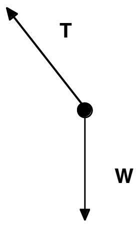
- draw diagrams ('**free body diagrams**') to show only the external forces on the body of interest

Newton's second law is a vector equation

- write components of Newton's second law in 2 (or 3) directions

Example. As the bus takes a steady turn with radius 8 m at constant speed, you notice that a mass on a string hangs at 30° to the vertical. How fast is the bus going? *Draw a diagram with physics*





We know:
tension in direction of string,
weight down,
acceleration horizontal
circular motion

The mass (and the bus) are in circular motion, so we can apply Newton 2:

$$F_{\text{horiz}} = ma = m\frac{v^2}{r}$$

(the acceleration is centripetal)

Only the tension has a horizontal component, so

$$T \sin 30° = m\frac{v^2}{r} \qquad \text{(i)}$$

Need one more eqn: mass is not falling down, ie

vertical acceleration = 0, so

$$T \cos 30° = mg \qquad \text{(ii)}$$

Again, we have two equations in two unknowns

Dived (i) by (ii) to eliminate T:

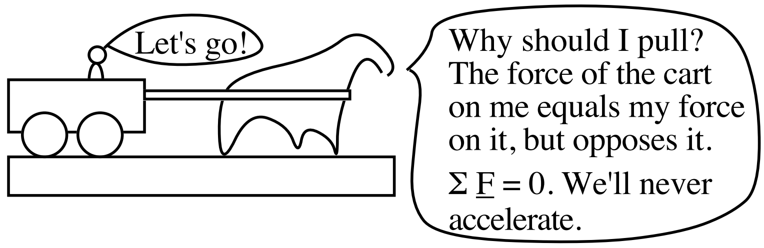
$$\tan 30° = m\frac{v^2}{r}\frac{1}{mg}$$

which we rearrange to give

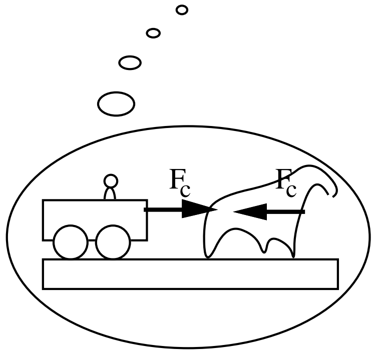
$$\to \quad v = \sqrt{gr \tan 30°} = 6.7 \text{ m/s} \to 20 \text{ kph.}$$

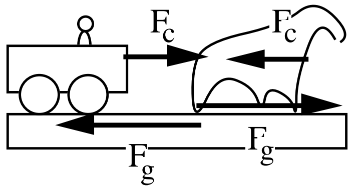
Don't stop yet! First, check the dimensions. Reasonable? Suggestions? Problems?

Problem. Horse and cart. Wheels roll freely.



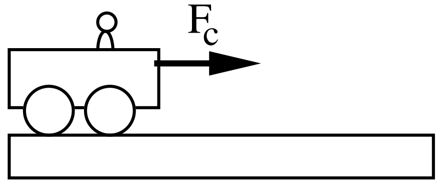
What would you say to the horse?





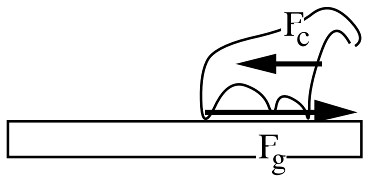
This is a good exercise in looking at the ***external forces***

Horizontal forces on cart (mass m_c)



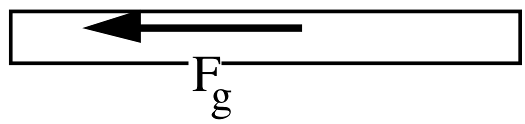
$F_c = m_c a_c = m_c a$

Horizontal forces on horse (mass m_h)



$F_g - F_c = m_h a$

Horizontal forces on Earth (mass m_E)

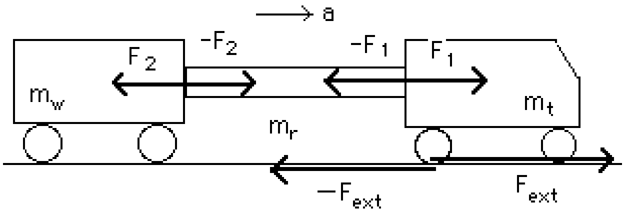


$m_E \gg m_h + m_c$

In principle, the earth accelerates to the left. But $m_E \gg m_h$ so $a_E \ll a_h$

"light" ropes etc. *Here, light means m << other masses*

Truck (m_t) pulls wagon (m_w) with rope (m_r). All have same **a** .



Let's apply Newton's second to each element:

i) wagon: $-F_2 = m_w a.$

ii) rope: $F_1 - F_2 = m_r a$

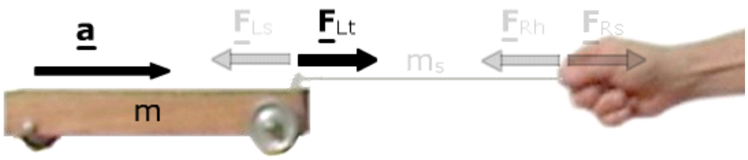
iii) truck: $-F_1 + F_{ext} = m_t a$

(ii)/(i) -> $\dfrac{F_1 - F_2}{-F_2} = \dfrac{m_r a}{m_w a}$

important result:

$\therefore$ if $m_r \ll m_w$, $F_1 = F_2$.

