The modern flute differs little from the mid-19th century design of Theobald Boehm. The modern flute can be played in almost 40,000 acoustically unique key configurations (fingerings), and each fingering plays a small number of different notes and possibly multiphonics (more than one note at the same time, or chords). Despite 150 years of flute playing worldwide, only several hundred fingerings have been investigated in detail.

The acoustic impedance at the input (blowing) hole of the flute tells a lot about the flute’s operating modes.

\[ Z(f) = \frac{P(f)}{U(f)} \]

Acoustic impedance (Z) is the ratio of acoustic air pressure (P) to air flow (U), and these quantities are strongly dependent on frequency. When the input impedance of a flute is relatively low, air flow is large at the input hole and resonance occurs at the corresponding frequency (i.e. a sound of a certain pitch is produced).

The deep minima of input impedance spectra in most cases correspond to playable musical notes. The frequency of minima determine the pitch of notes, and the depth of minima have a large bearing on the playability of these notes. Thus the development of an accurate acoustic impedance simulation has the potential to predict many of the features of the modern flute’s entire musical response.

Figure 1 shows the measured impedance spectra of the flute for two different fingerings\(^1\). The first fingering is the standard fingering for G4 and G5. Indeed, two deep minima exist at G4 and G5. To distinguish between these two notes, flutists vary their lip position and increase their blowing pressure to excite higher notes. The second fingering is the standard fingering for D6. The fingering also plays the note C5, and since C5 and D6 are not harmonically related (their frequencies not in a simple integer ratio), both notes can be played together as the multiphonic C5&D6.

To simulate acoustic impedance, a one-dimensional waveguide model of the flute is used to approximate its complex geometry. The flute is divided into a network of cylindrical pipes and conical horns — geometries for which theoretical calculations of impedance are well-established. Figure 2 demonstrates the calculation of acoustic impedance for the flute. Each bore segment and tone hole is approximated by a cylinder. Beginning at the far end of the flute, \( Z_1 \) — the input impedance of bore segment 1 — is calculated with \( Z_{L1} \) as its load. The input impedance of tone hole 2 is also calculated to give \( Z_2 \). \( Z_1 \) and \( Z_2 \) are then combined in parallel as the load to bore segment 3, and thus the calculation continues to the input of the instrument. The calculation is repeated for frequencies between 0.2 and 4 kHz to give an impedance spectrum for a particular fingering.

To simulate acoustic impedance as accurately as possible, a number of parameters must be adjusted to compensate for the inadequacies of a one-dimensional model. These parameters include the height of tone holes and the factors used to represent wall losses. Simpler geometries — such as cylindrical pipes, pipes with side holes and shortened flutes — were firstly studied to determine these parameters in an incremental fashion\(^2\). The resulting flute simulation is shown in figure 3. It compares the impedance spectrum of the one-dimensional flute model with the measured spectrum for the D6 standard fingering. Agreement between the two is excellent.
To evaluate the playability of impedance minima, the impedance spectra of 76 different fingerings were selected and measured for data analysis. 957 impedance minima were found in the measured spectra. To define the playability of these minima, Jane Cavanagh (principal flutist of the UNSW Orchestra) was asked to play and evaluate each of the 76 selected fingerings. Jane defined the playability of impedance minima on a scale of 0 - 3, and these results (along with the extracted physical parameters of each minimum) formed the training set of an expert system.

C5.0 was used to build a decision tree which classifies impedance minima as either playable or unplayable. Figure 4 shows this decision tree. Cubist was used to build a linear model which ranks playable minima on Jane’s scale. This expert system, together with the impedance simulation, predicts the playable notes of the flute (of which there are ~150,000) for every fingering in a ~12 hour computer process. For any particular fingering, pairs and triplets of predicted notes which are not harmonically related are predicted as multiphonics.

The predicted results of impedance calculations and data analysis are presented as a musically useful web service. Alternate fingerings and multiphonics are stored in a relational database, searchable in a structured manner. *The Virtual Boehm Flute* is permanently located at:

http://www.phys.unsw.edu.au/music/flute/virtual/