# Acoustics of a Tamtam

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**Chinese tamtams are characterized by a delayed "shimmer" due to high frequency modes of vibration excited by nonlinear coupling to modes of lower frequency. Although the exact nature of this coupling is not known, it can be described by a semi-quantitative theory which is consistent with the experimental data. Coupling between modes of high and low axial symmetry appears to depend upon a ring of hammered bumps. Modes of low frequency have decay times as long as 18 seconds, whereas modes of higher frequency decay more rapidly.**

1. **INTRODUCTION:** Among the many percussion instruments of Oriental origin that have been adopted into Western music, the tamtam is one of the most interesting acoustically. The sound of a large Chinese tamtam, which can be the loudest of any instrument in an orchestra, reaches full brilliance one or two seconds after being struck and may continue for up to one minute if the instrument is not damped. The tamtam is also widely used for creating special effects, such as the sound produced by the Herculean figure in the familiar screen trademark of the J. Arthur Rank Film Corporation.[1]

When the tamtam is struck somewhere near its center with a large padded mallet, the initial sound is one of very low pitch, but in a few seconds a louder sound of high pitch builds up, then slowly decays, leaving once again a lingering sound of a low pitch. The high-pitched sound fails to develop if the initial blow is not hard enough.

This paper describes the vibrational behaviour of the tamtam and attempts to analyze the conversion of energy from vibrational modes of low frequency to those of higher frequency. Due to the complexity of the vibrating system an exact theoretical description of the nonlinear mode conversion is not possible. Nevertheless, the semi-quantitative theory presented is consistent with the experimental measurements described.

## 2. THEORETICAL CONSIDERATIONS

### 2.1 The tamtam

Tamtams are of varying size up to about one meter in diameter. They are usually made of bronze (approximately 80% copper and 20% tin, with occasional traces of lead or iron). Although the center is usually raised slightly, they do not have a prominent central dome as do gongs and cymbals. They do, however, have one or more circles of hammered bumps and a fairly deep rim. They are considerably thinner than most large gongs. Our tamtam is shown in Fig. I.

### 2.2 Modes of vibration

Because of their large size, local variations in thickness, and numerous bumps and hammer marks, the modes of vibration of a tamtam show only faint resemblance to the normal modes of flat plate.[2,3] The low-frequency domain has several prominent axisymmetric modes, which absorb much of the energy of the initial blow.

Other families of modes of considerable interest are those that have numbers of radial nodes equal to the number (or an integer multiple) of hammered bumps in one of the circles. These modes would be favoured in the delayed sound if the bumps play a prominent role in the conversion of energy from axisymmetric to non-symmetric modes as suspected.

### 2.3 Nonlinear coupling

Any detailed analysis of the behaviour of the higher modes of the tamtam must, of course, await an understanding of the precise nature of the physical nonlinearities involved. It is possible, however, to write down some general results which are independent of the precise physics of the problem and which provide a framework against which the observed behaviour can be discussed.

In a linear system we can analyze the motion in terms of normal modes which are completely non-interacting. If $x_i(r,\theta)$ is the displacement associated with the $i$ th normal mode then, after the initial strike, we can write

$$M_i\ddot{x}_i + R_i\dot{x}_i + K_i x_i = 0 \tag{1}$$

where $M_i$, $R_i$ and $K_i$ are respectively a generalized mass, resistance and spring constant associated with this mode.

In the nonlinear system $K_i$ is not a constant but has the form

$$K_i = K_i^0 + K_i'(x_1, x_2, \ldots) \tag{2}$$

where $K_i'$ is a general nonlinear function of all the mode amplitudes. For such a system we can no longer simply separate the modes as in (1) but rather we must write down a complete equation for the whole system, which then has a form like

$$\sum_j M_j\ddot{x}_j + \sum_j R_j\dot{x}_j + \sum_j K_j^0 x_j + \sum_j K_j'(x_j, x_k, \ldots)\, x_j = 0 \tag{3}$$

If the frequency of mode $i$ is $\omega_i$ then (1) appears simply as the $\omega_i$ Fourier component of (3) when the nonlinear terms in $K_i'$ are neglected. More generally, however, we must retain these terms and so arrive at an equation of the form

$$M_i \ddot{x}_i + R_i \dot{x}_i + K_i^0 x_i = F_i\ (x_j, x_k \ldots) \quad (4)$$

where $F_i$ is essentially the sum of those terms in $\Sigma K_j' x_j$ with frequencies close to $\omega_i$. Any term in $F_i$ with phase equal to that of $x_i$ will simply modify the mode frequency $\omega_i$ while terms in $F_i$ in quadrature with $x_i$ will feed energy into or out of this $i$ th mode. The first effect is important in understanding the pitch change behaviour of certain Chinese gongs, on which we have commented elsewhere,[4] while the second effect will concern us primarily here.