Forces at opposite ends of light ropes etc are equal and opposite.



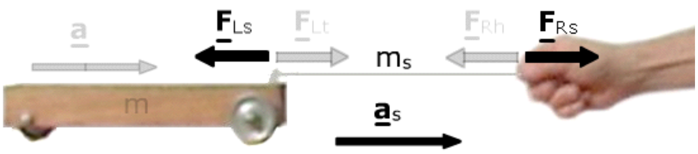
$|\mathbf{F}_{Lt}| = |\mathbf{F}_{Ls}| = F_L$

$|\mathbf{F}_{Rt}| = |\mathbf{F}_{Rs}| = F_R$

$F_L = ma$ (i)

$\Sigma F \text{ (to right)} = F_R - F_L$

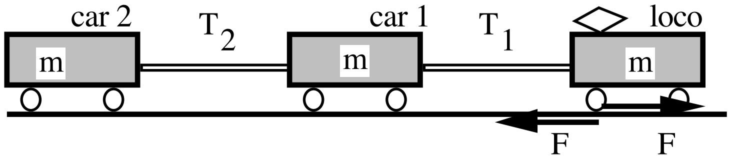
$F_R - F_L = m_s a_s$ (ii)



Again, see Physclips if this isn't clear

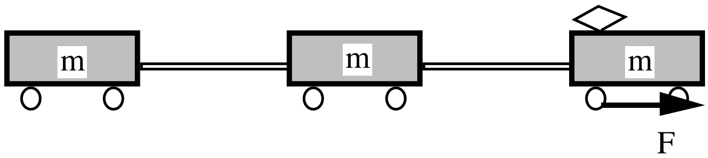
Tension. When the mass of a string, coupling etc is negligible, forces at opposite ends are equal and opposite and we call this the **tension** in the string.

Example. (one dimensional motion only) Consider a train. Wheels roll freely. Locomotive exerts horizontal force F on the track. What are the tensions T_1 and T_2 in the two couplings?



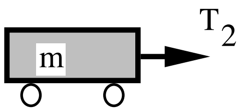
Whole train accelerates together with a.

Look at the external forces acting on the train (horiz. only).



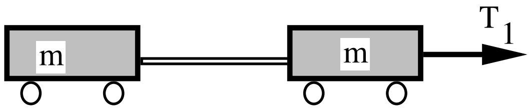
$$F = (m+m+m)a \rightarrow \quad a = F/3m$$

Look at external horiz forces on car 2:



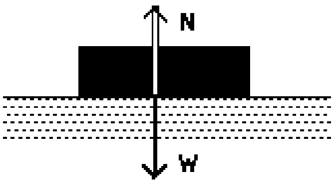
$$T_2 = ma = F/3$$

and on cars 2 and 1 together



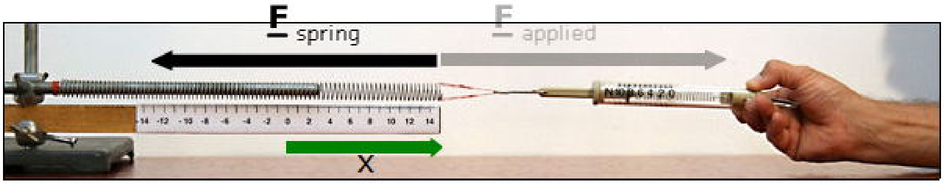
$$T_1 = 2ma$$

Newton's 3rd:



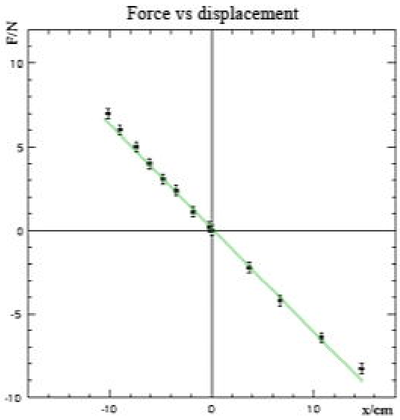
A question for you: How does floor "know" to exert N = W = mg?

Hookes law



From Physclips Ch 6.3

Mechanics > Weight and contact forces > 6.3 Hooke's law



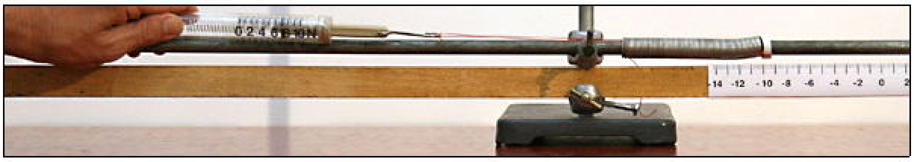
Empirical law:

$$F = -kx$$

where k is the **spring constant**

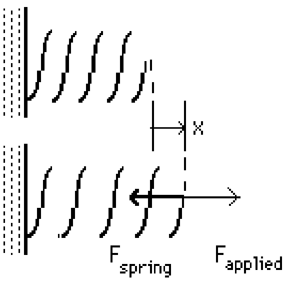
k has units of $N.m^{-1}$.

