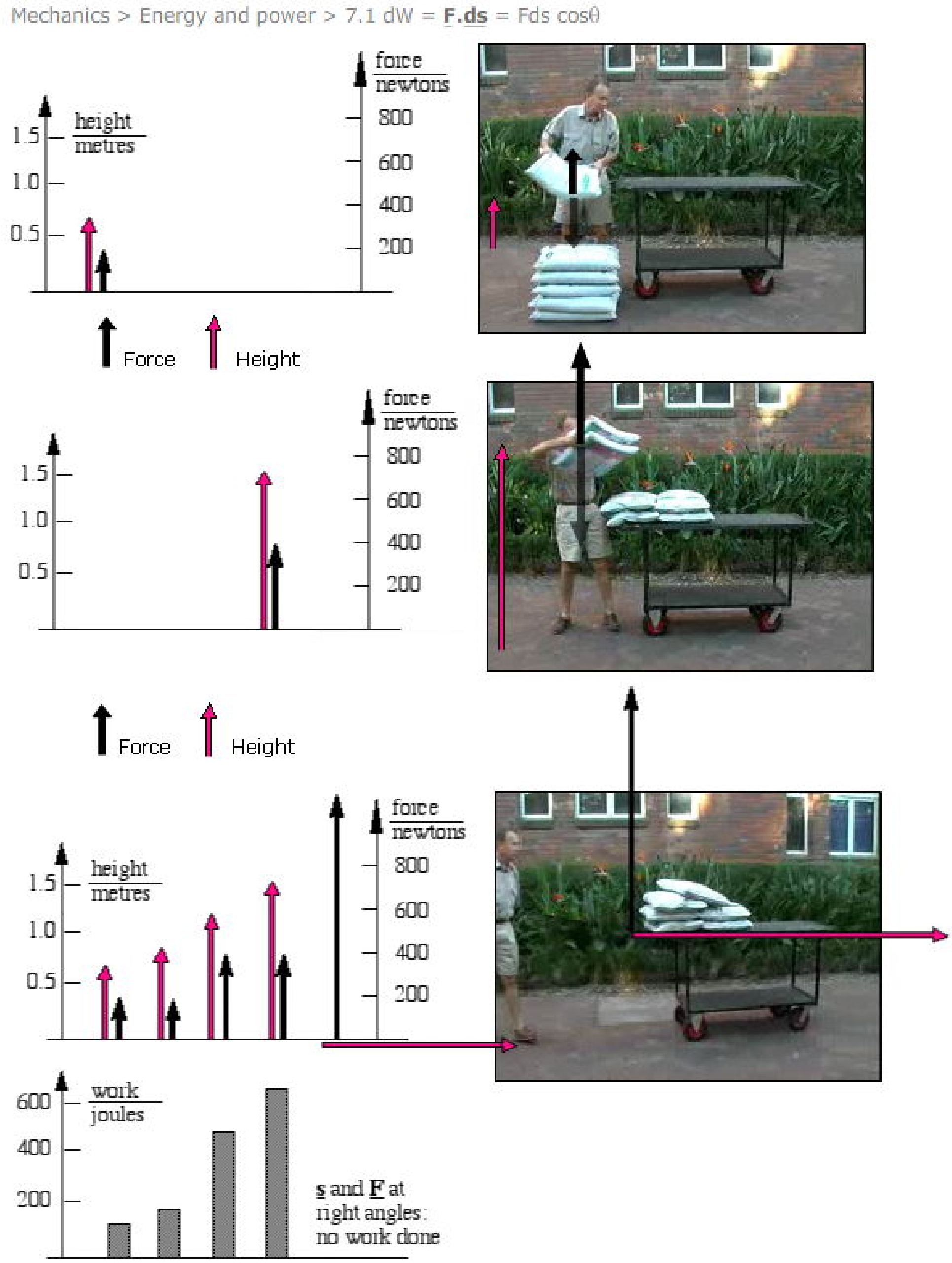
Work and Energy

(PHYS 1121 & 1131, UNSW, Session 1, 2011)

S&J chapters 7.1-7.8; 8.1-8.5; Physclips Ch 7

- (the dot product)
- **definition of work**
- **definition of kinetic energy –>**
  restatement of Newton 2
- **conservative and non-conservative forces**
- **potential energy**

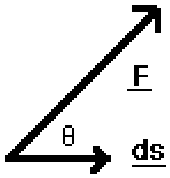
Sometimes, the physics sense of work is very like the use in normal language. This bloke is doing work



but the trolley isn't doing work. Why not? See Physclips, Work and Energy

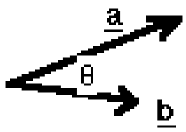
We need some new maths: The scalar product. *dot product*

Why? e.g. Work: scalar, related to $\underline{\mathbf{F}}$, $\underline{\mathbf{ds}}$ and θ. *because it makes maths easier*



$$dW = |\underline{\mathbf{F}}|\,|\underline{\mathbf{ds}}|\cos\theta$$

(later: also used for voltage $dV = |\underline{\mathbf{E}}|\,|\underline{\mathbf{ds}}|\cos\theta$ *etc)*



therefore *define* $\underline{\mathbf{a}}\,.\,\underline{\mathbf{b}} = ab\cos\theta \;\; (= \underline{\mathbf{b}}\,.\,\underline{\mathbf{a}})$ *pronounced "a dot b"*

Apply to unit vectors:

$\underline{\mathbf{i}}\,.\underline{\mathbf{i}} = 1\,.\,1\cos 0° = 1 = \underline{\mathbf{j}}\,.\underline{\mathbf{j}} = \underline{\mathbf{k}}\,.\underline{\mathbf{k}}$

$\underline{\mathbf{i}}\,.\underline{\mathbf{j}} = 1\,.\,1\cos 90° = 0 = \underline{\mathbf{j}}\,.\underline{\mathbf{k}} = \underline{\mathbf{k}}\,.\underline{\mathbf{i}}$