In the tamtam the initial strike with a large soft-headed mallet excites primarily the first mode of frequency $\omega_1$ for which the mode shape $x_1$ (r) is close to $a_1 J_0(kr)$ where $k$ is determined so that the Bessel function goes through its first zero near the edge of the gong. If we make the simplifying assumption that this is the only mode excited, then, by expanding $F(x_1)$ as a Taylor series without any explicit assumption about its form, and by noting that $\cos^n\theta$ has a leading term $\cos n\theta$, we see that the component in $F(x_1)$ with frequency $n\omega_1$ is proportional to $a_1^n$.

The normal modes of a tamtam are not harmonically related, but we can always define, for the $i$ th mode, an integer $n$ which is closest to the ratio $\omega_i/\omega_1$, and it is this $n$ th component of $F(x_1)$ that is most important in driving $x_i$. (We return to more complicated possibilities later.) There will always be some fraction of this driving force in quadrature with $x_i$ so that we can see immediately that the amplitude $a_i$ of the $i$ th mode grows like

$$da_i/dt = A_{1i}\, a_1^n \simeq A_{1i}\, a_1^{\omega_i/\omega_1} \quad (5)$$

Here $A_{1i}$, to which we return presently, is a coupling coefficient between modes $i$ and $1$.

In the more general case in which several modes are excited by the initial strike we must consider them not only separately but also in interaction. Thus, if two modes $\omega_j$ and $\omega_k$ are excited with amplitudes $a_j$ and $a_k$, we must consider all terms in $F(x_j, x_k)$ of the form $a_j^n a_k^m \cos(n\omega_j \pm m\omega_k)$ where $n\omega_j \pm m\omega_k \simeq \omega_i$.

Several such terms may combine to give a periodically-varying force amplitude near $\omega_i$ and a consequent complicated behaviour of $a_i$.

## 2.4 Time variation of the radiated spectrum

The lowest modes are probably damped largely by radiation, as we have said before. Their decay therefore follows a simple law like

$$a_1(t) = a_1(0)\exp(-t/\tau_1) \quad (6)$$

The upper modes, in contrast, are pumped by the fundamental through the nonlinear coupling, the strength of which for an $n$ th order coupling varies like $[a_1(t)]^n$. The $i$ th mode has its own decay time $\tau_i$, however and, if the pumping term were constant, its amplitude would grow like

$$a_1(t) = A_{1i} a_1^n \tau_i [1 - \exp(-t/\tau_i)] \quad (7)$$

If it is being pumped by the $n$ th harmonic of $\omega_1$, however, its form should be like

$$a_i(t) = A_{1i} a_1^n \tau_i [1 - (n\tau_i/\tau_1)]^{-1} \times [(\exp(-nt/\tau_1) - \exp(-t/\tau_i)] \quad (8)$$

The behaviour suggested by (6) and (8) does seem to agree with experience.

## 2.5 Variation of radiated spectrum with strike force

The forms of (5) and (8) suggest a possibility for analysis of the radiated sound as a function of the force of the strike exciting the tamtam. Indeed, if $\tau_i$ and $\tau_1$ remain unchanged by the force of the blow, which implies that the primary energy loss mechanisms for the fundamental mode remain radiation and internal losses rather than transfer to higher modes, we can conclude from (5) and (8) that

$$(a_i)_{max} \propto A_{1i}\, a_1^{\omega_i/\omega_1} \quad (9)$$

This equation, however, ignores the fact that, in general, mode $i$ will also be excited directly by the initial strike, in a manner which is linear and therefore exactly proportional to $a_1$. We therefore expect

$$(a_i)_{max} \propto B_i a_1 + A_{1i} a_1^{\omega_i/\omega_1} \quad (10)$$

where $B_i$ is another coupling coefficient. Such an equation is a worthwile basis for the analysis though it neglects the possibility of interactions in which two or more modes combine to excite $x_i$.