$k = 61 \pm 1\ N.m^{-1}$



Hooke's Law:

	Strain	proportional to	stress
i.e.	fractional deformation	$\propto$	$\frac{\text{Force}}{\text{area}}$



No applied force

(x = 0)

Under tension

x > 0

Note that $\underline{\mathbf{F}}_{spring}$ is in the *opposite* direction to x.

Experimentally (see Physclips example above), $|\underline{\mathbf{F}}_s| \propto |x|$ over small range of x

$$F = -kx \qquad \textbf{Hooke's Law.}$$

linear elastic behaviour - more in S2.

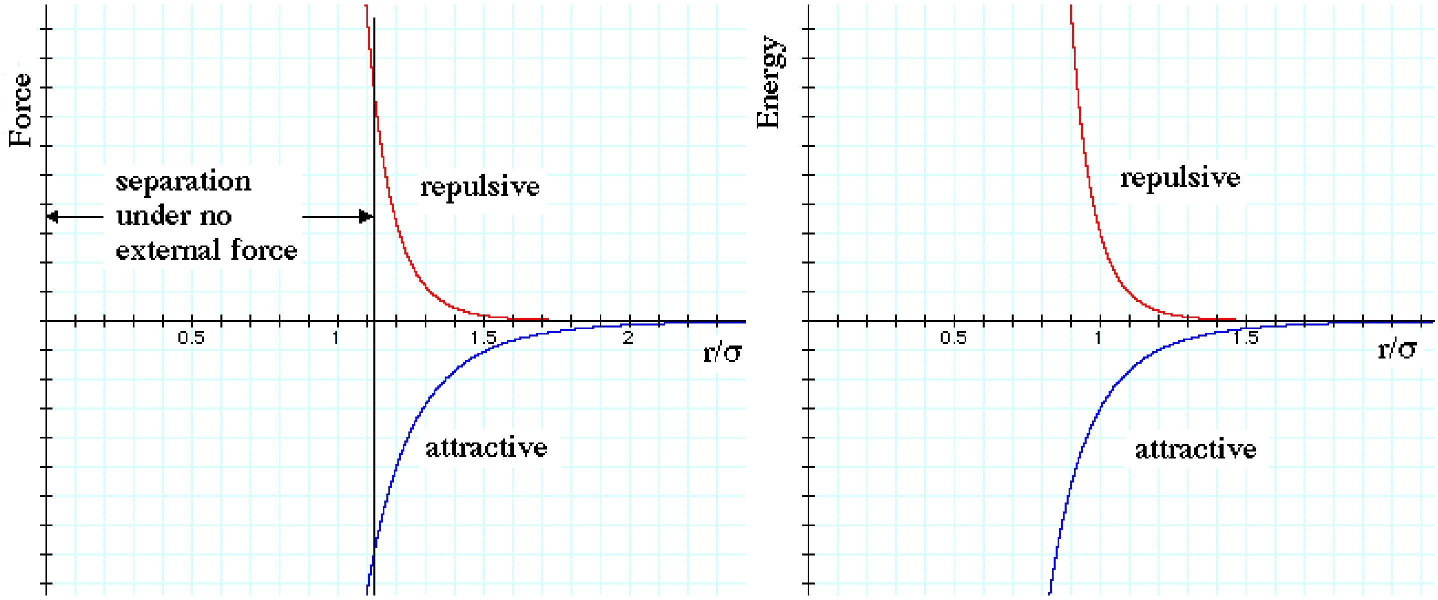
Linear approximations are v. useful!

www.animations.physics.unsw.edu.au/jw/elasticity.htm

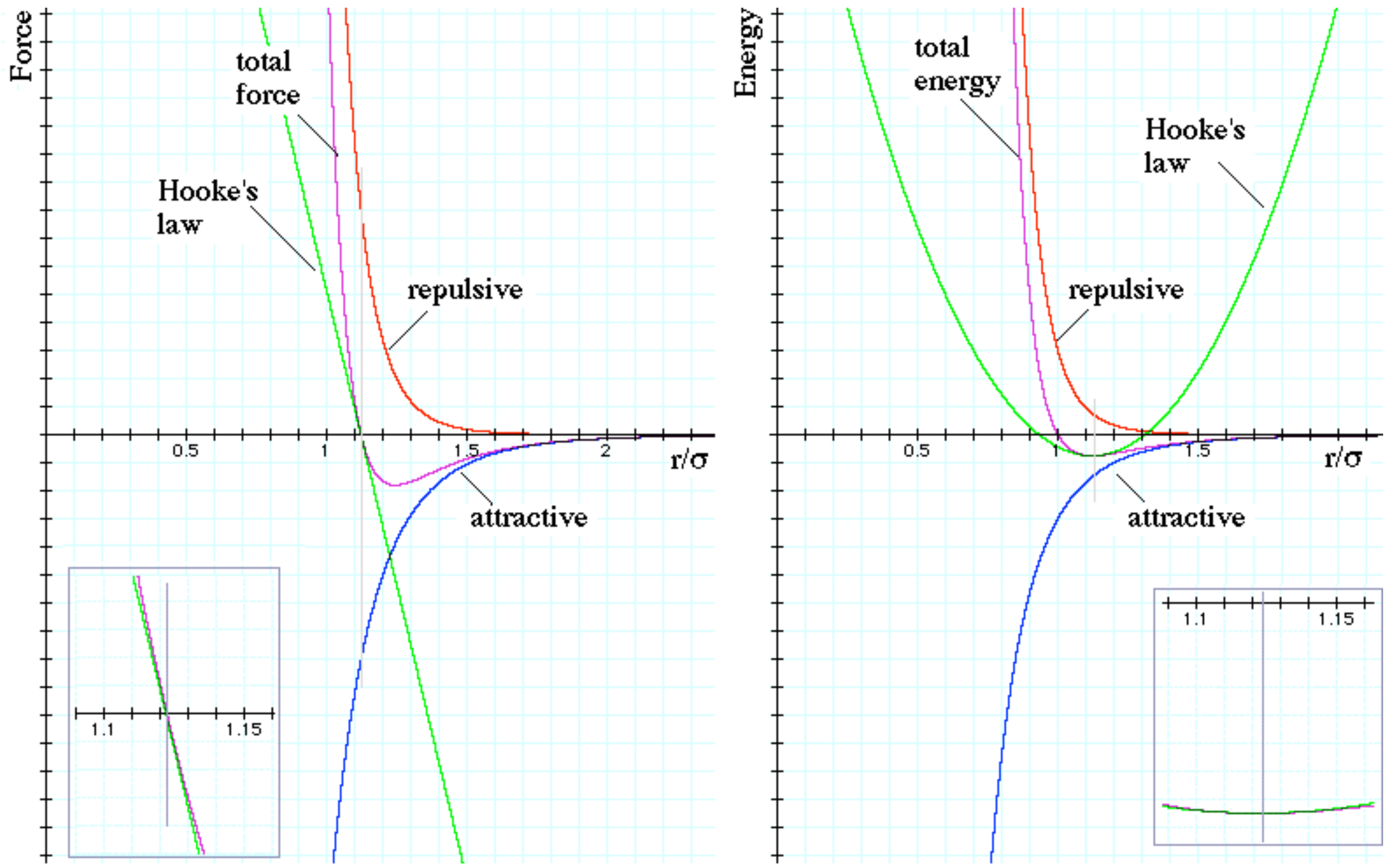
Why is linear elasticity so common?

