Question 1

i) \[ V = L^3 = L_o^3(1+\alpha(T-T_o))^3 \]
\[ = L_o^3(1+3\alpha(T-T_o)) \]
\[ \Delta V = \frac{L_o^3(1+3\alpha(T-T_o)) - L_o^3}{L_o^3} \]
\[ = 3\alpha(T-T_o) \]

ii) \[ \rho \equiv \frac{M}{V} = \frac{nm}{V} \] where \( n \) is number of moles in tank. Substituting from \( PV = nRT \),
\[ \rho = \frac{PAm}{RT} \] so
\[ \frac{\Delta \rho}{\rho_o} = \frac{\frac{PAm}{RT} - \frac{PAm}{RT_o}}{\frac{PAm}{RT_o}} = \frac{1 - \frac{1}{T_o}}{\frac{1}{T_o}} = \frac{T_o}{T} - 1 \]

iii) a) \[ PV = nRT \] so \( nm \propto V \).
   i) \[ \frac{\Delta V}{V} = 3\alpha(T-T_o) = 0.12\% \]
   b) \[ m \propto \rho \]
   ii) \[ \frac{\Delta \rho}{\rho_o} = \frac{T_o}{T} - 1 = -6.8\% \]

iv) \[ \frac{1}{2} m \bar{v}^2 = \frac{3}{2} kT \]
\[ \therefore \quad v_{r.m.s.} \equiv \sqrt{\bar{v}^2} = \sqrt{\frac{3kT}{m}} \]
\[ v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kT \cdot N_A}{mol \text{ wt}}} \]
\[ N_2 : \quad = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 293 \times 6.02 \times 10^{23}}{0.028}} = 510 \text{ ms}^{-1} \]
\[ H_2 = \quad = 1.9 \text{ kms}^{-1} \]

v) Earth: for hydrogen, although \( v_{rms} < v_{escape} \), the random motion of molecules means that occasionally hydrogen molecules in the Earth's atmosphere acquire escape velocity and are lost. Consequently nearly all hydrogen in Earth's atmosphere has escaped. For nitrogen, \( v_{rms} \ll v_{escape} \), so we have retained the nitrogen. Jupiter is colder and has a higher escape velocity: it has retained its hydrogen atmosphere. The moon has the lowest escape velocity and virtually no atmosphere.
Question 2

i) Newton's 2nd law: \( \Sigma \mathbf{F} = m \mathbf{a} \).
If the wheels roll freely, only the weight has a component in the direction of \( s \), so
\[ W \sin \theta = ma_0. \]
\( W = mg \) so \( a_0 = g \sin \theta \).

ii) Here, non-conservative forces do no work. (Friction acts between wheels and ramp, but there is no relative motion at contact, so no work is done.)
\[ \therefore \text{ Mechanical energy conserved} \]
\[ U_i + K_i = U_f + K_f \]
\[ mgh + 0 = 0 + \frac{1}{2}mv_f^2 \]
so \( v_f = \sqrt{2gh} \)

iv) \[ \text{bold curve includes air resistance} \]
\[ \text{area under curve} = v_f \]
Question 3

i) During the brief collision, large contact forces act, so external forces are neglected. So momentum of hammer plus brick is conserved. In the vertical direction: \( P_{\text{initial}} = P_{\text{final}} \).

\[
\frac{mv}{m+M} = (m+M)V,
\]

so \( V = \frac{m}{m+M}v \).

ii) From the proportionality of load to deformation, the clown's chest obeys Hooke's law: \( F = -kx \).

Here \( k = \frac{\text{weight of 100 kg man}}{\text{deformation}} = \frac{980 \text{ N}}{30 \text{ mm}} = 33 \text{ kN.m}^{-1} \). (3 marks)

iii) Hooke's law \( \rightarrow \) it acts like a spring, and provides a conservative restoring force. Therefore mechanical energy will be conserved in the deformation that follows the collision. Neglecting gravitational potential energy, we have:

\[
U_i + K_i = U_f + K_f \]

\[
0 + \frac{1}{2}(M+m)V^2 = \frac{1}{2}kx_{\text{max}}^2 + 0
\]

\[
M+m\left(\frac{m}{m+M}v\right)^2 = kx_{\text{max}}^2
\]

\[
\frac{m^2}{m+M}v^2 = kx_{\text{max}}^2 \quad (*)
\]

\[
m + M = \frac{m^2v^2}{kx_{\text{max}}^2}
\]

\[
M = \frac{m^2v^2}{kx_{\text{max}}^2} - m = 13 \text{ kg}.
\]

(Anyone whose sympathy for the clown prompts him/her to point out that this already compresses the chest by 4 mm, and that they OHS Officer should be warned that the total deformation is now 34 mm, should get a bonus mark.)

iv) \( M > 13 \text{ kg} \). From (*) if \( M < 13 \text{ kg} \), the energy of the brick after the collision will cause excessive deformation of the clown's chest.