## 2.6 Coupling coefficients

The analysis above is quite general for any non-linear system and is specialized to the tamtam by defining on a physical basis the coupling coefficients $A_i$ and $B_i$ for the modes involved. In a plate with radial symmetry, like a cymbal, the normal modes are relatively simple in analytical form and can be written as

$$x_{nm} = R_{nm}(r)\cos m\theta \quad (11)$$

It is then clear that an impact at the centre of the plate can excite only modes with circular symmetry $(m = 0)$ and also that these $m = 0$ modes can couple to other modes through the coefficients $A_{ij}$ only if those other modes also have $m = 0$. If, however, the strike is at the edge, then modes with all m values are excited but again the $A_{ij}$ are non-zero only for pairs of modes with the same m values.

The tamtam, however, does not have circular symmetry and the modes $x(r,\theta)$ cannot be written in separable form as in (11). This means that, in general, the initial excitation coefficients $B_i$ and the coupling coefficients $A_{jk}$ will be non-zero for all modes and pairs of modes. Considerations of symmetry and physical causes suggest, however, that for initial excitation at the centre, the $B_i$ and $A_{jk}$ will be large for all modes and pairs of modes having the same angular symmetry (or a multiple of it) as the tamtam itself. The $B_i$ will, however, be generally large for modes resembling the $J_0(kr)$ modes of a flat circular plate and small for the modes resembling $J_n(kr) \cos n\theta$. Non-linearity in the nearly flat central part of the tamtam will give significant coupling between the $J_0$-like modes, while nonlinearity at the ring of bumps will couple the $J_0$ modes strongly to $J_n$-like modes with appropriate angular symmetry.

## 3. EXPERIMENTAL STUDIES

### 3.1 Description of the tamtam

The tamtam used in these experiments was 95.5 cm in diameter and approximately 2 mm thick, with a rim about 3 cm deep. Close to the rim is a circle of 101 hammered bumps, and less prominent bumps lie in circles roughly ½ and ¾ of the overall diameter. It was fabricated in Japan and is shown in Fig. 1.

### 3.2 Modes of vibration

A Bruel and Kjaer Type 8001 impedance head and Type 4810 vibration exciter were used to measure the mechanical admittance as a function of driving frequency. The driving point force was kept constant by use of a Gen Rad 1569 level controller, and the accelerometer output was amplified and integrated with a Bruel and Kjaer Type 2651 charge amplifier connected through the tracking filter of a Gen Rad 1900A analyzer to a Gen Rad 1521B chart recorder. Several driving points were used for each plate, including the center, near the edge, and at half the radius.

We attempted to determine the modal configuration for as many plate resonances as possible. Normally this was accomplished by moving a small microphone in the near field of the radiated sound. The plate-to-microphone spacing was kept as small as possible, and the nodal lines were mapped by noting the change in phase when a node was passed.

The principal modes excited with center drive had frequencies of 39, 162, 195, 318, 854, 979, and 1000 Hz. The lowest mode (39 Hz), which was the only one having complete axial symmetry, was about an octave below the corresponding fundamental mode in a large (36-inch diameter) Paiste tamtam previously studied.[5] The 318-Hz mode was nearly axially symmetric, having 3 concentric nodal circles but with two partial radial nodes near the outer edge.

With the drive point near the edge, the most prominent resonances occurred at 77, 148, 176, 199, 1000, 1223, and 1383 Hz. Mode shapes suggest standing flexural waves around the circumference. The frequences of these modes can be fitted to an empirical relationship $f = cm^k$ where m is the number of radial nodes, c = 29 Hz, and k = 1.17. This behaviour is somewhat similar to the modes of a large orchestral cymbal.[6]

### 3.3 Sound spectra

Peak sound spectra were determined by means of a Bruel and Kjaer Type 1623 filter set to a 23% (⅓-octave) bandwidth. The relative maximum sound pressure levels in several bands are given in Table 1. The tamtam was excited in three different ways: center strike with and without "priming" (i.e., with a soft roll before striking), and scraping on the edge with a wooden drum stick. The maximum sound levels occur in the bands centered at 1000, 2000 and 4000 Hz.

### 3.4 Determination of modal coupling

In order to attempt to determine some of the coupling coefficients in (3) and (4), the tamtam was driven in steady state at its center to a moderately large amplitude in one of its nearly axisymmetrical modes; the one selected was the one at 162 Hz. An accelerometer was attached to the tamtam at points lying near the edge, at half the radius, and at three quarters of the radius. Motion was detected at several frequencies including 252, 333, 490, 978 and 2890 Hz. The amplitude of the mode with f = 333 Hz was determined to be proportional to the square of the f = 162 Hz mode, which is fairly typical for a mode near the second harmonic of the driving force. No definite relationship could be established between the amplitudes of the other modes and that of the driving force due to our inability to drive the tamtam hard enough in the steady state.

