Atomic Physics and Search for Variation of Fundamental Constants

V. A. Dzuba
School of Physics, UNSW
Theoretical arguments for fundamental constants to vary:

- **Extra space dimensions** (Kaluza-Klein theories, Superstring and M-theories, etc.). Extra space dimensions is a common feature of theories unifying gravity with other interactions. Any change in size of these dimensions would manifest itself in the 3D world as variation of fundamental constants.

- **Scalar fields** (Bekenstein theory, etc.). Fundamental constants appear as expectation values of some scalar fields which don’t have to be stationary in the non-stationary Universe.
Search for variation of fundamental constants

- Big Bang Nucleosynthesis
- Cosmic Microwave Background Radiation
- Quasar Absorption Spectra
- Oklo natural nuclear reactor
- Analysis of meteorite data
- Atomic clocks

1 Based on analysis of atomic spectra
Which Constants?

- Since variation of **dimensional** constants cannot be distinguished from variation of **units**, it only makes sense to consider variation of **dimensionless** constants.
- Quasar absorption spectra depends on the fine structure constant $\alpha = \frac{e^2}{hc} = \frac{1}{137.036}$
- Atomic clocks:

<table>
<thead>
<tr>
<th>Optical transitions</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microwave transitions</td>
<td>$\alpha, \frac{m_e, q}{\Lambda_{QCD}}$</td>
</tr>
</tbody>
</table>
Quasar absorption spectra

Earth — Gas cloud — Quasar

Light

α
Quasar absorption spectra

One needs to know $E(\alpha^2)$ for each line to do the fitting.
Alkali Doublet Method
(Varshalovich, Potekhin, Ivanchik, et al)

Fine structure interval

\[ \Delta_{FS} = E(p_{3/2}) - E(p_{1/2}) = A(Z\alpha)^2 \]

If \( \Delta_Z \) is observed at red shift \( Z \) and \( \Delta_0 \) is FS measured on Earth then

\[ \frac{\Delta \alpha}{\alpha} = \frac{1}{2} \left( \frac{\Delta Z}{\Delta_0} - 1 \right) \]

Ivanchik et al, 1999: \( \Delta\alpha/\alpha = -3.3(6.5)(8) \times 10^{-5} \).
Murphy et al, 2001: \( \Delta\alpha/\alpha = -0.5(1.3) \times 10^{-5} \).
Many Multiplet Method
(Flambaum, Webb, Murphy, et al)

Advantages:
• Order of magnitude gain in sensitivity
• Statistical: all lines are suitable for analysis
• Many opportunities to study systematic errors

δω >> δΔ_{FS}!
Complication: no simple formula for $\omega(\alpha)$.
Solution: use atomic calculations!

For $\alpha$ close to $\alpha_0$, $\omega = \omega_0 + q(\alpha^2/\alpha_0^2 - 1)$

$q$ is found by varying $\alpha$ in computer codes:

$q = d\omega/dx = [\omega(0.1) - \omega(-0.1)]/0.2, \quad x = \alpha^2/\alpha_0^2 - 1$

In atomic units $e=1$, $h=1$, $\alpha=1/c$

Variation of $\alpha$ corresponds to variation of speed of light and $\alpha=0$ corresponds to non-relativistic limit!
\[ \omega = \omega_0 + q \left( \frac{\alpha^2}{\alpha_0^2} - 1 \right) \]
## Atoms of interest

<table>
<thead>
<tr>
<th>Z</th>
<th>Atom / Ion</th>
<th>Transitions</th>
<th>( N_{\text{ve}} ) (^1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>C I, C II, C III</td>
<td>p-s</td>
<td>4, 3, 2</td>
</tr>
<tr>
<td>8</td>
<td>O I</td>
<td>p-s</td>
<td>4</td>
</tr>
<tr>
<td>11</td>
<td>Na I</td>
<td>s-p</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>Mg I, Mg II</td>
<td>s-p</td>
<td>2, 1</td>
</tr>
<tr>
<td>13</td>
<td>Al II, Al III</td>
<td>s-p</td>
<td>2, 1</td>
</tr>
<tr>
<td>14</td>
<td>Si II, Si IV</td>
<td>p-s</td>
<td>3, 1</td>
</tr>
<tr>
<td>16</td>
<td>S II</td>
<td>s-p</td>
<td>4</td>
</tr>
<tr>
<td>20</td>
<td>Ca II</td>
<td>s-p</td>
<td>1</td>
</tr>
<tr>
<td>22</td>
<td>Ti II</td>
<td>s-p, d-p</td>
<td>3</td>
</tr>
<tr>
<td>24</td>
<td>Cr II</td>
<td>d-p</td>
<td>5</td>
</tr>
<tr>
<td>25</td>
<td>Mn II</td>
<td>s-p, d-p</td>
<td>1</td>
</tr>
<tr>
<td>26</td>
<td>Fe II</td>
<td>s-p, d-p</td>
<td>7</td>
</tr>
<tr>
<td>28</td>
<td>Ni II</td>
<td>d-p</td>
<td>9</td>
</tr>
<tr>
<td>30</td>
<td>Zn II</td>
<td>s-p</td>
<td>1</td>
</tr>
</tbody>
</table>