Consider the intermolecular forces F and energies U and separate into attractive and repulsive components:

See homework problem on interatomic forces



We'll do energy later, and we'll see:



Mass and weight

(inertial) mass m defined by F = ma

observation:

near earth's surface and without air,

all bodies fall with same a (= –g) (*so far. This is still subject to experimental test!*)

weight W = – mg

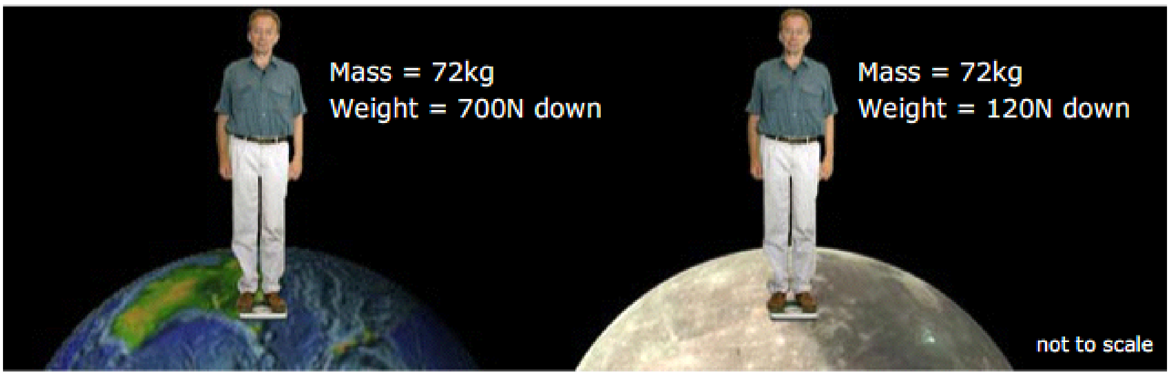
What is your weight?

Mechanics > Weight and contact forces > 6.2 Weight versus mass

mass determines "resistance to accelerate" (inertia)

$$m \equiv \frac{\underline{\mathbf{F}}_{\text{total}}}{\underline{\mathbf{a}}}$$ in kilograms (kg)
or slugs* (Liberia; Myanmar; USA)

$\underline{\mathbf{W}}$ = mg (down) in newtons (N)
or pounds force* (Liberia; Myanmar; USA)



Warning: do not confuse mass and weight, or their units

kg –> mass N –> force ($kg.m.s^{-2}$)

kg wt = weight of 1 kg = mg = 9.8 N

Why is $W \propto m$? *Why is $m_g \propto m_i$?*

ma = F = W = (Grav field).(grav. property of body)

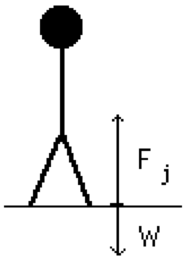
- Mach's Principle
- Principle of General Relativity
- Interactions with vacuum field,

not in our syllabus, but v. interesting questions

Example Grav. field on moon $g_m = 1.7\ \mathrm{ms}^{-2}$. An astronaut weighs 800 N on Earth, and, while jumping, exerts 2kN while body moves 0.3 m. What is his weight on moon? How high does he jump on earth and on moon?