Since the steady state experiment failed to establish a quantitative relationship between modes of low and high frequency, it was decided to use percussive excitation with a soft mallet instead. An accelerometer was attached to the tamtam at approximately half way between the center and edge, and a microphone was positioned about one meter distant. The outputs from the accelerometer and the microphone were recorded on two tracks of a tape loop, and the tamtam was struck near its center with blows of varying strength. The tapes were then played back through a tunable filter (Gen Rad 1900A) so that the vibration amplitude and sound radiation at various frequencies could be measured.

Fig. 2 shows the peak accelerometer voltage for vibrations at several frequencies vs that of the nearly axisymmetrical mode at 162 Hz. The accelerometer voltages can be divided by $\omega^2$ and some constant scale factor to obtain the amplitudes in (10). Although the data points scatter considerably, it is possible to draw a family of curves having slopes of one for small amplitude, increasing in steepness at larger amplitude, as predicted by (10).

### 3.5 Sound buildup and decay

By coupling the filter output to a Gen Rad 1521B graphic level recorder, the sound buildup and decay times in various one-third octave bands could be recorded. Buildup and decay times (60 dB) are given in Table II. The maximum writing speed of the recorder is 200 dBs$^{-1}$ which corresponds to a 0.3 s rise time for 60 dB. Hence buildup times below 0.4 s are not significant. Priming the tamtam before striking appears to lower the buildup times in the 4000, 8000, and (most likely) in the 16000 Hz bands, although the signal-to-noise ratio in the 16000 Hz band was most often insufficient to permit accurate measurements.

The buildup and decay of vibrations at various frequencies during the first 0.4 s is shown in Fig. 3. These waveforms were recorded with an accelerometer placed approximately half way between the center and edge.

Fig. 4 shows both the acceleration, as above, and the sound reaching a microphone about one meter from the tamtam. The slow buildup of the high-frequency modes is apparent in these figures.

## 4. DISCUSSION OF RESULTS

### 4.1 Modes of vibration and sound spectra

The tamtam has many modes of vibration. When it is excited near the edge, the modes appear to result from the excitation of standing flexural waves around the circumference. Excitation at the center emphasizes axisymmetric modes of low frequency which, however, couple to many other modes of higher frequency. Thus, when the tamtam is struck near the center with a hard blow, the initial sound spectrum is dominated by the low and middle-frequency bands. Modes of higher frequency build up in amplitude more slowly, so that a second or two after striking the sound spectrum is dominated by the bands centered around 1000 and 2000 Hz. The still-later developing modes of higher frequency contribute substantially to the timbre but not much to the total sound level.

Modes of high frequency not only build up more slowly but also decay more rapidly than modes of low frequency (see Table II). The persistence of the low-frequency modes, in fact, usually makes it necessary to damp the tamtam at the appropriate time after striking in a musical performance.

### 4.2 Modal coupling

We propose that the high-frequency modes are excited by two mechanisms: directly by the strike and indirectly by coupling to modes of lower frequency. It is very difficult to determine the respective coupling coefficients $B_i$ and $A_{li}$ in (10) by experiment, however.

When the tamtam is driven at 162 Hz (the frequency of a prominent axisymmetric mode) the amplitude of the nearly-harmonic mode at 333 Hz is observed to be nearly proportional to the square of the 162 Hz mode, which is consistent with (10). However, no definite relationship could be established between the amplitudes of the other modes.

When the tamtam is struck with a large mallet, a comparison of modal amplitudes shows fair resemblance to the behaviour predicted by (10). For soft to medium blows, the first term dominates, and the modal amplitudes increase in proportion to the amplitude of the axisymmetric mode at 162 Hz. The coefficient $B_i$ decreases with frequency, and thus the curves of higher frequency are displaced downward in Fig. 2. For hard blows, the second term in (10) begins to take on increasing importance because of the exponent $\omega_i/\omega_1$ to which $a_i$ is raised. This is demonstrated by the upward curvature in Fig. 2.