Scalar product by components

$\underline{\mathbf{a}}\,.\,\underline{\mathbf{b}} = (a_x\,\underline{\mathbf{i}} + a_y\,\underline{\mathbf{j}} + a_z\,\underline{\mathbf{k}}).(b_x\,\underline{\mathbf{i}} + b_y\,\underline{\mathbf{j}} + b_z\,\underline{\mathbf{k}})$ *expand out to give nine terms… ugh*

$= (a_xb_x)\,\underline{\mathbf{i}}\,.\underline{\mathbf{i}} + (a_yb_y)\,\underline{\mathbf{j}}\,.\underline{\mathbf{j}} + (a_zb_z)\,\underline{\mathbf{k}}\,.\underline{\mathbf{k}}$ *where* $\underline{\mathbf{i}}\,.\underline{\mathbf{i}} = \underline{\mathbf{j}}\,.\underline{\mathbf{j}} = \underline{\mathbf{k}}\,.\underline{\mathbf{k}} = 1$

$+ (a_xb_y + a_yb_x)\,\underline{\mathbf{i}}\,.\underline{\mathbf{j}} + (..)\,\underline{\mathbf{j}}\,.\underline{\mathbf{k}} + (..)\,\underline{\mathbf{k}}\,.\underline{\mathbf{i}}$ *and these terms are all zero, so*

$\underline{\mathbf{a}}\,.\,\underline{\mathbf{b}} = a_xb_x + a_yb_y + a_zb_z$ *which is an important result*

And, at no extra charge, we get a useful geometrical tool

Problem. Find the angle between

$\underline{\mathbf{a}} = 4\,\underline{\mathbf{i}} - 3\,\underline{\mathbf{j}} + 7\,\underline{\mathbf{k}}$

$\underline{\mathbf{b}} = 2\,\underline{\mathbf{i}} + 5\,\underline{\mathbf{j}} - 3\,\underline{\mathbf{k}}$

Hooee! Imagine doing this by geometry. Let's use dot product which, using the result above, we can two ways:

$ab\cos\theta = \underline{\mathbf{a}}\,.\,\underline{\mathbf{b}} = a_xb_x + a_yb_y + a_zb_z$

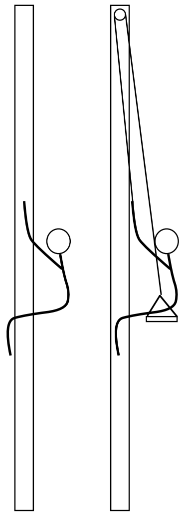
$$\cos\theta = \frac{a_xb_x + a_yb_y + a_zb_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}\sqrt{b_x^2 + b_y^2 + b_z^2}} = \frac{4*2 - 3*5 - 7*3}{\sqrt{4^2 + 3^2 + 7^2}\sqrt{2^2 + 5^2 + 3^2}}$$

= .... *hit the calculator to give:*

$\rightarrow \theta = 122°$

Is it easier for the sailor to climb the mast using the halyard (a rope passing through a pulley at the top of the mast)?

Why?



Neglecting acceleration:

Without rope:

$\underline{\mathbf{W}} = \underline{\mathbf{F}}_{\text{feet}} + \underline{\mathbf{F}}_{\text{hands}}$

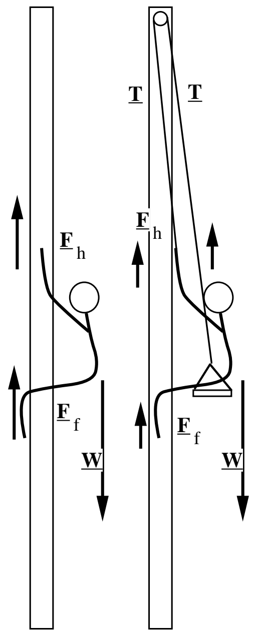
With rope:

$\underline{\mathbf{W}} = \underline{\mathbf{F}}_{\text{feet}} + \underline{\mathbf{F}}_{\text{hands}} + \underline{\mathbf{T}}$

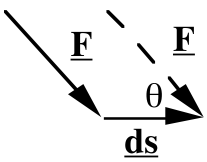
but $\underline{\mathbf{F}}_{\text{hands}} = \underline{\mathbf{T}}$ so

$\underline{\mathbf{W}} = \underline{\mathbf{F}}_{\text{feet}} + 2\,\underline{\mathbf{F}}_{\text{hands}}$

During the moment when $\underline{\mathbf{F}}_{\text{feet}} = 0$, your hands apply 50% less force! But how do you "pay for" the reduction in force? Let's introduce work



Definition of work



When force varies, use differential displacement $\underline{\mathbf{ds}}$

$dW = F\,ds\cos\theta = \underline{\mathbf{F}}.\underline{\mathbf{ds}}$

we can think of this in two ways:

*(F) (ds cosθ) –> F * component of ds // F, or*

*(F cosθ) (ds) –> ds * component of F // ds*

$$W = \int_0^L F\cos\theta\, ds$$

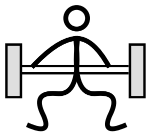
if F & θ are constant, we get $W = FL\cos\theta$

But this is the baby version: forces do vary!

SI Unit: 1 Newton x 1 metre = 1 Joule

SIMPLE MACHINES (pulleys, levers, screws, inclined planes etc)

Example. How much work is done by lifting 100 kg vertically by 1.8 m very slowly?



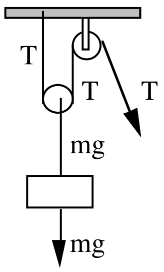
Slow $\therefore$ $F_{applied} \cong mg$

W = mg d cos 0° = mgh : more later

= 1.8 kJ.

Not a lot – how much if you walk up one flight of stairs?

