LETTERS TO THE EDITOR

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Nonlinearity, chaos, and the sound of shallow gongs

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Experimental studies on several orchestral gongs of the tamtam and cymbal families suggest that two separate nonlinear mechanisms contribute to the evolution of the sound. The first mechanism is an upward cascade of energy from the low-frequency modes initially excited into high-frequency modes, caused by coupling between tension and shear stresses at regions of sharp change in shape of the gong. The second is a transition from simple periodic nonlinear modal motion to multiple fractional subharmonics, or even chaotic motion, which fills out the radiated spectrum at frequencies between those of the normal linear modes. Each of these mechanisms has considerable hysteresis, so that the spectrum of the radiated sound evolves over a period of several seconds. Measurements using high-level sinusoidal excitation have elucidated some of the features of this behavior.

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INTRODUCTION

For some time, we have been interested in the unusual acoustics of the large orchestral gong, or Chinese tamtam.[1] The sound begins with a dull, low-frequency thump which then, over a time of about 1 s, blossoms into a bright shimmering high-frequency wideband blaze of sound, dying away over as long as 10 s. It might, perhaps, be supposed that the sound is of wide bandwidth from the beginning, with the upper partials initially masked by the very large amplitude of the lower modes. Analysis of the sound establishes clearly, however, that there is a slow buildup and decay of radiated energy in the higher partials of the sound, while the energy radiated by the lower modes simply decays. This is shown quantitatively in the two spectra of Fig. 1, the first of which refers to the sound immediately after the strike, and the second to the sound about 3 s later.

Detailed time-resolved analysis of the spectrum of the radiated sound, or of the vibrations of the gong itself, has proved extremely difficult because of the close spacing of the spectral peaks. This leads to a beating behavior in an analysis of moderately narrow bandwidth (10 or 50 Hz),[1] (Ref. 1), while a time-resolved analysis at smaller bandwidth shows that individual peaks shift in frequency as well as in amplitude during the evolution of the sound. A comparison between the mechanical admittance of a geometrically simpler gong (the Turkish gong described later) at its center and the spectrum of its radiated sound when struck in the center with a soft beater reveals that the sound contains a closer distribution of spectral peaks than does the linear mode spectrum. This observation could perhaps be accounted for by incidental excitation of nonaxially symmetric modes, but it is not clear that this is the whole explanation.

As a result of further studies, we are now able to report experimental and theoretical results that throw some light upon the phenomena responsible for the characteristic sound of gongs of this type, and perhaps of orchestral cymbals as well. It seems to us to be worthwhile to report these results now, since a detailed understanding will probably not come rapidly.

I. GONGS AND CYMBALS

Gongs and cymbals exhibiting the effects in which we are interested—which means that we eliminate those gongs that are more nearly bells—all have more or less the form of shallow axially symmetric shells. The orchestral gong, or Chinese tamtam, which shows the effects in the most pronounced manner, is shown diagrammatically in Fig. 2(a). It is a very nearly flat shell, 1–2 mm in thickness and about a meter in diameter, which is stiffened by a rolled-down circumferential ring, a series of rings of hammered isolated bumps, and a central striking dome. It is not immediately clear to what extent these details contribute to the vibrational behavior, but observation shows that the energy initially

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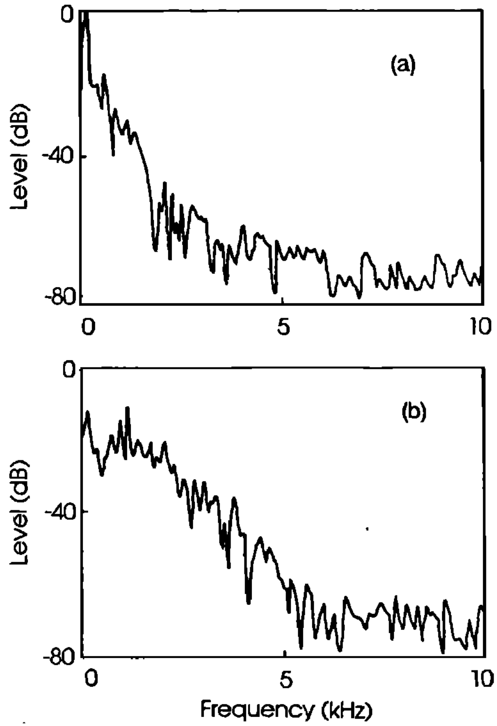

FIG. 1. Frequency spectra of the radiated sound from a Chinese tamtam (a) immediately after the strike and (b) after a delay of about 3 s. The reference level is arbitrary, but the same for both plots.

imparted to circularly symmetric low-frequency modes, by striking the central dome with a large soft hammer, is progressively transferred to modes having their maximum vibrational amplitudes near the periphery of the gong.