The variables in Fig. 2 are the peak voltages from the accelerometer attached to the tamtam after filtering through a narrow-band filter (10 Hz bandpass at 162 Hz, 50 Hz at other frequencies). No effort was made to determine actual modal amplitudes. If the axes in Fig. 2 were actual amplitudes, the vertical separation of the curves of different frequencies would be substantially greater, since the acceleration must be divided by $\omega^2$ to obtain the amplitude. The shapes of the curves would remain as in Fig. 2, however.

The buildup and decay waveforms in Figs. 3 and 4 show considerable evidence of beats. These may be due, in part, to interference between two or more modes lying close together in frequency, but they are also suggestive of multiple or cascade excitation processes. During the transfer of energy from low-frequency modes to those of high frequency, modes of intermediate frequency would be expected to build up and decay intermittently, thus appearing as beats.

## 5. CONCLUSIONS

The distinctive timbre of a tamtam arises from the relatively slow buildup of modes of vibration having high frequencies. This slow buildup occurs because energy is fed to these modes from the modes of low frequency excited initially. The nature of the non-linear coupling between modes is not well understood at present, but the large number of hammered bumps spaced around the tamtam appear to play a significant role in transferring energy from axisymmetric to radially symmetric and asymmetric modes. The harder the blow, the more significant the non-linear coupling becomes.

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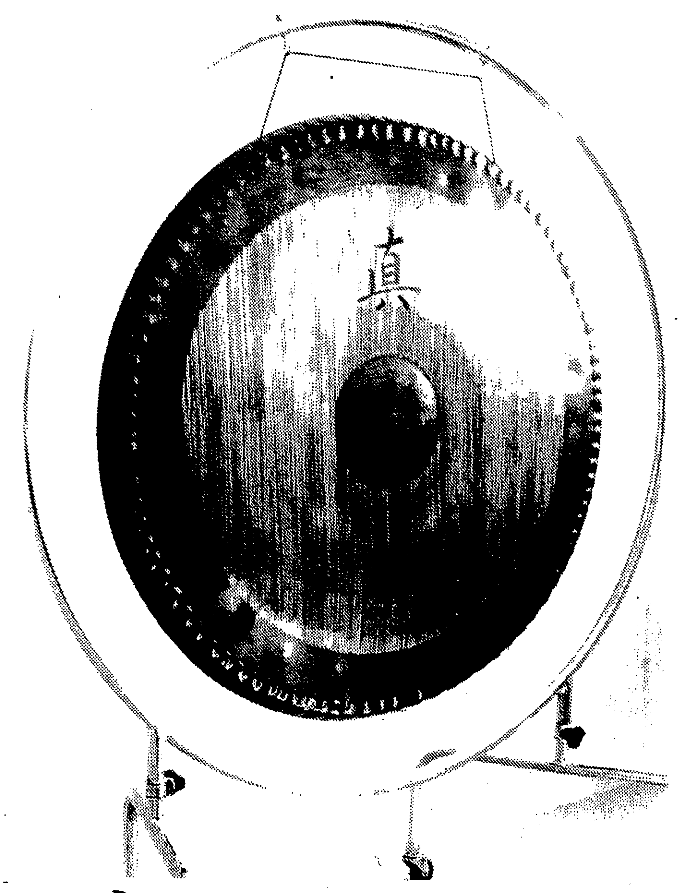