$$mg_E = W_E \rightarrow \quad m = \frac{800\ \mathrm{N}}{9.8\ \mathrm{ms}^{-2}} = 82\ \mathrm{kg}$$

$$W_m = mg_m = 82\mathrm{kg}\ 1.7\mathrm{ms}^{-2} = 140\ \mathrm{N}$$



Vertical (y) motion with constant acceleration. While feet are on ground,

$\Sigma F = 2\ \mathrm{kN} - W_E$

$= 1.2\ \mathrm{kN}$ (Earth)

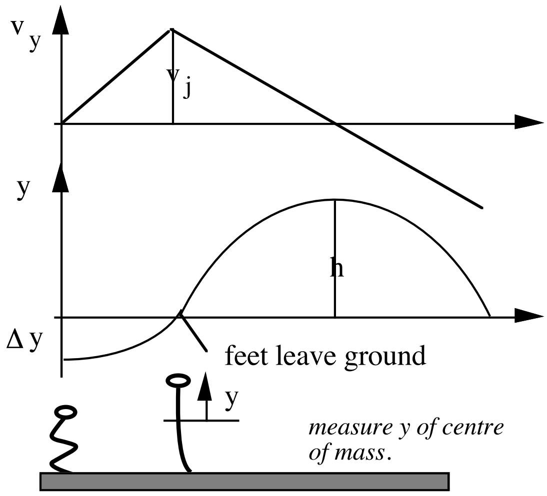
Moon:

$\Sigma F = 2\ \mathrm{kN} - mg_m = 1.9\ \mathrm{kN}$

Jump has two parts:

feet on ground $\left(a = \dfrac{\Sigma F}{m}\right)$ $\quad v_i = 0, v_f = v_j$

feet off ground $a = -g$ $\quad v_i = v_j, v_f = 0$



While on ground:

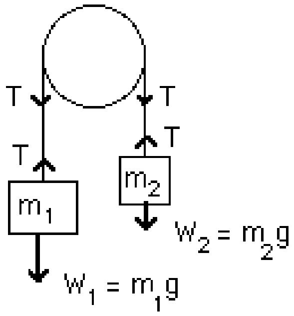
$$v_j^2 - v_o^2 = 2a_j \Delta y = 2\frac{\Sigma F}{m}\Delta y$$

Earth $\rightarrow v_j = 3.0\ \mathrm{ms}^{-1}$ $\qquad$ Moon $\rightarrow v_j = 3.7\ \mathrm{ms}^{-1}$

While above ground:

$$v^2 - v_j^2 = -2gh \quad \rightarrow \quad h = \frac{v_j^2}{2g}$$

$h_E = 0.5$ m. $\quad h_m = 4$ m



Example (an important problem)

Light pulley, light inextensible string. What are the accelerations of the masses?

A number of different conventions are possible. The important thing is to define yours carefully and use it consistently.

Let a be acceleration (down) of m_1 = acceleration (up) of m_2.

Take up as positive, look at the free body diagram for m_1: we are only interested in the vertical direction. By assumption, the acceleration is down. T is up. m_1g is down.

So Newton 2 for m_1 gives: $T - m_1g = -m_1a$

And Newton 2 for m_2: $T - m_2g = +m_2a$

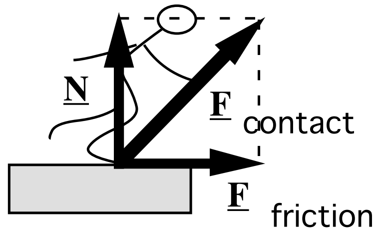
subtract: $-m_1g + m_2g = -m_1a - m_2a$

$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

(Check: if $m_1 = m_2$, $a = 0$. If $m_2 = 0$, $a = g$.)

Alternatively, we might have said

a_1 be the acceleration up of m_1 and a_2 be the acceleration of m_2. Inextensible string so $a_1 = a_2$. etc.