Yet it is harder to do, because the force is inconveniently large. Consider:



If the rope and pulleys are light, and if the accelerations are negligible, then

Force on LH pulley

$ma \cong 0 = 2T - mg$

$\therefore$ $T = mg/2$

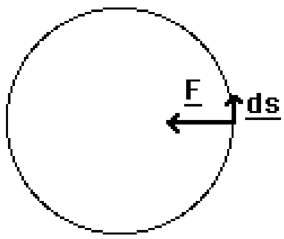
If mass rises by D, word done = mgD.

But rope shortens on both sides of rising pulley,

if mass rises by D, rope must be pulled 2D, so

work done = T * 2D = mgD

We do the same work with less force by covering *more distance*.



Example. What is the work done by gravity in a circular orbit?

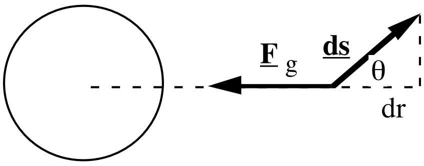
$W = \int F\, ds \cos\theta$

$= 0$

Historically important: no work to do!

Example. $F_{grav} \propto 1/r^2$. How much work is done to move m = 1 tonne from earth's surface (r = 6500 km) to r = ∞?

$$W = \int F\, ds \cos\theta$$



$$= \int F\, dr$$

$$F = -F_{grav} = \frac{Cm}{r^2}$$

more later, but for now, what is the constant C? What do we know?

On earth's surface, we've dropped objects so we know that know that a = F/m = – 9.8 ms^{-2}

$\therefore$ C = $(9.8\ ms^{-2})(6.5\ 10^6\ m)^2 = 4.1\ 10^{14}\ m^3s^{-2}$

$$W = \int_{6500km} \frac{Cm}{r^2}\, dr$$

note: potential energy proportional to – 1/r

$$= -Cm\left(\frac{1}{\infty} - \frac{1}{6.510^6 m}\right)$$

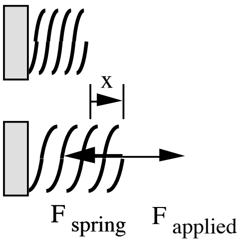
Not equal to mgh. More on this later

=

$= 6.3\ 10^{10}$ J = 63 GJ.

Worse: rockets very inefficient: as we'll see later

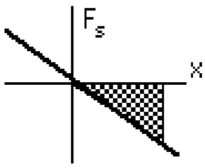
Work to deform spring



No applied force

(x = 0)

Hooke's law:



Work done **by** spring = $\int F_{spring}.dx$

$$= \int -kx.dx = -\frac{1}{2}kx^2 + 0$$

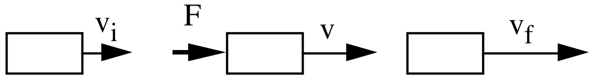
Work done **on** spring = $\int F_{applied}.dx$

$$= \int kx.dx = +\frac{1}{2}kx^2$$

(= work stored in spring)

The work-energy theorem

(Total) force F acts on mass m in x direction.



Work done by F $= \int_i^f F dx$ *(use F = ma)*

$$= \int_i^f m \frac{dv}{dt} dx = \int_i^f m \frac{dx}{dt} dv$$

$$= \int_i^f m\, v.dv = \left[\frac{1}{2} mv^2 \right]_i^f$$

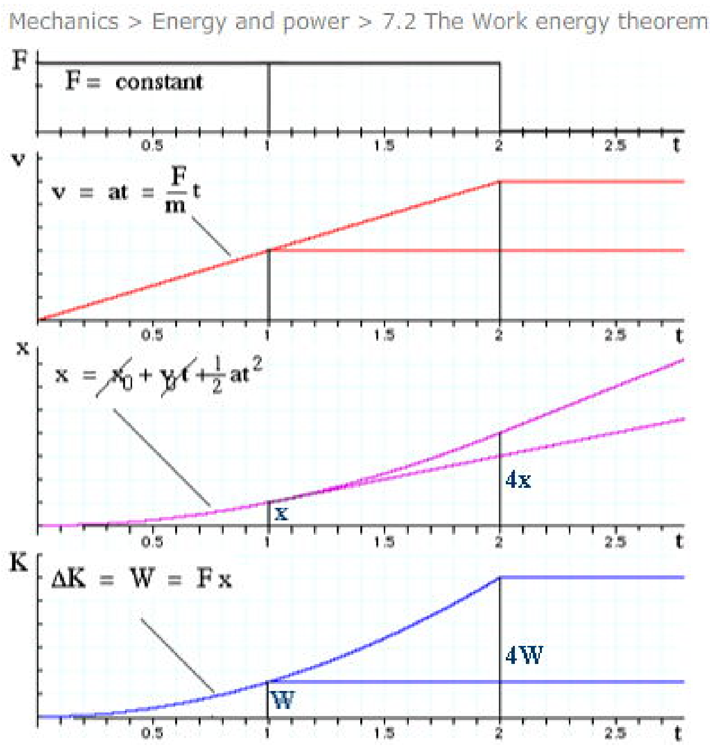
Work done by F $= \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 \equiv \Delta K$

Define **kinetic energy** $K \equiv \frac{1}{2} mv^2$

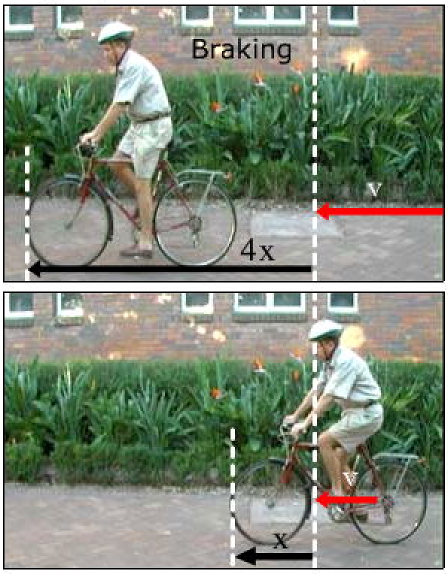
Increase in kinetic energy of body = work done by **total** force acting on it.