*Fig. 1 The large tamtam used in these experiments.*

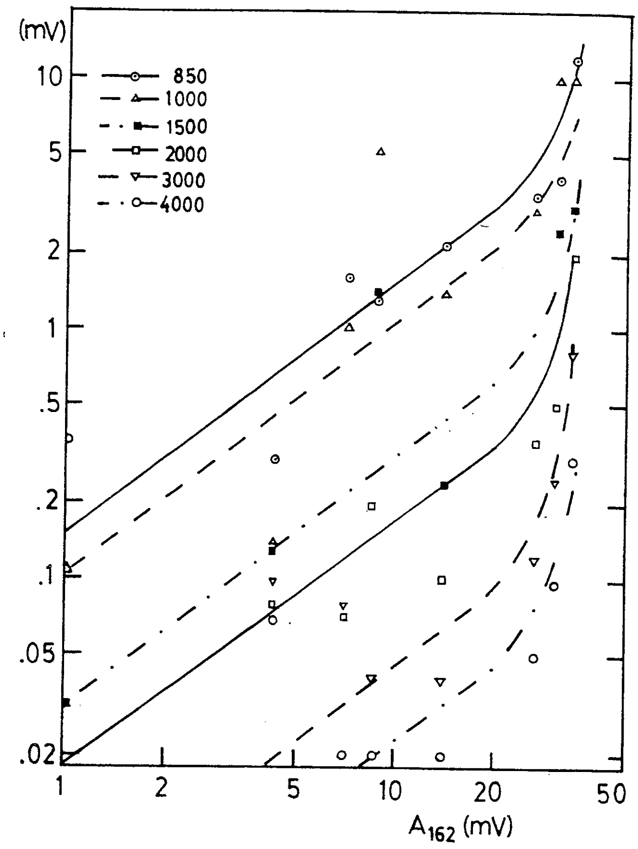


*Fig. 2 Vibrational amplitudes, $a_f$, in six frequency bands as functions of the amplitude, $a_{162}$, of the axisymmetric mode at 162 Hz. The amplitudes given are the peak voltages recorded from an accelerometer attached to the tamtam. The bandwidth at 162 Hz was 10 Hz, and 50 Hz at all other frequencies.*

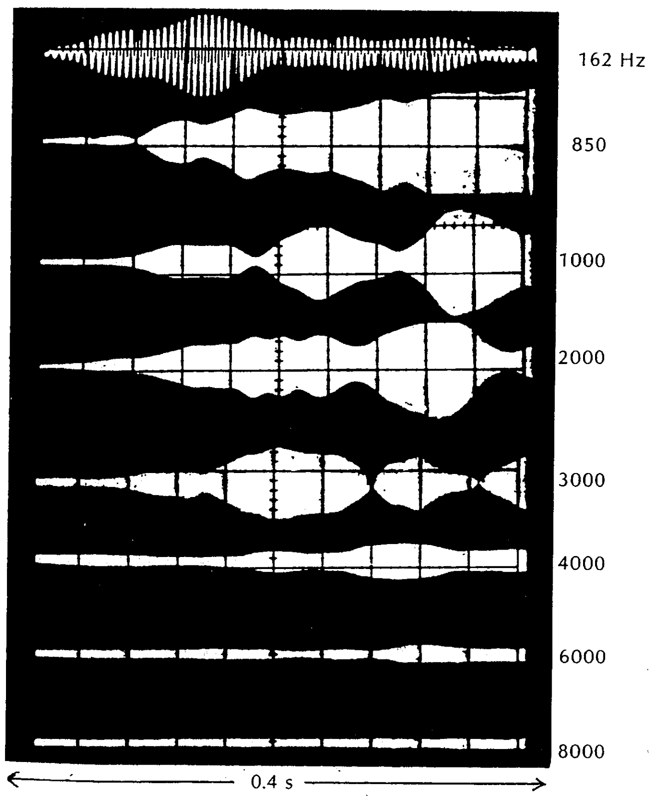


*Fig. 3 Buildup and decay of vibrations in different frequency bands during the first 0.4 s.*

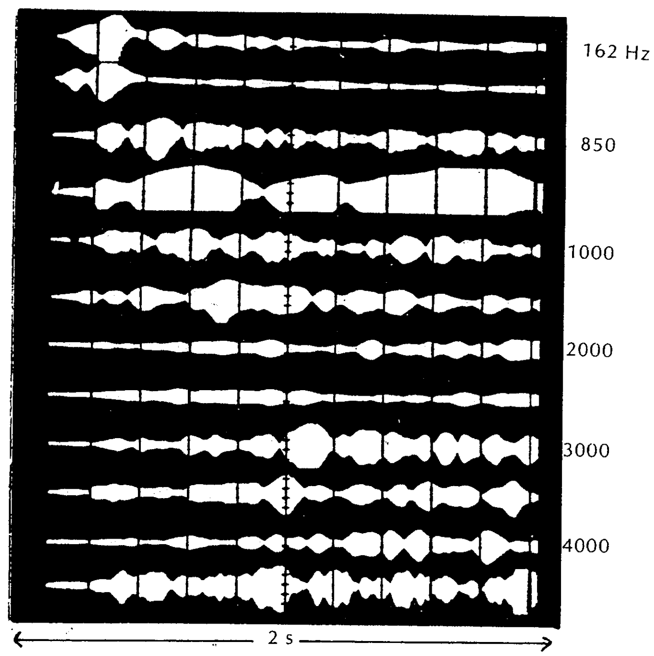


*Fig. 4 Buildup and decay of radiated sound (upper curve at each frequency) and acceleration (lower curve at each frequency) in six different frequency bands.*