\(^1N_{\text{ve}}\) – number of valence electrons
Methods of Atomic Calculations

<table>
<thead>
<tr>
<th>$N_{ve}$</th>
<th>Method</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Correlation Potential Method</td>
<td>0.1-1%</td>
</tr>
<tr>
<td>2-6</td>
<td>Configuration Interaction + Many-Body Perturbation Theory</td>
<td>1-10%</td>
</tr>
<tr>
<td>2-15</td>
<td>Configuration Interaction</td>
<td>10-20%</td>
</tr>
</tbody>
</table>

These methods cover all periodic system of elements.

They were used for many important problems:
- Saving Standard Model from PNC in Cs.
- Predicting spectrum of Fr, etc., etc., etc.
Fine structure anomalies and level crossing

Energies of “normal” fine structure doublets as functions of $\alpha^2$

$$\Delta E = A(Z\alpha)^2$$
Fine structure anomalies and level crossing

Energies of “normal” fine structure triplets as functions of $\alpha^2$

$$\Delta E = A(Z\alpha)^2$$
Fine structure anomalies and level crossing

Energies of strongly interacting states as functions of $\alpha^2$

$$\Delta E = A(Z\alpha)^2$$
Implications to study of $\alpha$ variation

- Not every fine structure interval can be used in the analysis based on formula $\Delta E = A(Z\alpha)^2$ (not good!).
- Strong enhancement is possible (good, but for atomic clocks only).
- Level crossing may lead to instability of calculations (bad!).
Problem: level pseudo crossing

Energy levels of Ni II as functions of $\alpha^2$

Values of $q = dE/d\alpha^2$ are sensitive to the position of level crossing
Problem: level pseudo crossing

Values of \( q = \frac{dE}{d\alpha^2} \) are sensitive to the position of level crossing.

Solution: matching experimental \( g \)-factors
Pb II: g-factors don’t help

Two $^3D_{3/2}$ states are strongly mixed, but g-factors do not depend on mixing.

Solution: perform calculations with extremely high accuracy.
## Results of calculations

### Anchor lines

<table>
<thead>
<tr>
<th>Atom</th>
<th>$\omega_0$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mg I</td>
<td>35051.217</td>
<td>86</td>
</tr>
<tr>
<td>Mg II</td>
<td>35760.848</td>
<td>211</td>
</tr>
<tr>
<td>Mg II</td>
<td>35669.298</td>
<td>120</td>
</tr>
<tr>
<td>Si II</td>
<td>55309.3365</td>
<td>520</td>
</tr>
<tr>
<td>Si II</td>
<td>65500.4492</td>
<td>50</td>
</tr>
<tr>
<td>Al II</td>
<td>59851.924</td>
<td>270</td>
</tr>
<tr>
<td>Al III</td>
<td>53916.540</td>
<td>464</td>
</tr>
<tr>
<td>Al III</td>
<td>53682.880</td>
<td>216</td>
</tr>
<tr>
<td>Ni II</td>
<td>58493.071</td>
<td>-20</td>
</tr>
</tbody>
</table>

### Negative shifters

<table>
<thead>
<tr>
<th>Atom</th>
<th>$\omega_0$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ni II</td>
<td>57420.013</td>
<td>-1400</td>
</tr>
<tr>
<td>Ni II</td>
<td>57080.373</td>
<td>-700</td>
</tr>
<tr>
<td>Cr II</td>
<td>48632.055</td>
<td>-1110</td>
</tr>
<tr>
<td>Cr II</td>
<td>48491.053</td>
<td>-1280</td>
</tr>
<tr>
<td>Cr II</td>
<td>48398.862</td>
<td>-1360</td>
</tr>
<tr>
<td>Fe II</td>
<td>62171.625</td>
<td>-1300</td>
</tr>
</tbody>
</table>