*This is a **theorem**, ie a tautology*
because it is only true by definition of KE and by Newton 2.
*∴ restatement of Newton 2 in terms of energy. **Not** a new law*

Work energy theorem (baby version)



Energy and power > 7.2 The Work energy theorem



important road safety lesson

doubling the speed $v \to 2v$ would give $K \to 4K$ four times as much kinetic energy

so same braking force must act over 4 times the distance

Power. is the rate of doing work

Average power $\bar{P} \equiv \dfrac{W}{\Delta t}$

Instantaneous power $P = \dfrac{dW}{dt}$

SI unit: 1 Joule per second ≡ 1 Watt (1 W)

Example Jill (m = 60 kg) climbs the stairs in Matthews Bldg and rises 50 m in 1 minute. How much work does she do against gravity? What is her average output power? (neglect accelerations)

$W = \int \underline{\mathbf{F}} \cdot \underline{\mathbf{ds}} = \int F_y \, dy$ *(only y displacement matters, because mg acts in (-ve) y direction)*

$F_y \cong mg$

$W = mg \int dy = mg \, \Delta y$

$= 29 \text{ kJ}$ *(cf $K = \frac{1}{2} mv^2 \sim 20 - 40$ J)*

$\bar{P} \equiv \dfrac{W}{\Delta t} = \dfrac{mg \, \Delta y}{\Delta t} = 490 \text{ W}$

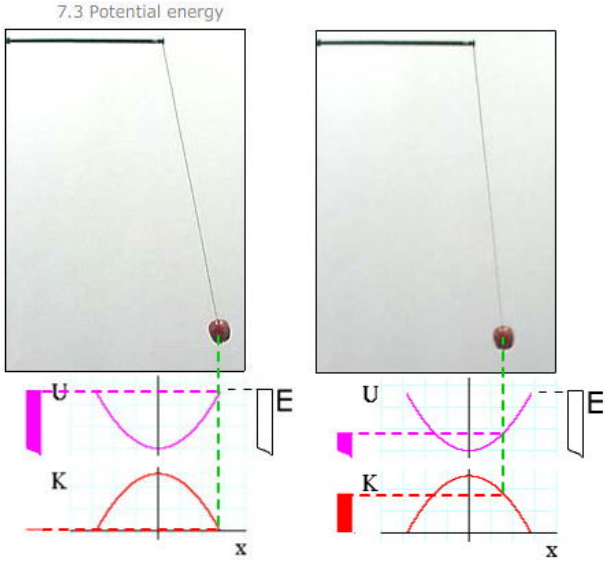
(to give a scale, humans can produce 100s of W,
car engines several tens of kW)
(1 horsepower ≡ 550 ft.lb.s^{-1} = 0.76 kW)

Potential energy.

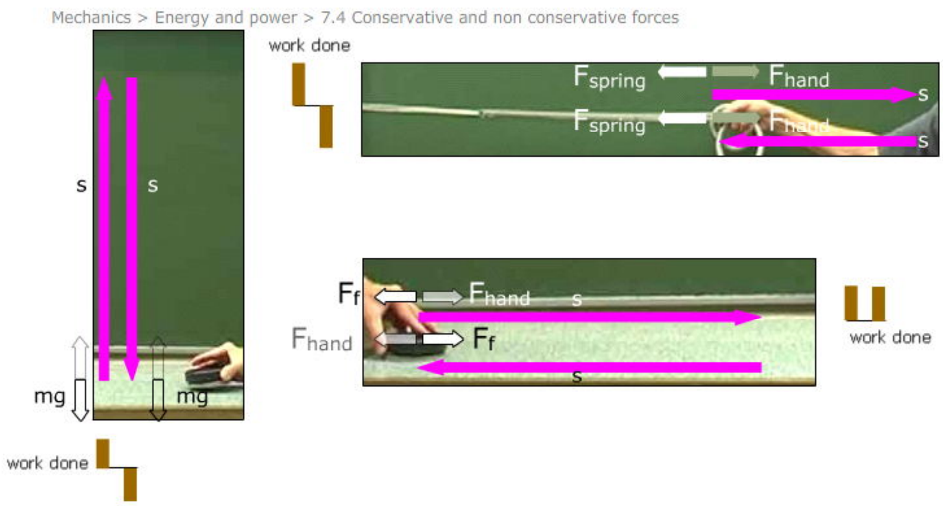
e.g. Compress **spring**, do W on it, but get no K. Yet can get energy out: spring can expand and give K to a mass. –> Idea of stored energy.

e.g. **Gravity**: lift object (slowly), do work but get no K. Yet object can fall back down and give back K.

Recall $W_{\text{against grav}} = mg\,\Delta y$ i.e. $W = W(y)$



But: Slide mass slowly along a surface. Do work against **friction**, but can't recover this energy mechanically. Not all forces "store" energy. Look at these three diagrams:

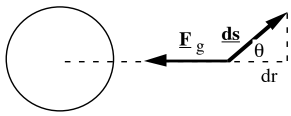


For the spring and gravity, when we change the direction of the displacement the force *doesn't* change direction, so the sign of the work done changes, so, round a closed path, the work done is zero.

For friction, when we change the direction of the displacement the force *does* change direction, so the sign of the work done doesn't changes, so friction does negative work, and we do positive work against it.

So we have two very different sorts of forces.

Conservative and non-conservative forces (same examples)



$$W_{\text{against gravity}} = -\int_i^f F_g dr \cos\theta$$

$$= -\int_i^f F_g\, dz$$

$$= mg \int_i^f dz$$

$$= mg\,(z_f - z_i)$$ *in* ***uniform*** *field*

W is uniquely defined at all $\underline{\mathbf{r}}$, i.e. W = W($\underline{\mathbf{r}}$)

If $z_f - z_i$ are the same, W = 0.