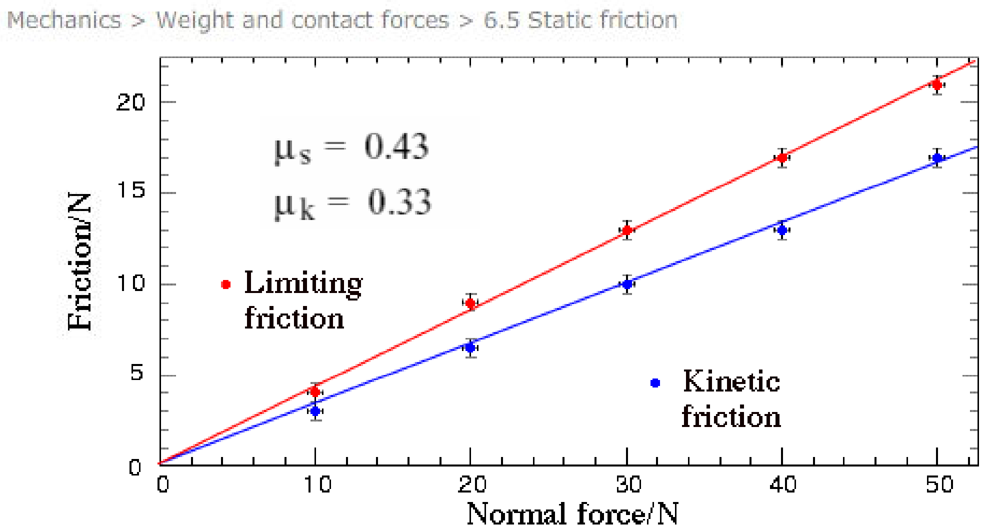


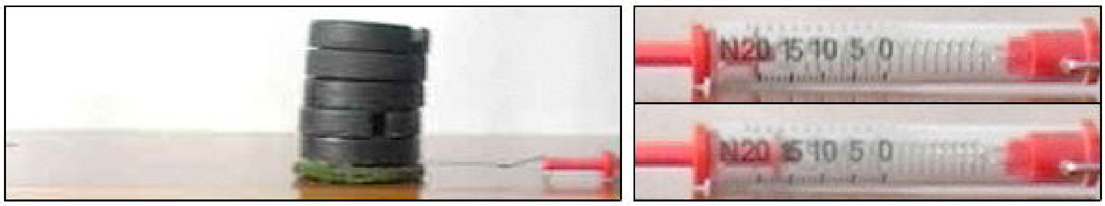
Contact forces

The normal component of a contact force is called the **normal force** **N**. The component in the plane of contact is called the **friction force** $\underline{\mathbf{F}}_f$.

Normal force: at right angles to surface, is provided by deformation.

- If there is relative motion, **kinetic** friction (whose direction opposes relative motion)
- If there is *no* relative motion, **static** friction (whose direction opposes applied force)





See the experiment on Physclips Ch 6.5

Define coefficients of kinetic (k) and static (s) friction:

$|F_f| = \mu_k N$ $\qquad\qquad$ $|F_f| \leq \mu_s N$ $\qquad\qquad$ *n.b.:* $\leq$

Friction follows this **approximate** empirical law

μ_s and μ_k are approx. independent of N and of contact area.

Often $\mu_k < \mu_s$.

(It takes less force to keep sliding than to start sliding.)

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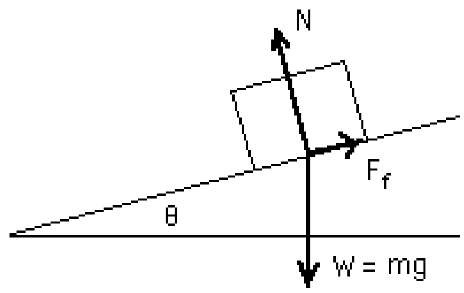


Example. θ is gradually increased to θ_c when sliding begins. What is θ_c? What is a at θ_c? (*as before, free body diagram, N2:*)

Newton 2 in normal direction:

$$N - mg\cos\theta = 0 \qquad \text{(i)}$$

Newton 2 in direction down plane:

$$mg\sin\theta - F_f = ma. \qquad \text{(ii)}$$

No sliding: $a = 0$ so total force is zero.

$\therefore$ (ii) $\Rightarrow$ $mg\sin\theta = F_f \leq \mu_s N$

(i) $\Rightarrow$ $= \mu_s mg\cos\theta$

$mg\sin\theta \leq \mu_s mg\cos\theta$

$\tan\theta \leq \mu_s$, $\theta_c = \tan^{-1}\mu_s$ *useful technique for μ_s*

Sliding at $\theta = \theta_c$: $a > 0$

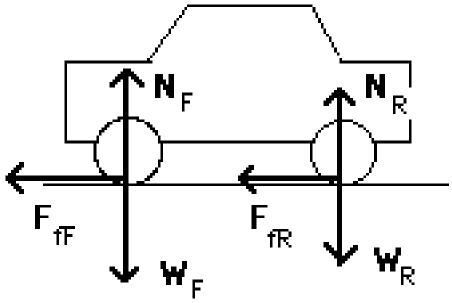
$$\therefore \text{(ii)} \Rightarrow a = g\sin\theta_c - \frac{F_f}{m}$$

$$= g\sin\theta_c - \frac{\mu_k N}{m}$$

(i) $\Rightarrow$ $= g\sin\theta_c - \mu_k g\cos\theta_c$

we had $\theta_c = \tan^{-1}\mu_s$ so

$$a = g\cos\theta_c(\mu_s - \mu_k)$$



Example. Rear wheel drive car, 3 kN weight on **each** front wheel, 2 kN on rear. Rubber-road:

$\mu_s = 1.5$, $\mu_k = 1.1$

(mass of car)*g = weight = 2(3 kN + 2 kN) = 10 kN

m = 1 tonne

Neglect rotation of car during accelerations. Assume that brakes produce 1.8 times as much force on front wheels as on back (*why? Look at brake discs and pads*). (i) What is max forward acceleration without skidding? What is maximum deceleration for (ii) not skidding? (iii) 4 wheel skid?

(i) $F_{fRs} \leq \mu_s N_R = \mu_s W_R$ on each rear wheel

$= 1.5 \text{x} 2 \text{ kN} = 2.9 \text{ kN}$ on each rear wheel

$$a_{max} = \frac{F}{m} = \frac{2 * 2.9\text{kN}}{1000\text{kg}} = 5.8 \text{ ms}^{-2}$$

Stopping.

