**Question 1**

A tank is in the shape of a cube, length $L_0$ at temperature $T_0$. Linear coefficient of thermal expansion $\alpha$. It contains air with molar mass $m$. Its pressure remains constant at $P_A$.

i) Derive an expression for $V(T)$. Using $\alpha(T-T_0) \ll 1$, make your expression a linear function of temperature. Express it as a fractional increase in volume, $\Delta V/V_0$.

ii) From the equation of state, derive an expression for $\rho(T)$, the density of air in the tank as a function of temperature. Express your answer as a fractional increase in volume, $\Delta \rho/\rho_0$, where $\rho_0$ is the volume at $T_0$. Simplify the expression.

iii) Take $\alpha = 2.0 \times 10^{-5}$ K$^{-1}$, $T_0 = 293$ K and $\Delta T = 20$ K. Calculate the percentage change in the mass of gas in the tank due to (a) the change in volume of the tank alone (ie neglect density change) and (b) the change in density of the gas alone (ie neglect volume change). State the sign of the change in each case.

Take the average temperature of the Earth's atmosphere to be 290 K.

iv) Calculate the root mean square velocity $v_{rms}$ for Nitrogen (molar mass 28 kg/kmol) and Hydrogen (2 kg/kmol) in the Earth's atmosphere. (5 marks)

v) The escape velocity for the Earth is 11 km.s$^{-1}$. Comment briefly on the significance of your results for part (iv) for the composition of the atmospheres of the Earth, the Earth's moon, and Jupiter. (Three four clear sentences should suffice.) (3 marks)

**Question 2**

A ramp is shaped as shown. The upper, straight portion makes angle $\theta$ to the horizontal. The initial elevation of the car (mass $m$) is $h$ above the final horizontal portion. The wheels of the toy car rotate without friction on the axles and have negligible mass. It has no motor and is released from rest.

a) Neglect air resistance for the following parts.

i) With the aid of a diagram showing clearly the forces acting, derive an expression for the initial acceleration $a_0$.

ii) Showing all reasoning, derive an expression for the final speed $v_f$ of the car.

iii) Sketch the acceleration $a(t)$, speed $v(t)$ and position $s(t)$ measured along the ramp as shown. On all graphs, show the times $t_1$, $t_2$ and $t_3$ at which the car passes points $s_1$, $s_2$ and $s_3$.

On both the $a(t)$ and $v(t)$ graphs, show features related to both $a_0$ and $v_f$.

On both the $s(t)$ graph, show a feature related to $v_f$.

b) Now consider air resistance, which is a force that increases with speed $v$.

iv) On each of the graphs you have just drawn, sketch approximately the behaviour you would expect in the presence of air resistance.

Clearly distinguish the two curves in each case.
Question 3

a) In a circus performance, a clown lies on his back with a brick, mass M, on his chest. An assistant uses a hammer with a mass m = 1.0 kg, to crack the brick. The head of the hammer is travelling vertically down at v = 20 ms⁻¹. The mass of the handle is negligible. The collision between hammer and brick is of extremely short duration. However, because the brick cracks at the surface, the collision is completely inelastic.

i) Derive an expression for the velocity V of the brick plus hammer immediately after the collision with the brick.

ii) In an earlier part of the performance, a selection of audience members with different weights has stood on the clown's chest. The deformation of the chest is proportional to the weight of the person standing, and a 100 kg man produces a depression of 30 mm in his chest. Derive an expression for the spring constant of the clown's chest.

iii) The Occupational Health and Safety Officer for the circus decides that the breaking brick trick should not depress the clown's chest more than 30 mm beyond the resting position of the brick before the collision. Derive a value for the required mass M of the brick. You may neglect the gravitational potential energy associated with deformation of the clown's chest.

iv) Express your answer to part (iii) as an inequality. Describe the reason for the direction of the inequality.

Caution. Do not try this exercise at home.