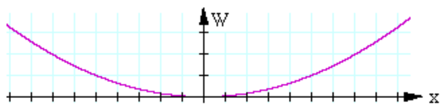
∴ Work done against gravity round a closed path = 0 This is the definition

Gravity is a **conservative force**

Spring

$$W_{\text{against spring}} = -\int_i^f F_{\text{spring}}.dx$$

$$= -\int_i^f -kx.dx$$



$$= \frac{1}{2}\, k(x_f^2 - x_i^2)$$

W is uniquely defined at all x, i.e. W = W(x)

$x_f = x_i \;\Rightarrow\; W = 0.$

∴ Work done round a closed path = 0

Spring force is a **conservative force**

so it has stored or potential energy: symbol U.

Friction

$$dW_{\text{against fric}} = -F_f\, ds \cos\theta$$

but $\underline{\mathbf{F}}_f$ always has a component *opposite* $\underline{\mathbf{ds}}$

∴ dW always ≥ 0. *(we never get work back)*

∴ cannot be zero round closed path, ∴ W ≠ W($\underline{\mathbf{r}}$)

∴ friction is a **non-conservative force**

Note that direction of friction (dissipative force) is always against motion. Direction of g doesn't change

Potential energy

For a **conservative** force **F** (i.e. one where work done against it, W = W(**r**)) we can define potential energy U by $\Delta U = W_{against}$. i.e.

$$\Delta U = -\int_i^f F\,dr\cos\theta$$

Same examples: **spring**

$$\Delta U_{spring} = -\int_i^f F_{spring}.dx$$

$$=$$

$$= \frac{1}{2}k(x_f^2 - x_i^2)$$

Choice of zero for U is arbitrary.

Here U = 0 at x = 0 is obvious, so

$$U_{spring} = \frac{1}{2}kx^2$$

From energy to force:

$U = -\int F\,ds$ where ds is in the direction // F

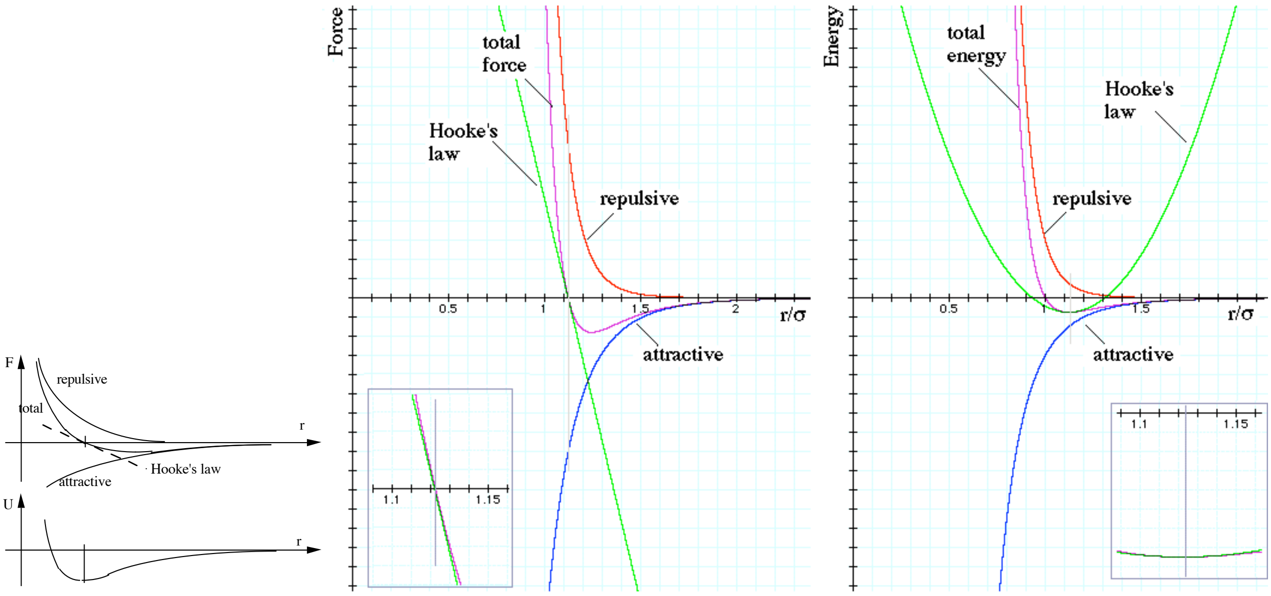
$$F = -\frac{dU}{ds}$$

in fact $F_x = -\frac{dU}{dx}$, $F_y = -\frac{dU}{dy}$, $F_z = -\frac{dU}{dz}$

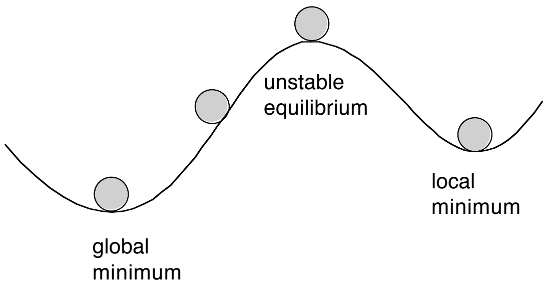
Spring: $U_{spring} = \frac{1}{2}kx^2 \quad \therefore \quad F_{spring} = -kx$

Gravity: $U_g = mgz \quad \therefore \quad F_g = -\frac{dU}{dz} = -mg$

Energy of interaction:


F
repulsive
total
r
Hooke's law
attractive
U
r
Force
total
force
Hooke's
law
repulsive
0.5
1
1.5
2
r/σ
attractive
1.1
1.15
Energy
total
energy
Hooke's
law
repulsive
0.5
1
1.5
r/σ
attractive
1.1
1.15

Energy diagrams and equilibria:



Treat this as y(x) for a particle in a uniform gravitational field, we can see U(x) and imagine the direction of force (–dU/dx).