For all wheels, $F_{fs} \leq \mu_s N = \mu_s W$

$F_{fF} = 1.8\ F_{fR}$. $\mu_s W_F = 1.5\ \mu_s W_R$,

$\therefore$ front wheels skid first, when $F_{fF} > \mu_s W_F$. 2 front and 2 rear wheels.

max total friction = (front + rear) = $\left(2 + \frac{2}{1.8}\right) .\mu_s W_F$ (brakes -> 1.8 times as much force on front wheels)

= 1400 kg.wt = 14 kN

ii) $a_{max} = \frac{F_{max}}{m} = \frac{14 \text{ kN}}{1000 \text{ kg}} = 14 \text{ ms}^{-2}$

iii) $a = \frac{\Sigma F_k}{m} = \frac{\Sigma \mu_k W}{m} = ... = 11 \text{ ms}^{-2}$

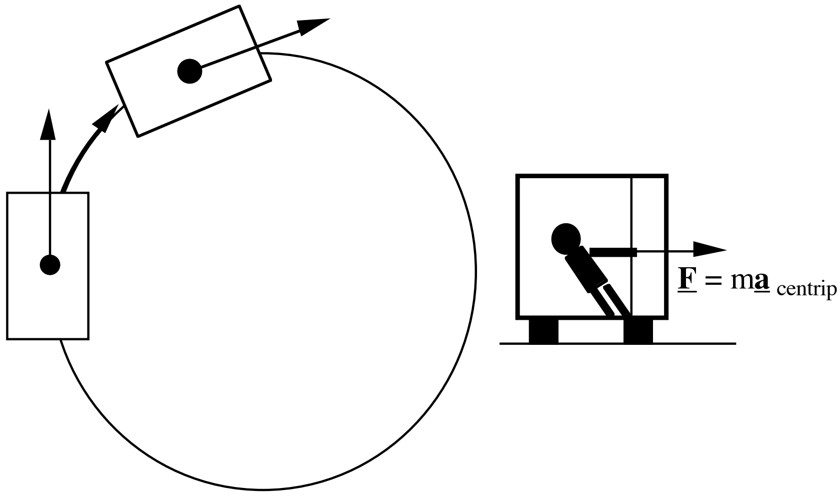
Questions:

Does area of rubber-road contact make a difference?

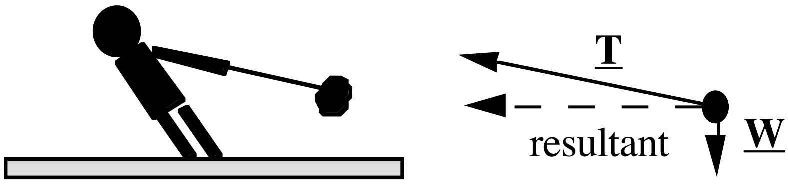
Does the size of the tire make a difference?

Centripetal acceleration and force

Circular motion with ω = const. and v const. eg bus going round a corner



Or consider a hammer thrower

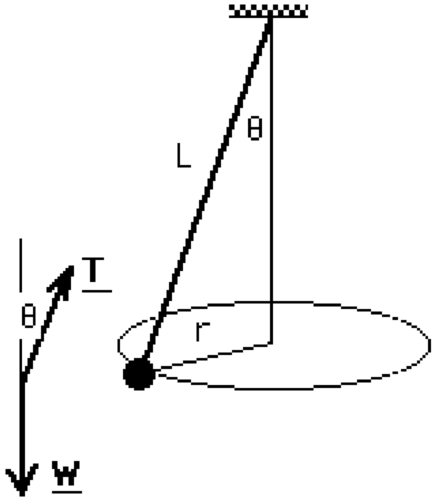


Resultant force produces acceleration in the horizontal direction, towards the centre of the motion

Centripetal force, centripetal acceleration

Example Conical pendulum. (Uniform circular motion.) What is the frequency?

Free body diagram



Apply Newton 2 in two directions:

Vertical: $a_y = 0 \quad \therefore \quad \Sigma F_y = 0$

$\therefore \quad T \cos\theta - W = 0$

$$T = \frac{mg}{\cos\theta}$$

Horizontal:

$$\frac{mv^2}{r} = ma_c = T \sin\theta$$

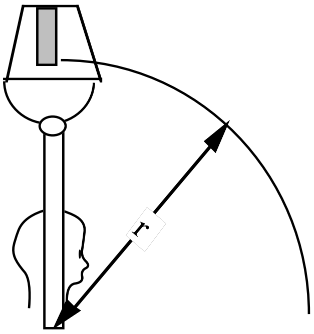
$$= \frac{mg \sin\theta}{\cos\theta}$$

$$\therefore \quad \frac{v^2}{r} = g \tan\theta$$

$$\therefore \quad v = \sqrt{rg \tan\theta}$$

$$\therefore \quad \frac{2\pi r}{\text{period}} = \sqrt{rg \tan\theta}$$

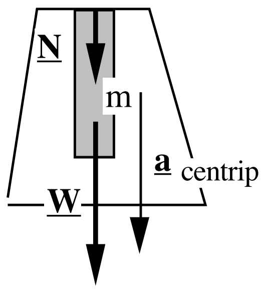
$$\therefore \quad f = \frac{1}{\text{period}} = \frac{1}{2\pi}\sqrt{\frac{g \tan\theta}{r}}$$



Example. Foolhardy lecturer swings a bucket of bricks in a vertical circle. How fast should he swing so that the bricks stay in contact with the bucket at the top of the trajectory?

Draw diagram & identify important variables

pose question mathematically.



W and **N** provide centripetal force.

