The Ideal Gas law

I. Pressure

A cylinder contains an ideal gas that is at room temperature. The cylinder is sealed with a piston of mass M and cross sectional area A that is free to move up and down without friction. No gas can enter or leave the cylinder. The piston is at rest. Atmospheric pressure (i.e. the pressure of the air surrounding the cylinder) is \( P_0 \).

A. In the space provided, draw a free-body diagram for the piston. Make sure the label for each force indicates:
- the type of force
- the object on which the force is exerted
- the object exerting the force

B. In the space provided, draw an arrow to indicate the direction of the net force of the piston. If the net force is zero, state so explicitly.

C. Is the force exerted on the gas inside the cylinder greater than, less than, or equal to the force exerted on the piston by the air outside the cylinder? Explain.

Write an equation that relates all the forces on your free-body diagram. \( \text{(Hint: How are these forces related to the net force?)} \)

\[
\begin{align*}
P_0 A + Mg &= PA \\
P_0 &= \text{pressure outside (atmospheric pressure)} \\
P &= \text{Pressure of gas inside cylinder}
\end{align*}
\]

\( \text{net force} = 0 \Rightarrow \text{upwards and downwards forces cancel} \)
D. Is the pressure of the gas in the cylinder greater than, less than, or equal to atmospheric pressure? Explain.

Greater than. $P$ inside needs to exert a greater force as it needs to balance $Mg$ as well as $P_0$

Determine the value of the pressure of the gas in the cylinder in terms of the given quantities. (Hint: Which of the forces that acts on the piston can help you find the pressure of the gas?)

$$P = \frac{P_0 A + Mg}{A} \quad \Rightarrow \text{just substitute into } P = \frac{F}{A}$$

$$= P_0 + \frac{Mg}{A}$$

E. A second cylinder contains a different sample of ideal gas at room temperature, as shown in the diagram. The two cylinders and their pistons are identical.

Is the pressure of the gas in the second cylinder greater than, less than, or equal to the pressure in the cylinder above? If you cannot tell, state so explicitly. Explain.

It is the same. It does not depend on the type of ideal gas, the same equation applies.

II. Pressure and temperature

A. A cylinder of the type described in section I contains a fixed amount of gas. Initially, it is in thermal equilibrium with an ice-water bath. The pressure, volume and temperature of the gas are $P_{\text{initial}}, V_{\text{initial}}$, and $T_{\text{initial}}$, respectively.

The cylinder is then removed from the ice-water bath and placed into boiling water. After the system has come to thermal equilibrium with the boiling water the pressure, volume and temperature are $P_{\text{final}}, V_{\text{final}}$, and $T_{\text{final}}$.

1. Is $T_{\text{final}}$ greater than, less than, or equal to $T_{\text{initial}}$?

$$T_{\text{final}} > T_{\text{initial}}$$

If this is conducted at one atmosphere then $T_{\text{initial}} = 0^\circ C$, and as long as the water is still boiling $T_{\text{final}} = 100^\circ C$

2. Is $P_{\text{final}}$ greater than, less than, or equal to $P_{\text{initial}}$? Explain.

The same. The equation $P = P_0 + \frac{Mg}{A}$ still applies.
The forces have not been changed and so the pressure has not changed.
Is your answer consistent with your answer to part D of section I? If not, resolve any inconsistencies.

Yes. Students will need to answer this one themselves based on their own answers.

3. Is $V_{\text{final}}$ greater than, less than, or equal to $V_{\text{initial}}$? Explain.

\[ V_{\text{final}} > V_{\text{initial}} \]
The gas expands as its temperature increases. This comes about due to the ideal gas law
\[ PV = nRT \Rightarrow \frac{V}{T} = \frac{nR}{P} = \text{constant}. \]

Is your answer consistent with the ideal gas law (i.e., the relationship $PV = nRT$)?
If not, resolve any inconsistencies.

Yes. See answer above. Students will need to resolve their own inconsistencies here, based on their answers.

B. In the process you considered in part A above, which variables are held constant and which are allowed to change? Explain how you can tell.

$P, n, R$ are held constant. $V, T$ change.

$P$ is constant as applied force is constant

$n$ is constant as molecules do not enter or leave the cylinder

$R$ by definition is constant.

$T$ changes as in thermal equilibrium with water baths at different temperatures.

$V$ changes because it has to. See arguments above.

C. Consider the following student dialogue.

Student 1: “According to the ideal gas law, the pressure is proportional to the temperature. Since I increased the temperature of the gas, the pressure must go up.”

Student 2: “That’s right. Since no gas entered or left the system, the volume did not change. So the pressure must have increased.”

Do you agree with either of the students? Explain your reasoning.

No. Pressure is constant as it is due to $P_0$ and the mass of the piston.
When the temperature changes if the number of molecules stays constant $PV$ must change but this can either be due to $P$ changing, $V$ changing or both changing.
III. Pressure and temperature (again)

A. A cylinder of the type described in section I contains a fixed amount of gas and now the piston is fixed in place. Initially, it is in thermal equilibrium with an ice-water bath. The pressure, volume and temperature of the gas are $P_{\text{initial}}$, $V_{\text{initial}}$, and $T_{\text{initial}}$, respectively.