Minima give stable equilibria: stable with respect to small perturbations. Maxima give unstable equilibria.

Similar energy diagrams in chemistry and elsewhere.

Conservation of mechanical energy (sometimes!)

Recall: Increase in K of body = work done by **total** force acting on it. *(restatement of Newton 2)*

But, if all forces are conservative, work done **by** these forces $= -\Delta U$ *(definition of U)*

$\therefore$ if only conservative forces act, $\Delta K = -\Delta U$

We define mechanical energy

$$E \equiv K + U$$

so, if only conservative forces act, $\Delta E = 0$.

we can make this stronger.

Work done by **non-conservative forces**

Define internal energy U_{int} where

$$\Delta U_{int} = -\text{ Work done } \textbf{by} \text{ n-c forces}$$

(= + Work done **against** n-c forces)

Recall def^n of K: ΔK = work done by Σ force

$$\therefore \quad \Delta K = -\Delta U - \Delta U_{int}$$

$$\therefore \quad \Delta K + \Delta U + \Delta U_{int} = 0$$

If n-c forces do no work, then $\Delta U_{int} = 0$, so:

If non-conservative forces do no work,

$$\Delta E \equiv \Delta K + \Delta U = 0$$

or: **mechanical energy** E **is conserved**

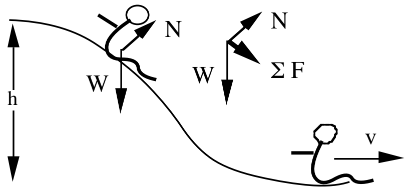
Equivalent to Newton 2, but useful for many mechanics problems where integration is difficult.

State the principle carefully!

Never, ever write: "~~kinetic energy = potential energy~~"

3 reasons why not: It's not true. In general, it gives the wrong answer. It makes examiners angry.

Classic problem. Child pushes off with v_i. How fast is the s/he going at the bottom of the slide? Neglect friction (*a non-conservative force*).



i) *By Newton 2 directly:*

$$v = \int_{top}^{bottom} a\,dt = \int_{top}^{bottom} \frac{F}{m} = \int_{top}^{bottom} g\cos\theta\,dt = ...$$

ii) *Using work energy theorem (Newton 2 indirectly):*

Non-conservative forces do no work, ∴ mechanical energy is conserved, i.e.

$$\Delta E = \Delta K + \Delta U = 0 \qquad E_f = E_i$$
$$K_f - K_i + U_f - U_i = 0 \quad \textit{or} \quad K_f + U_f = K_i + U_i$$

either way we get

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i = 0$$

rearrange –> $$v_f = \sqrt{v_i^2 + 2g(y_i - y_f)}$$

Conservation of energy

observation: for many forces, $W = W(\mathbf{\underline{r}})$, i.e. the work done by or against these forces is a function only of position. Therefore, for these forces only, it's useful to define $U = U(\mathbf{\underline{r}})$.

observation: for all systems yet studied, U_{int} is a state function, i.e. $U_{int} = U_{int}$(measured variables)

Hence idea of internal energy. e.g.:

Friction, ($-U_{int}$) = heat produced when work is done against friction.

Air resistance ($-U_{int}$) is sound and heat.

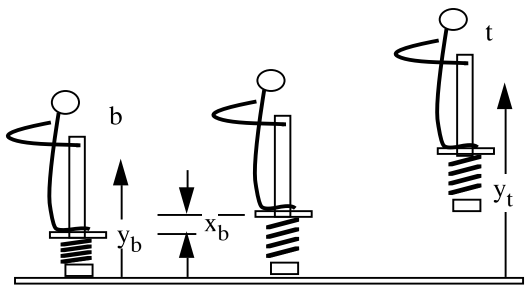
Combustion engines and animals: $+U_{int}$ comes from chemical energy

$$\Delta K + \Delta U + \Delta U_{int} = 0$$

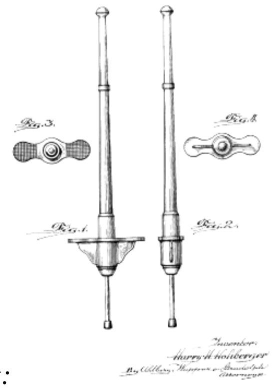
is statement of Newton 2 plus definitions of K, U, U_{int}.

The statement that ΔU_{int} is a state function is the first law of thermodynamics. It is a law, ie falsifiable. More on this in Heat.

Example. Freda ($m = 60$ kg) rides pogo stick ($m << 60$ kg) with spring constant $k = 100$ kN.m^{-1}. Neglecting friction, how far does spring compress if jumps are 50 cm high?



patent extract:



Non-conservative forces do no work, ∴ mechanical energy is conserved, i.e.

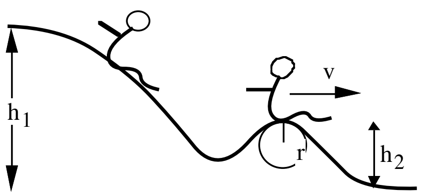
$E_{bottom} = E_{top}$

$K_b + U_b = K_t + U_t$ *($U = U_{grav} + U_{spring}$)*

$\frac{1}{2} mv_{horiz}^2 + (mgy_b + \frac{1}{2} kx_b^2) \cong \frac{1}{2} mv_{horiz}^2 + (mgy_t + \frac{1}{2} kx_t^2)$

$mg(y_t - y_b) \cong \frac{1}{2} kx_b^2$

$\therefore \quad x_b \cong \sqrt{\frac{2mg(y_t - y_b)}{k}}$ *substitute* $\cong 80$ mm.



Example. Slide starts at height h_1. Later there is a hump with height h_2 and (vertical) radius r. What is the minimum value of $h_2 - h_1$ if slider is to become airborne? Neglect friction, air resistance.

Over hump, $a_c = \frac{v^2}{r}$ (down) . Airborne if $g < a_c$, i.e. if $v_2^2 > gr$.