The large Turkish gong, made by Avedis Zildjian and shown in Fig. 2(b), is of much simpler geometry, but shows much the same vibrational phenomena. It is about 60 cm in overall diameter and consists of a shallowly dished spherical-cap shell surrounded by a wide conical flange. The shell material is bronze about 2 mm in thickness, and the depth of the dish is about 5 mm. The whole gong is heavy and relatively rigid, and the striking hammer is heavy and only moderately soft. Observation shows that vibration is largely confined to the central spherical-cap shell, as is to be expected given the stiffness of a conical-shell flange.[2]

Finally, the cymbal, shown in Fig. 2(c), is again essentially a spherical-cap shell, 1–2 mm in thickness and with a depth of about 15 mm, in the center of which is a more deeply curved and somewhat thicker dome. In playing, such a cymbal is usually clashed against an identical partner in such a way as to excite edge vibrations, while other similar cymbals may be struck near their edges with hard sticks. In normal playing, the cymbal is supported at its center, so that circularly symmetric modes are not largely excited. It is, however, these modes with which this report is concerned, so that all our measurements were designed to excite and detect only the axially symmetric modes.

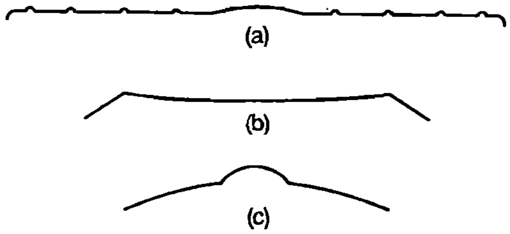

FIG. 2. Cross sections of (a) the Chinese tamtam, (b) the Turkish gong, and (c) the orchestral cymbal, referred to in this study. The diagrams are only approximately to scale.

We have framed our discussion in terms of nonlinear coupling between modes that are essentially those associated with the ordinary small-amplitude linear behavior of the system. This approach is justified because the nonlinearity is small enough that, at least up to the onset of chaotic behavior, these modes continue to be identifiable, although their frequencies may be shifted and they may be accompanied by harmonics of their normal frequencies. The nonlinearity is, in all cases, associated with geometrical distortions of the structure rather than with elastic nonlinearity of the materials from which it is made.

II. MODE COUPLING

Because of the complicated geometry of these instruments, we initially carried out studies on simpler systems, with geometry similar to that of a cross section of a gong, in order to examine phenomena that might contribute to the sound produced. In particular, we showed that, both in the case of a taut string passing at an angle over a bridge of finite compliance[3] and in the case of a thin bar with two symmetrically placed kinks,[4] there is a nonlinear coupling which allows energy to be transferred from one mode to others at frequencies two or three times that of the original mode.

Our experiments in these two cases demonstrated that, if the string was initially excited so that the second or third mode had nearly zero amplitude, then the amplitude of this mode grew to a maximum after a time of about 0.1 s and then decayed. The maximum amplitude of the initially missing mode varied as either the second or third power, respectively, of the initial amplitude of the fundamental, though its maximum amplitude was always small. If the dimensions of the bar were chosen so that two modes had frequencies nearly in the ratio 2:1, then similar effects could be clearly observed.

The theory underlying these processes[3,4] shows that we may expect them to be present also in shells having sharp changes in slope, and perhaps under other conditions as well, and thus provides the beginning of an understanding of the mode-conversion process. In the case of the Turkish gong, the shell has circular symmetry, so there can be no coupling between circularly symmetric and angularly dependent modes. The rings of hammered bumps on the tamtam, on the other hand, destroy the circular symmetry and provide a mechanism by which circularly symmetric modes can couple to modes having the angular symmetry of the bump rings. This mode-conversion mechanism thus appears to be important in the generation of some features of the characteristic sound of gongs, particularly the high-frequency "sheen" which develops in the later part of the sound.