The cylinder is then removed from the ice-water bath and placed into boiling water. After the system has come to thermal equilibrium with the boiling water the pressure, volume and temperature are $P_{\text{final}}$, $V_{\text{final}}$, and $T_{\text{final}}$.

1. Is $T_{\text{final}}$ greater than, less than, or equal to $T_{\text{initial}}$?
   $T_{\text{final}}$ is greater than $T_{\text{initial}}$.

2. Is $P_{\text{final}}$ greater than, less than, or equal to $P_{\text{initial}}$? Explain.
   $P_{\text{final}}$ is greater than $P_{\text{initial}}$.

3. Is $V_{\text{final}}$ greater than, less than, or equal to $V_{\text{initial}}$? Explain.
   $V_{\text{final}}$ is equal to $V_{\text{initial}}$.

Is your answer consistent with the ideal gas law (i.e., the relationship $PV = nRT$)? If not, resolve any inconsistencies.

B. In the process you considered in part A above, which variables are held constant and which are allowed to change? Explain how you can tell.

$V$, $n$ and $R$ are constant. $P$ and $T$ change.
IV. Pistons and work

A fixed quantity of ideal gas is contained within a metal cylinder that is sealed with a movable, frictionless, insulating piston. (The piston can move up or down without the slightest resistance from friction, but no gas can enter or leave the cylinder. The piston is heavy, but there can be no heat transfer to or from the piston itself.) The cylinder is surrounded by a large container of water with high walls as shown.

![Diagram of piston and cylinder]

**Step 1.** Start of Process #1: The water container is gradually heated, and the piston very slowly moves upward. At time $B$ the heating of the water stops, and the piston stops moving when it is in the position shown in the diagram below:

![Diagram showing piston position at time B]

**Step 2.** Now, empty containers are placed on top of the piston as shown. Small lead weights are gradually placed in the containers, one by one, and the piston is observed to move down slowly. While this happens, the temperature of the water is nearly unchanged, and the gas temperature remains practically constant. (That is, it remains at the temperature it reached at time B, after the water had been heated up.)
Step 3. At time $C$ we stop adding lead weights to the container and the piston stops moving. (The weights that were added until now are still in the containers.) The piston is now found to be at exactly the same position it was at time $A$.

Step 4. Now, the piston is locked into place so it cannot move; the weights are removed from the piston. The system is left to sit in the room for many hours, and eventually the entire system cools back down to the same room temperature it had at time $A$. When this finally happens, it is time $D$. 
A During the process that occurs from time $A$ to time $B$, is positive work done on or by the environment or is no net work done?

Positive work is done on the environment by the gas.

(Work done by the gas is given by: $W = \int P \cdot dV$

Work done on the gas is given by: $W = -\int P \cdot dV$

Different textbooks use different definitions of '$W$'.

B During the process that occurs from time $A$ to time $B$, the gas absorbs $x$ Joules of energy from the water. Does the total kinetic energy of all of the gas molecules increase/decrease by more than/less than/equal to $x$ or remains unchanged?

It increases by less than $x$ Joules as $\Delta E_{\text{int}} = \text{total KE} = \text{heat added} + \text{work done on the gas}$

C During the process that occurs from time $B$ to time $C$, does the total kinetic energy of all gas molecules increase, decrease or remain unchanged?

Total KE remains unchanged as $\Delta E_{\text{int}} = \frac{1}{2} n R \Delta T$ and $T$ does not change.

D During the process that occurs from time $B$ to time $C$, is there any net heat transferred between the gas and the water? If yes, please specify from where to where, if no, please state that explicitly.

Yes. In this case positive work is done on the gas. The total energy does not change so heat flows from the gas to the water.

E During the process that occurs from time $C$ to time $D$, $y$ Joules of heat transfer occurs from the gas to the water. Does the total kinetic energy of all the gas molecules increase/decrease by more than/less than/equal to $y$ or remains unchanged?

It decreases by $y$ Joules: decreases as the temperature decreases, no work is done in this case as $V$ is constant.

F For the process occurring from time $A$ to time $D$, is the net work done by the gas on the environment equal to, less than or greater than zero?

The net work is equal to zero as the piston is in the same position.

G Is the heat transfer for the process occurring from time $A$ to time $D$ equal to, less than or greater than zero?

The total heat transfer must be zero as the total change in internal energy is zero and so is the work.

<table>
<thead>
<tr>
<th></th>
<th>$Q$</th>
<th>$W_{\text{on gas}}$</th>
<th>$\Delta E_{\text{int}} \equiv \text{KE}$</th>
<th>$\text{water}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \rightarrow B$</td>
<td>$+$</td>
<td>$-$</td>
<td>$+ (\Delta E_{\text{int}} &lt; Q)$</td>
<td>$+$</td>
</tr>
<tr>
<td>$B \rightarrow C$</td>
<td>$-$</td>
<td>$+$</td>
<td>$0$</td>
<td>$\sim 0$</td>
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<tr>
<td>$C \rightarrow D$</td>
<td>$-$</td>
<td>$0$</td>
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Part I to III of this worksheet comes from “Tutorials in Introductory Physics” by L. McDermott, P. Shaffer and the physics education group at the University of Washington.