Question 4

i) In one sentence, give a formal definition of temperature.

ii) Define heat capacity. If your definition is an equation, define the terms in it.

iii) Define specific heat (= specific heat capacity). If your definition is an equation, define the terms in it.

iv) A car has a fuel tank made of aluminium. A motorist completely fills this tank at 20 °C, at which temperature the tank has a volume of 34 litres. He then leaves the car at the service station, parked in the sun. The car is painted black and, during the day, the temperature of the tank and the fuel in it rises to 45 °C (you may assume that the temperature is uniform). Calculate the amount of fuel that you would expect to overflow from the tank. The linear coefficient of expansion for aluminium is α_{Al} = 2.3 \times 10^{-5} °C⁻¹. The volumetric coefficient of expansion for the fuel is β_{fuel} = 1.40 \times 10^{-3} °C⁻¹.

v) Pneumatic or air suspension has some advantages (and some disadvantages) in comparison with springs. In this question we consider an idealised version of air suspension. A volume V₀ of air at atmospheric pressure P₀ and temperature T₀ is sealed in a piston of area A that slides without leaks or friction in a cylinder.

The air may be considered as an ideal gas with molar mass 0.029 kg.kmol⁻¹. The piston is then loaded with a mass m, that includes the mass of the piston. The system is allowed to reach mechanical and thermal equilibrium at T₀.

Showing your working, derive an expression for h₀, the equilibrium height of the piston in the cylinder as shown in the sketch in terms of the parameters given above and the gas constant.

vi) For the suspension system in part (v), the stiffness (ie the ratio of force to displacement, just like the spring constant of a spring) depends on the speed of the displacement. Would the system be stiffer for a rapid displacement or a slow one? Explain your answer in a few short, clear sentences.
Question 5

i) An athlete runs up a mountain. His altitude increases at a steady rate of 0.55 m s⁻¹. His climbing is 20% efficient: that is, for each Joule of biochemical energy he converts, 20% is converted into increased gravitational potential energy. The athlete has a mass of 80 kg and a total skin area of 1.8 m². His skin temperature Tₐ is uniform and 34°C. His surroundings have a temperature Tₛ that is uniform and 30°C. Assume that both the skin and the surroundings have emissivities of 0.8. As in the original Olympics, the athlete is nude. (If you are under 18 years of age, the athlete is wearing a small costume, whose effects we shall neglect.)

a) Showing your working, calculate the rate at which he is converting biochemical energy into heat.

b) Showing your working, calculate the nett rate of heat loss by radiation.

c) If all the heat produced by his body were lost by sweating, so that his body temperature stays constant, and if all of his sweat evaporates, calculate the rate of water loss. Convert your answer into litres per hour. Assume that the latent heat of evaporation of water at skin temperature is 2.5 MJ kg⁻¹.

d) Would your answer to (c) be an overestimate or an underestimate of the real rate of water loss? Explain your answer in a sentence or so.

e) Suppose that (perhaps because of dehydration), his sweating slowed such that the total rate of heat loss by the athlete fell to 500 W. Assume that he continues producing heat at the rate you calculated in (a). Calculate how long it would take for his body temperature to rise by an average of 2°C. (The specific heat of the body is 4 kJ kg⁻¹ °C⁻¹.)

ii) An ideal gas, initially with pressure P₀, volume V₀ and temperature T₀ is compressed isothermally to P₁, V₁ and T₀ (step A). The gas then expands adiabatically to P₀, V₂ and T₂ (step B). It then returns isobarically to its original state P₀, V₀ and T₀ (step C).

a) Sketch a P,V diagram for this process. On the axes, indicate P₀, P₁, V₀, and V₁. Also label the steps A, B and C and indicate their direction with arrows.

b) Q is the heat added to the gas, W is the work done by the gas, and ΔU is the change in its internal energy. In the table provided, indicate with the symbols +, – and 0 whether the terms are positive, negative or zero for each step.

<table>
<thead>
<tr>
<th>Step</th>
<th>Q</th>
<th>W</th>
<th>ΔU</th>
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<tbody>
<tr>
<td>A isotherm</td>
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<td>B adiab</td>
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<td>C isobar</td>
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| Whole cycle   |    |    |      | (Σ)