III. VIBRATION OF SPHERICAL-CAP SHELLS

Rossing and Fletcher[5] have examined some aspects of the role of curvature in the vibrational behavior of spherical-cap shells, concentrating on shifts in the normal-mode frequencies with amplitude which were later explained in detail by Fletcher.[6] Our present interest is in rather more general aspects of the behavior. Again, we have tried to simplify the problem by concentrating on individual modes and their interactions.

Perhaps the most important observations we have made in relation to this problem arose from measurements of the response of the Turkish gong, and later of the orchestral cymbal, to a sinusoidal force applied at its center (using a Bruel & Kjaer shaker, model 4810, and either an impedance head, model 8001, or a subminiature accelerometer, model 4374). This initially gave information on the frequency of the axially symmetric modes and showed that their distribution in frequency was less dense than the observed density of spectral peaks in the analyzed large-amplitude sound of the gong, as we have already remarked. The first axially symmetric shell resonance was found at 96 Hz, close to the frequency expected from an approximate theory developed by Fletcher,[6] with higher resonances near 200, 416, 664, and 992 Hz. Most of these resonances had small satellite peaks, the origin of which was not elucidated.

In examining the individual resonances for nonlinear behavior at relatively large exciting forces, however, we discovered not simply a frequency shift but, instead, a catastrophic change in behavior. At a critical amplitude, and over a very small frequency range near the admittance peak, the periodic motion of the gong suddenly developed large-amplitude subharmonic components. The behavior was critically dependent on the frequency, amplitude, and immediate history of the excitation, but subharmonics of orders 2, 3, and 4 were easily evoked. Associated with these subharmonics were all multiples of their frequencies. Sometimes the number of peaks on the spectrum was so large that unambiguous assignment of fractional frequencies was not possible, and since the unresolved background intensity was also large, the motion might reasonably be described as chaotic. Subjectively, the sound produced by the gong when vibrating in one of these multiple-subharmonic modes was very similar to the fully developed after-ring of the gong when excited by a vigorous blow. The orchestral cymbal, when excited at its center, exhibited very similar behavior.

FIG. 3. Measured orbit in (displacement, velocity) phase space for the Turkish gong in a vibration regime showing bifurcation to a subharmonic of order 2. The excitation frequency is 91 Hz.

Further insight into this phenomenon was obtained by measuring both the velocity and displacement of the center of the gong. The velocity was obtained by integrating the signal from a subminiature accelerometer (B&K type 4374) mounted on the gong, while the displacement signal was obtained from a capacitive transducer (B&K type MM0004) placed close to the gong surface. As a precaution, the gong was driven through a compliant spring, rather than directly by the shaker, so that its behavior would not be influenced by the nonzero mechanical impedance of that device. Figure 3 shows the orbit in phase space (displacement, velocity) of the characteristic point representing the center of the gong in a regime in which the subharmonic of order 2 has just become large. The basic orbit has a complex form, rather than being a simple ellipse, thus reflecting the presence of strong harmonics or order 2, 3, and higher. The orbit has, in addition, split into two, reflecting the presence of a strong subharmonic of order 2. The thickness of the traces for these two individual orbit components suggests further small-scale splitting, but this effect may be caused simply by motion of the gong as a whole on its supports.

A further study was made by strobing the intensity of the oscilloscope beam once in each cycle of the force, at a defined phase, thus producing a Poincaré map of the motion. For a simple orbit, this map consists of a single point, while the map of the bifurcated orbit of Fig. 3 produces a pair of points. Maps with three points were produced in other cases, while, for a higher forcing amplitude, the map sometimes degenerated to a widely spread cluster of closely spaced points. This is indicative of chaotic behavior, and we might expect the Poincaré map to exhibit the fine structure associated with a strange attractor. Our maps did not show any such structure, but this can perhaps be attributed to the presence of other unrelated small motions, such as the swinging of the gong on its supports.

Figure 4 shows the region of (force, frequency) space in which various types of behavior were found for the gong. Each regime of subharmonic or chaotic behavior is narrowly defined in frequency and lies close to one of the primary linear admittance maxima of the gong, so that the vibrations in each regime are essentially independent and of different spatial distribution. This close correspondence with the admittance maxima is consistent with the view, which we explore later, that the bifurcation and chaotic behavior is better considered to be associated with large vibrational amplitudes rather than with large forces, although the two quantities are clearly closely related. The maximum amplitude achieved in the figure is less than 2 mm, which is only a quarter of the height of the shell. In the case of the cymbal, subharmonic behavior was observed for amplitudes around 1 mm, which is an even smaller fraction of the shell height.