### Positive shifters

<table>
<thead>
<tr>
<th>Atom</th>
<th>$\omega_0$</th>
<th>$q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fe II</td>
<td>62065.528</td>
<td>1100</td>
</tr>
<tr>
<td>Fe II</td>
<td>42658.2404</td>
<td>1210</td>
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<tr>
<td>Fe II</td>
<td>42114.8329</td>
<td>1590</td>
</tr>
<tr>
<td>Fe II</td>
<td>41968.0642</td>
<td>1460</td>
</tr>
<tr>
<td>Fe II</td>
<td>38660.0494</td>
<td>1490</td>
</tr>
<tr>
<td>Fe II</td>
<td>38458.9871</td>
<td>1330</td>
</tr>
<tr>
<td>Zn II</td>
<td>49355.002</td>
<td>2490</td>
</tr>
<tr>
<td>Zn II</td>
<td>48841.077</td>
<td>1584</td>
</tr>
</tbody>
</table>

Also, many transitions in Mn II, Ti II, Si IV, C II, C IV, N V, O I, Ca I, Ca II, Ge II, O II, Pb II

Complicated behaviour of atomic spectra provides opportunity to study systematic errors!
Results of the analysis

- Murphy et al, 2003: Keck telescope, 143 systems, 23 lines, 0.2<z<4.2
  \[ \Delta \alpha/\alpha = -0.543(116) \times 10^{-5} \]

- Quast et al, 2004: VLT telescope, 1 system, Fe II, 6 lines, 5 positive q-s, one negative q, z=1.15
  \[ \Delta \alpha/\alpha = -0.4(1.9)(2.7) \times 10^{-6} \]

- Srianand et al, 2004: VLT telescope, 23 systems, 12 lines, Fe II, Mg I, Si II, Al II, 0.4<z<2.3
  \[ \Delta \alpha/\alpha = -0.06(0.06) \times 10^{-5} \]
Atomic clocks

Cesium primary frequency standard:

\[ \nu = 9\,192\,631\,770\,\text{Hz}\]

HFS of 6s:

- F=4
- F=3

Also: Rb, Cd\(^+\), Ba\(^+\), Yb\(^+\), Hg\(^+\), etc.

E.g. \(\nu(\text{Hg}^+) = 40\,507\,347\,996.841\,59(14)(41)\,\text{Hz}\)
Optical frequency standards:

<table>
<thead>
<tr>
<th>Z</th>
<th>Atom</th>
<th>Transition</th>
<th>Frequency</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>Ca</td>
<td>$^1S_0-^3P_1$</td>
<td>455 986 240 494 144(5.3) kHz</td>
<td>Degenhardt et al, 2005</td>
</tr>
<tr>
<td>38</td>
<td>Sr$^+$</td>
<td>$^1S_0-^3P_1$</td>
<td>434 829 121 311(10) kHz</td>
<td>Ferrari et al, 2003</td>
</tr>
<tr>
<td>49</td>
<td>In$^+$</td>
<td>$^1S_0-^3P_0$</td>
<td>1 267 402 452 899 920(230) Hz</td>
<td>von Zanthier et al, 2005</td>
</tr>
<tr>
<td>70</td>
<td>Yb$^+$</td>
<td>$^2S_{1/2}-^2F_{7/2}$</td>
<td>642 121 496 772 300(600) Hz</td>
<td>Hosaka et al, 2005</td>
</tr>
</tbody>
</table>

Also: Al$^+$, Sr, Ba$^+$, Yb, Hg, Hg$^+$, Tl$^+$, Ra$^+$, etc.

Accuracy about $10^{-15}$ can be further improved to $10^{-18}$!
Opportunities:

Comparing rates of different clocks over long period of time can be used to study time variation of fundamental constants!

Optical transitions: $\alpha$

Microwave transitions: $\alpha$, $m_e$, $m_q/\Lambda_{QCD}$
Advantages:

• Very narrow lines, high accuracy of measurements.

• Flexibility to choose lines with larger sensitivity to variation of fundamental constants.

• Simple interpretation (local time variation).
Calculations to link change of frequency to change of fundamental constants:

Microwave transitions: analytical formula or atomic calculations.

\[ A_s = A_0 \alpha^2 F(\alpha Z) \]

Optical transitions: atomic calculations (as for quasar absorption spectra).

\[ \omega = \omega_0 + q(\alpha^2/\alpha_0^2 - 1) \]
## Results for variation of fundamental constants

<table>
<thead>
<tr>
<th>Source</th>
<th>Clock$_1$/Clock$_2$</th>
<th>$d\alpha/dt/\alpha\left(10^{-15} \text{ yr}^{-1}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Marion et al, 2003</td>
<td>Rb(hfs)/Cs(hfs)</td>
<td>0.05(1.3)$^a$</td>
</tr>
<tr>
<td>Bize et al, 2003</td>
<td>Hg+(opt)/Cs(hfs)</td>
<td>-0.03(1.2)$^a$</td>
</tr>
<tr>
<td>Fisher et al, 2004</td>
<td>H(opt)/Cs(hfs)</td>
<td>-1.1(2.3)$^a$</td>
</tr>
<tr>
<td>Peik et al, 2004</td>
<td>Yb+(opt)/Cs(hfs)</td>
<td>-0.2(2.0)</td>
</tr>
<tr>
<td>Bize et al, 2004</td>
<td>Rb(hfs)/Cs(hfs)</td>
<td>0.1(1)$^a$</td>
</tr>
</tbody>
</table>