No non-conservative forces act so

$E_2 = E_1$

$U_2 + K_2 = U_1 + K_1$

$mgy_2 + \frac{1}{2} mv_2^2 = mgy_1 + \frac{1}{2} mv_1^2$

$\frac{1}{2} mv_2^2 = mg(y_1 - y_2)$

$(y_1 - y_2) = \frac{v_2^2}{2g} > \frac{gr}{2g} = \frac{r}{2}$

Example Bicycle and rider (80 kg) travelling at 20 m.s^{-1} stop without skidding. $\mu_s = 1.1$. What is minimum stopping distance? How much work done by friction between tire and road? Between brake pad and rim? Wheel rim is ~300 g with specific heat ~ 1 kJ.kg^{-1}, how hot does it get?

friction –> deceleration –> stopping distance

$$|a| = \frac{F_f}{m} \leq \frac{\mu_s N}{m} = \mu_s g$$

$$|a| \leq \mu_s g$$

$$v_f^2 - v_i^2 = 2as \quad -> \quad s = \frac{v_f^2 - v_i^2}{2a}$$

$$s \geq \left| \frac{-v_i^2}{2\mu_s g} \right| = 19 \text{ m}$$

Work done by friction between tire and road?

No skidding, $\therefore$ no relative motion, $\therefore$ W = 0.

Between pad and rim? Here there is relative motion.

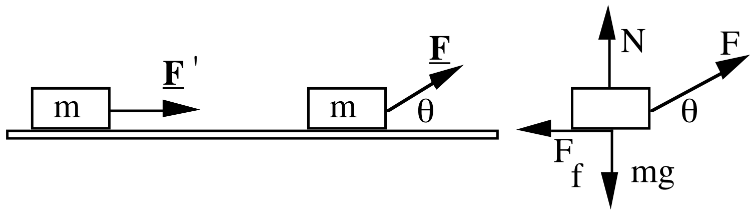
All K of bike & rider –> heat in rim and pad

$$W = \Delta K = K_f - K_i = -16 \text{ kJ}$$

$$Q = mC\Delta T \quad ......$$

$\Delta T \sim 50$ °C (*Heat and this definition come later in the syllabus*)

Example Which way is it easier to drag an object?



Suppose we move at steady speed, a = 0. Which requires less F? Which requires less work?

mechanical equilibrium –> horizontal $F\cos\theta = F_f$

vertical $N + F\sin\theta = mg$

sliding –> $F_f = \mu_k N$

$F\cos\theta = \mu_k N$

eliminate N –> $F\cos\theta = \mu_k(mg - F\sin\theta)$

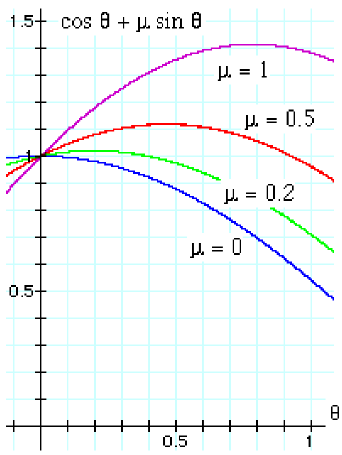
$$F = \frac{\mu_k mg}{\cos\theta + \mu_k \sin\theta}$$

when $\theta = 0$, $F' = \mu_k mg$

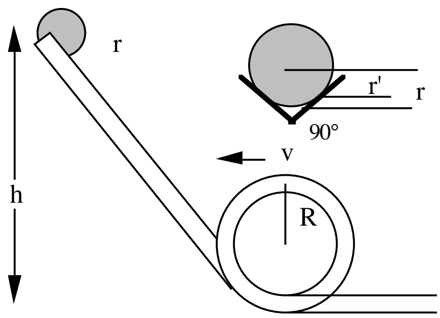
$F < F'$ if $\cos\theta + \mu_k\sin\theta > 1$, i.e. if μ_k large & θ small

Work done = $Fs\cos\theta = F_f s$

$= \mu_k N s = \mu_k s(mg - F\sin\theta)$ *decreases* with θ

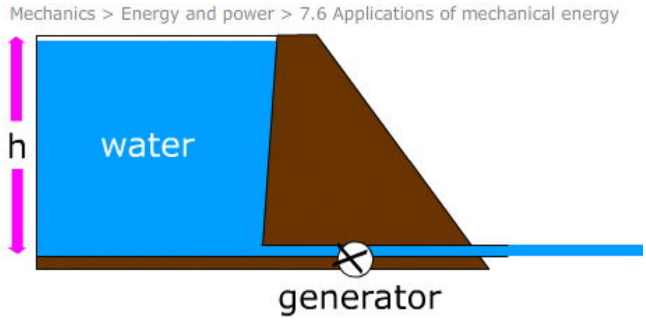


Puzzle There's a prize for the first completely correct answer to this one.



How high should h be so that it can loop the loop? Note the cross section of the track. h and R are measured from the rolling positions of the centre of the ball

Example. A hydroelectric dam is 100 m tall. Assuming that the turbines and generators are 100% efficient, and neglecting friction, calculate the flow of water required to produce 10 MW of power. The output pipes have a cross section of 5 m^2.