Exploration of these phenomena showed also that there was considerable hysteresis associated with the onset of the subharmonic regime. Transition from the normal regime, once a power level had been set for the excitation, could take as long as several seconds, and the regime then persisted for many seconds if the excitation was reduced or even set to zero.

Clearly, a well-developed multiple-subharmonic vibra-

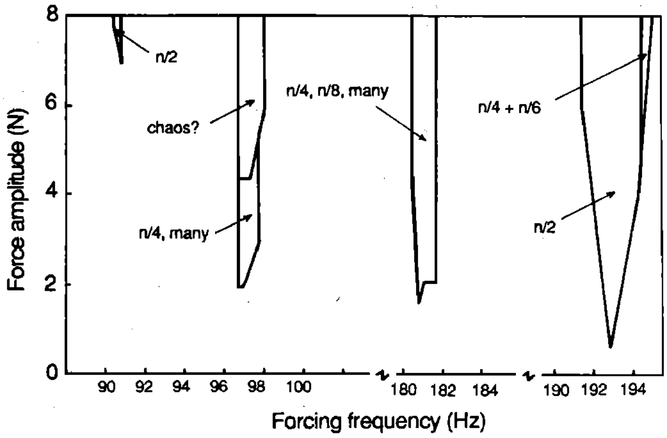

FIG. 4. Plot of the regions in (frequency, force) space for which complex behavior was found for the Turkish gong. The excitation frequency is, in each case, close to one of the major admittance maxima, near 96 and 200 Hz, respectively.

tion regime is capable of producing many of the observed acoustic phenomena associated with the gong sound, particularly if subharmonics associated with more than one gong mode are excited. It seems plausible that a large-amplitude impulsive excitation with a soft-headed hammer could take the vibration amplitude of several of the low-frequency axially symmetric modes to values above those necessary for generation of subharmonics and thus produce the observed rich and dense acoustic spectrum.

IV. THEORY

Elsewhere[5,6] we have discussed certain aspects of the axially symmetric vibration of spherical-cap shells. In particular, we have shown that the inherent nonlinearity gives rise to changes of mode frequency with amplitude.[6] If the thickness of the shell is greater than about twice the height of the arch of the shell, then the mode frequency rises with increasing amplitude, while, if the shell thickness is small, the mode frequency initially falls with increasing amplitude. This analysis explains the frequency shifts observed for the modes of the gongs studied, but this is only a minor part of the situation.

As we have discussed elsewhere,[6] the behavior of the mode displacement x_n associated with the nth axisymmetric mode of a shallow spherical shell can be described approximately by an equation of the form

$$\frac{d^2x_n}{dt^2} + 2k_n\frac{dx_n}{dt} + \omega_n^2(x_n + \alpha_n x_n^2 + \beta_n x_n^3) = F_n(t), \tag{1}$$

where ω_n is the small-amplitude mode frequency, k_n represents the mode damping, and F_n is the applied force, properly weighted for mode shape. The parameters α_n and β_n can be expressed in terms of shell diameter, curvature, and thickness.[6] This equation with $F = 0$, properly describes the shift of vibration frequency with mode amplitude. If F is taken to be a combination of terms of the type

$$\gamma_{n,ijk\cdots} x_i x_j x_k \cdots \sin(\omega_i \pm \omega_j \pm \omega_k \cdots), \tag{2}$$

with coefficients γ determined by the geometry of the shell, and particularly by any abrupt changes of slope, as discussed for the kinked bar,[4] then this describes the coupling and energy transfer between modes. It must be admitted, however, that the explicit forms of the γ coefficients have not been worked out for the shell.