Question 4

i) Temperature is that quantity that is equal in two bodies in thermal equilibrium. (1)

ii) The heat capacity \( C \) of a body is the heat required to raise its temperature by unit temperature, or

Heat capacity \( C = \frac{Q}{\Delta T} \) where \( Q \) is the heat added and \( \Delta T \) is the temperature change resulting.

iii) The specific heat \( c \) of a substance is the heat required to raise the temperature of unit mass (or one mole) of the substance by unit temperature, or Specific heat \( c = \frac{Q}{m\Delta T} \) where \( Q \) is the heat added, \( m \) is the mass and \( \Delta T \) is the temperature change resulting, or equivalent.

iv) The tank expands proportionally in all directions, so the increase in its volume is given by

\[
\Delta V_{\text{tank}} = \beta_{\text{steel}}V_{20} = 3\alpha_{\text{steel}}V_{20}\Delta T.
\]

The fuel expands by \( \Delta V_{\text{fuel}} = \beta_{\text{fuel}}V_{20}\Delta T \).

Neglecting effects due to pressure increase (these will be small) the amount overflowing is

\[
\Delta V = \Delta V_{\text{fuel}} - \Delta V_{\text{tank}} = \beta_{\text{fuel}}V_{20}\Delta T - 3\alpha_{\text{steel}}V_{20}
\]

\[
= (1.40 \times 10^{-3} \text{ °C}^{-1} - 3 \times 2.3 \times 10^{-5} \text{ °C}^{-1})(35 \text{ l})25\text{°C}
\]

\[
= 1.1 \text{ litres}.
\]

v) Let the initial volume of gas contain \( n \) moles. \( P_A V_0 = nRT_0 \), where \( R \) is the gas constant.
At the new thermal equilibrium, the pressure \( P \) satisfies \( PV = PAh_0 = nRT_0 \).

Mechanical equilibrium requires \( mg = (P - PA)A \) so \( P = PA + mg/A \)

Combining these equations and rearranging:

\[
h_0 = \frac{nRT_0}{PA} = \frac{PAV_0}{PA} = \frac{PAV_0}{(PA + mg/A)A}
\]

\[
h_0 = \frac{PAV_0}{PA + mg} = \frac{V_0/A}{1 + mg/PAA} \text{ or an equivalent expression.}
\]

vi) The pneumatic 'spring' is stiffer for a fast deformation. If it is slow enough, the process will be isothermal, so \( PV^\gamma = \text{constant} \). If it is rapid enough, the process will be adiabatic so \( PV^\gamma = \text{constant} \). \( \gamma \), the ratio of specific heats, is greater than 1 so, for an adiabatic process a given change in \( V \) produces a larger change in \( P \) and so a larger force.
Question 5

i) a) 20% efficient, so rate of heat production = 4 times rate of mechanical power.

\[ H = 4 \frac{d}{dt} mgh = 4mg \frac{dh}{dt} \]

\[ = 4 \times (80 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (0.55 \text{ m/s}) = 1.7 \text{ kW} \]

b) Rate of heat lost from skin by radiation – rate of heat absorbed by skin from surroundings

\[ H_{\text{rad}} = A \sigma (e_o T_o^4 - e_s T_s^4) \]

\[ = (1.8 \text{ m}^2) \times (5.67 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4}) \times 0.8 \times ((307 \text{ K})^4 - (303 \text{ K})^4) \]

\[ = 37 \text{ W} \]

(No longer in PHYS1131 syllabus)

c) Energy to evaporate water = \( L \cdot m_{\text{sweat}} \) where \( m_{\text{sweat}} \) is the mass of sweat evaporated

Rate of heat lost by evaporation \( H = L \frac{dm_{\text{sweat}}}{dt} \)

Rate of sweating = \( H/L = (1.7 \text{ kW})/(2.5 \text{ MJ.kg}^{-1}) = 7.8 \times 10^{-4} \text{ kg.s}^{-1} = 2.8 \text{ kg/hr} = 2.8 \text{ litres/hour.} \)

d) Under most conditions (except perhaps very dry conditions) some sweat is lost because it falls off or is wiped away, so the rate of water loss would be higher than calculated here.

e) Rate of heat produced – rate of heat lost = \( H_{\text{new}} = H - 500 \text{ W} = 1.2 \text{ kW}. \)

Defn of heat capacity: \( Q = mc \Delta T \)

\[ t = \frac{Q}{H_{\text{new}}} = \frac{mc \Delta T}{H_{\text{new}}} = \frac{(80 \text{ kg})(4 \text{ kJ.kg}^{-1} \text{C}^{-1})(2 \text{C})}{1.2 \text{ kW}} = 500 \text{ s} = 9 \text{ minutes} \]

ii)

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<th>Step</th>
<th>Q</th>
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