Nett effect: ~ stationary water lost from ***top*** of dam, water appears with speed v at bottom.

need power… $\frac{dW}{dt}$ *time derivative…* Let flow be $\frac{dm}{dt}$.

$$dW \equiv \begin{matrix}\text{work done}\\ \text{by water}\end{matrix}$$

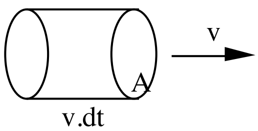
$$= -\begin{matrix}\text{Work done}\\ \text{on water}\end{matrix} = -\begin{matrix}\text{energy increase}\\ \text{of water}\end{matrix}$$

$$dW = -dE = -dK - dU$$

$$= -\left(\frac{1}{2}dmv^2 - 0\right) - (0 - dm.gh) = dm\left(gh - \frac{v^2}{2}\right)$$

$$P = \frac{dW}{dt} = \frac{dm}{dt}\left(gh - \frac{v^2}{2}\right)$$

Problem: v depends on $\frac{dm}{dt}$



$$\frac{dV}{dt} = \frac{A.(v.dt)}{dt} = Av$$

Density: $\rho \equiv \frac{\text{mass}}{\text{volume}} = \frac{m}{V}$ so $m = \rho V$

$$\frac{dm}{dt} = \rho\frac{dV}{dt} = \rho A v$$

$$P = \rho A v\left(gh - \frac{v^2}{2}\right)$$

$v^3 - (2gh)v + \frac{2P}{\rho A} = 0$ *we can solve a cubic, but it's messy. It's a one sig fig problem, so try an approximation*

Neglect v^3 –> $v = \frac{P}{gh\rho A} = 2$ m/s

and indeed we see that v^3 << other terms. Think about this: if you were designing the generator, would you have made the kinetic energy term comparable with the potential energy or work terms?

Flow = vA = 10 m^3/s –> 10 tonne/s

Some quantitative examples:

How much work is required to accelerate a car

i) from 0 to 10 km/hr?

ii) from 100 to 110 km/hr?

(As asked, this is work done by *total* force: it includes *negative* work done by air resistance)

Work energy theorem

$$W_{total} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

i) $\frac{1}{2}(1000\ \text{kg})\left(\frac{10\ 000\text{m}}{3\ 600\ \text{s}}\right)^2 - 0 = 4\ \text{kJ}$

ii) $W_{total} = \ldots \qquad = 80\ \text{kJ}$

$$dW = dK = d\left(\frac{1}{2}mv^2\right) = mv\,dv$$

Energy density:

Small rechargeable NiCad:

600 mA.hr and 1.25 V

–> (0.6 A)(3600 s)(1.25 V) = 3 kJ

(3 kJ)/(20 g) = 150 kJ/kg = 0.15 MJ/kg

Car battery:

Up to 100 Amp hours @ 12 V –> 4 MJ

< 0.5 MJ/kg

warning: don't try to extract this quickl

Lithium ion:

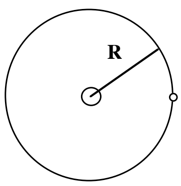
100 W.hour/kg -> 0.36 MJ/kg (some quote up to 0.9 MJ/kg)

	MJ/litre	MJ/kg
Petrol	29	45
LPG	22	34
Ethanol	19	30
Diesel	40	63

Speeding bullet

$$\frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2 \sim \frac{1}{2}(500\ \text{m/s})^2 = 0.1\ \text{MJ/kg}$$

Example: What is the intensity of solar radiation? $P_{sun} = 3.9\ 10^{26}$ W. Earth is 150 million km from sun.



$$\text{Intensity} \equiv \frac{P}{4\pi r^2} = ... = 1.38\ \text{kWm}^{-2}$$

called 'solar constant'

above atmosphere, $\perp$ *radiation*

PhysclipsWS > Travelling Waves > 3.3 Radiation and intensity

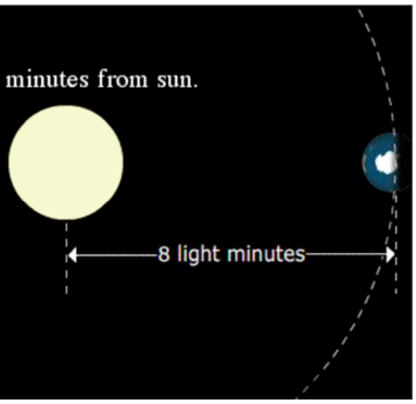
Example: $I_{solar} = 1.4\ \mathrm{kW.m^{-2}}$ at earth, 8 light minutes from sun.

Power of sun $= I.\ 4\pi r^2$

$= (1.4\ \mathrm{kW.m^{-2}}) 4\pi (8 \text{ minutes} * \text{speed of light})^2$

$= 4 \times 10^{26}$ Watts

$$I = \frac{P}{4\pi r^2} \quad \rightarrow \quad I \propto \frac{1}{r^2}$$

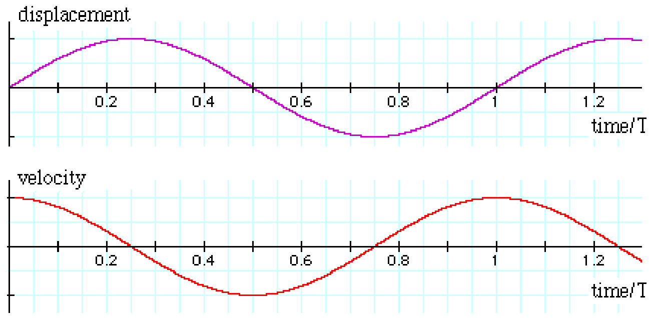


Energy in SHM **(Not in Mechanics syllabus but need in PHYS1231)**

See Oscillations in Physclips

$$x = A \sin \omega t \qquad v = \frac{dv}{dt} = A\omega \cos \omega t$$

$$U = \frac{1}{2} kx^2 \qquad K = \frac{1}{2} mv^2$$



$$E = U + K = \frac{1}{2} kA^2 \sin^2 \omega t + \frac{1}{2} mA^2\omega^2 \cos^2 \omega t$$

but $\omega = \sqrt{\frac{k}{m}}$ so $k = m\omega^2$

$$E = \frac{1}{2} m\omega^2A^2 \sin^2 \omega t + \frac{1}{2} mA^2\omega^2 \cos^2 \omega t$$

$$= \frac{1}{2} m\omega^2A^2 (\sin^2 \omega t + \cos^2 \omega t)$$

$$A\omega = v_{max}$$

$$E = K_{max} = U_{max}$$