$mg + N = ma_c$

For contact, we need

$N \geq 0$

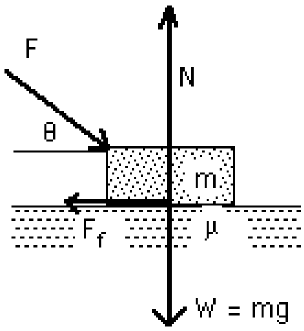
so $ma_c \geq mg$

how to express a_c?

$$a_c = \frac{v^2}{r} = r\omega^2 = r\left(\frac{2\pi}{T}\right)^2$$ *T is easy to measure*

$$T = 2\pi\sqrt{\frac{r}{a_c}} \leq 2\pi\sqrt{\frac{r}{g}}$$

$r \sim 1\text{m} \rightarrow T \leq 2$ s.



Example.

Apply force F at θ to horizontal. Mass m on floor, coefficients μ_s and μ_k. For any given θ, what F is required to make the mass move?

Eliminate 2 unknowns N and F_f –> $F(\theta, \mu_s, m, g)$

Stationary if $\quad F_f \le \mu_s N \qquad (1)$

Newton 2 vertical: $\quad N = mg + F\sin\theta \quad (2)$

Newton 2 horizontal: $\quad F\cos\theta = F_f \qquad (3)$

(1,3) –> stationary if $F\cos\theta \le \mu_s N$

$$F\cos\theta \le \mu_s(mg + F\sin\theta) \qquad \textit{(using (2))}$$

$$F\left(\cos\theta - \mu_s\sin\theta\right) \le \mu_s mg \qquad (*)$$

note importance $\left(\cos\theta - \mu_s\sin\theta\right)$

if $\left(\cos\theta - \mu_s\sin\theta\right) = 0,$ (F *very* large)

$$\theta = \theta_{crit} = \tan^{-1}(1/\mu_s).$$

If $\theta < \theta_c$, then $\left(\cos\theta - \mu_s\sin\theta\right) > 0$

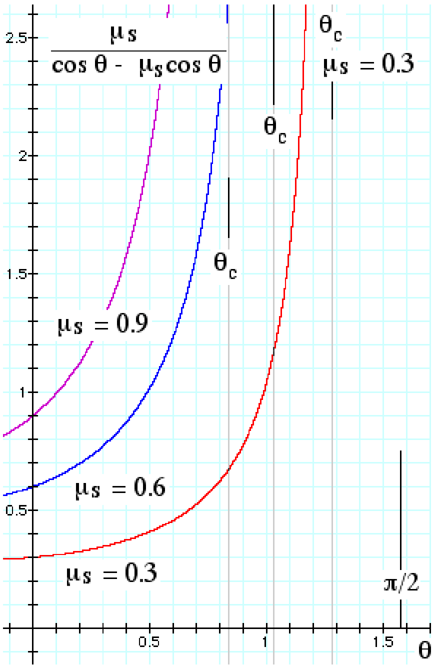
stationary if $\quad F \le \dfrac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$

i.e. moves when $F > F_{crit} = \dfrac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$

What if $\left(\cos\theta - \mu_s\sin\theta\right) = 0$?

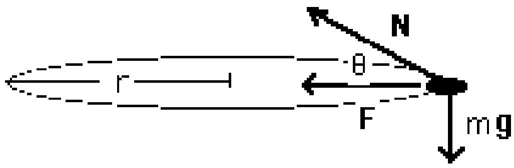
$\theta = \theta_c, \Rightarrow$?

(*) $\Rightarrow$ stationary if $\quad F \le \dfrac{\mu_s mg}{\cos\theta - \mu_s\sin\theta}$ i.e. stationary no matter how large F becomes.



$\frac{\mu_s}{\cos\theta - \mu_s \cos\theta}$
θ_c
$\mu_s = 0.3$
θ_c
θ_c
$\mu_s = 0.9$
$\mu_s = 0.6$
$\mu_s = 0.3$
$\pi/2$
θ
2.5
2
1.5
1
0.5
0.5
1
1.5

Example Plane travels in horizontal circle, speed v, radius r. For given v, what is the r for which the normal force exerted by the plane on the pilot = twice her weight? What is the direction of this force?



Centripetal force $F = m\frac{v^2}{r} = N\cos\theta$

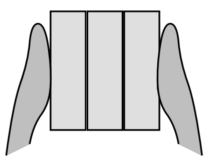
Vertical forces: $N\sin\theta = mg$

eliminate θ: $N^2 = m^2\left(\frac{v^4}{r^2} + g^2\right)$

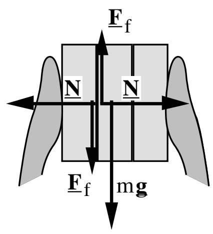
$$\left(\frac{N^2}{m^2} - g^2\right) = \frac{v^4}{r^2} \quad \text{–>} \quad r = \frac{v^2}{\sqrt{\frac{N^2}{m^2} - g^2}}$$

$$\sin\theta = \frac{mg}{N} = \frac{1}{2}$$

∴ 30° above horizontal, towards axis of rotation



Question. Three identical bricks. What is the minimum force you must apply to hold them still like this?



Vertical forces on middle brick add to zero:

$$2F_f = mg$$

Definition of μ_s

$$F_f \le \mu_s N$$

$$\therefore \quad N \ge \frac{F_f}{\mu_s} = \frac{mg}{2\mu_s}$$

Bricks not accelerating horizontally, so normal force from hands = normal force between bricks.

∴ (each) hand must provide $\ge \frac{mg}{2\mu_s}$ horizontally.

Vertically, two hands together provide 3mg.