If we take $F(t) = F\sin(\omega t)$, then (1) represents the behavior of a single mode of the shell under a sinusoidal exciting force, when coupling to other modes is neglected. This is a first approximation to our experimental situation. With this assumption, (1) is essentially the well-known Duffing equation, with an added term quadratic in x. The behavior of this equation has been studied extensively, from both analytical and numerical viewpoints, and it is known that it exhibits harmonic distortion, bifurcation leading to generation of subharmonics of various orders, and chaotic behavior.[7–10]

This suggests the origin of the bifurcation and chaotic behavior on which we have reported. There is, however, a problem, since both the general literature, and our own numerical explorations of (1), suggest that chaotic behavior occurs only when the amplitude of the mode vibration approaches the height of the shell arch, while we have observed these phenomena for amplitudes about an order of magnitude less than this. Certainly, however, the phenomena are much more pronounced in the tamtam, which has a very small curvature, than in the more deeply dished gongs, which is in general agreement with expectations. We have not yet seen a way out of this problem. It could be that the presence of coupling between modes, of the form given in (2), accounts for the greater sensitivity of the system, but we have no evidence for this.

It is perhaps significant that impulsive excitation of a low-frequency mode to large amplitude, as in normal striking of the gong, can generate, through terms of the form given in (2), sinusoidal exciting forces at frequencies which may be close to those of other normal modes of the shell. These modes are then driven as though by an external sinusoidal force of defined frequency and phase, and can be expected to exhibit bifurcation and chaotic behavior if the force amplitude is large enough. This gives a qualitative explanation of the similarity of sound quality between a normally struck gong and the same gong excited sinusoidally into its chaotic regime.

V. CONCLUSIONS

This study does not yet present a complete understanding of the processes leading to the characteristic sound of cymbal-like gongs. However, we have demonstrated that there are two major processes occurring. The first of these is frequency multiplication due to coupling between tensional and shear stresses and leading to the transfer of energy from low- to high-frequency modes, after a characteristic time delay. The second is the splitting of major vibrational modes into components having fractional frequency ratios, and even into chaotic vibrations. This effect, also with a characteristic induction time and hysteresis behavior, transfers energy from major modes to their subharmonics and is responsible for the observed close spacing of peaks in the low-frequency spectrum of the radiated sound. Together, these two mechanisms contribute an unmatched complexity to the sound of these percussion instruments.

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Determining the extent of coarticulation: Effects of experimental design[a)]

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The purpose of this letter is to explore some reasons for what appear to be conflicting reports regarding the nature and extent of anticipatory coarticulation, in general, and anticipatory lip rounding, in particular. Analyses of labial electromyographic and kinematic data using a minimal-pair paradigm allowed for the differentiation of consonantal and vocalic effects, supporting a frame versus a feature-spreading model of coarticulation. It is believed that the apparent conflicts of previous studies of anticipatory coarticulation might be resolved if experimental design made more use of contrastive minimal pairs and relied less on assumptions about feature specifications of phones.

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The purpose of this letter is to explore some reasons for what appear to be conflicting reports of the nature and extent of anticipatory coarticulation (Kozhevnikov and Chistovich, 1966; Lubker and Gay, 1982; Bell-Berti and Harris, 1982; Engstrand, 1981; Perkell, 1986; Sussman and Westbury, 1981). Generally, these results have been claimed to support one of two conflicting positions: One view, which we will refer to as the look-ahead model (e.g., Henke, 1967), is that articulatory features of a target phone migrate to preceding phones to an extent that depends on the feature composition of the latter. Another view, which we will refer to as frame theory (Bell-Berti and Harris, 1982), presumes that anticipatory coarticulation is due to the coproduction of neighboring segments (Fowler, 1980) in a relatively fixed temporal frame. It is our position that (1) the apparent conflicts depend in substantial part on assumptions made about the feature specification of phones, and (2) experimental design can obviate the need for such assumptions. In particular, we will refer to anticipatory lip rounding, although we believe that we are addressing the general phenomena of anticipatory coarticulation (Bladon and Al-Bamerni, 1982).

Previous studies of lip rounding (e.g., Bell-Berti and Harris, 1979, 1982; Benguerel and Cowan, 1974; Daniloff and Moll, 1968; Engstrand, 1981; Lubker and Gay, 1982) have employed alveolar consonant strings before rounded vowels on the assumption that these consonants are unspecified with regard to lip configuration. Thus the presence of electromyographic (EMG) activity or protrusive lip movement during these consonants has been presumed to indicate the onset of vowel-conditioned lip activity. However, if this activity is inherent to the production of the consonants themselves, then the onset of anticipatory vowel-related lip rounding cannot be determined unless the experimental de-

[a)] Portions of this letter were presented at the 103rd Meeting of the Acoustical Society of America [Gelfer *et al.*, J. Acoust. Soc. Am. Suppl. 1 **71**, S104–S105 (1982)].