$^a$assuming $m_q/\Lambda_{QCD} = \text{Const}$

Combined results:

\[
d/dt \ln\alpha = -0.9(2.9) \times 10^{-15} \text{ yr}^{-1}
\]

\[
d/dt \ln(m_q/\Lambda_{QCD}) = -4 \ (10) \times 10^{-15} \text{ yr}^{-1}
\]
Search for enhancement

If \( \omega = \omega_0 + q(\alpha^2/\alpha_0^2 - 1) \) then \( \Delta \omega/\omega_0 = 2q/\omega_0 \Delta \alpha/\alpha \)

\[ K = 2q/\omega_0 \]

is an enhancement factor.

For a transition between excited states:

\[ K = 2\Delta q/\Delta \omega \]

We should look for sufficiently different states
(large \( \Delta q \)) separated by small energy interval!

For atomic clocks \( K = 1 - 2 \) (no enhancement!).
Dysprosium miracle

Dy: $4f^{10}5d6s \ E=19797.96\ldots \ \text{cm}^{-1}, \ q=6000 \ \text{cm}^{-1}$

$4f^95d^26s \ E=19797.96\ldots \ \text{cm}^{-1}, \ q=-23000 \ \text{cm}^{-1}$

Interval $\Delta \omega = 10^{-4} \ \text{cm}^{-1}$

Enhancement factor $K = 10^8$ (!), i.e. $\Delta \omega / \omega_0 = 10^8 \Delta \alpha / \alpha$

Preliminary result (Budker et al., Berkeley)

$|d\ln \alpha / dt| < 4.3 \times 10^{-15} \ \text{yr}^{-1}$

Problem: states are not narrow!
We have:

- Atomic clocks: narrow states (good!), no enhancement (bad!).

  Is there anything in between?
  (narrow states + strong enhancement ?)

- Dysprosium: broad states (bad!), HUGE enhancement (good!).
For a “normal” multiplet:

- Lande rule: $\Delta E_{J,J_1} = AJ$
- $A < 0$, if $n_e > n_p$
- $A = c(Z\alpha)^2$
Fine structure anomaly in Te I

Real energy levels of the $p^4$ ground state configuration of Te I as functions of $\alpha^2$

$E^{(3P_1)} - E^{(3P_0)} = 5 \text{ cm}^{-1}$!

Enhancement factor $K = 100$
i.e. $\Delta \omega/\omega_0 = 100 \Delta \alpha/\alpha$

Also, all states are metastable!
More suggestions …

<table>
<thead>
<tr>
<th>Atom</th>
<th>State₁</th>
<th>State₂</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ce I</td>
<td>⁵H₃</td>
<td>¹D₂</td>
<td>2000</td>
</tr>
<tr>
<td></td>
<td>³H₄</td>
<td>³D₂</td>
<td>13000</td>
</tr>
<tr>
<td>Nd I</td>
<td>⁵K₆</td>
<td>⁷L₅</td>
<td>950</td>
</tr>
<tr>
<td></td>
<td>⁷L₅</td>
<td>⁷K₆</td>
<td>10⁵</td>
</tr>
<tr>
<td>Sm I</td>
<td>⁵D₁</td>
<td>⁷G₂</td>
<td>300</td>
</tr>
<tr>
<td>Gd II</td>
<td>⁸D₁₁/₂</td>
<td>¹⁰F₉/₂</td>
<td>1800</td>
</tr>
<tr>
<td>Tb I</td>
<td>⁶H₁₃/₂</td>
<td>⁸G₉/₂</td>
<td>600</td>
</tr>
</tbody>
</table>

Conclusion

• Analysis of quasar absorption spectra indicate that $\alpha$ might be smaller in early epoch. However, discrepancy between different groups must be resolved.

• Comparing the rates of different atomic clocks puts strong constraints on the variation of fundamental constants. Fast progress in the field promises new interesting results.

• All results involving optical atomic transitions were obtained using our calculations.
Publications:

- V. A. Dzuba, V. V. Flambaum, PRA 61, 034502 (2000).
- V. A. Dzuba, V. V. Flambaum, PRA, 72, 052514 (2005).
- S. G. Karshenboim et al, physics/0511180.