Acoustic Admittance of Air-Driven Reed Generators

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Summary

Measurements are reported of the acoustic admittance of a damped metal reed, over a frequency range including the reed resonance, for a range of blowing pressures and for configurations in which the reed is either opened or closed by the blowing pressure. The measurements confirm the results of previous theoretical analysis (N. H. Fletcher, [1]). Each configuration shows a blowing pressure below which the conductance is always positive and no acoustic generation can take place. Above this pressure a reed that is blown open shows negative conductance in only a small frequency range just above the reed resonance and a reactance that is always inductive. A reed that is blown closed shows a negative conductance from zero frequency up to the resonance and a reactance that is always capacitive.

Akustische Admittanz von luftbetriebenen Zungengeneratoren

Zusammenfassung

Es wird über Messungen der akustischen Admittanz von gedämpften Metallzungen in einem Frequenzbereich, welcher die Zungenresonanz einschließt, berichtet, und zwar für Blasdrucke und Konfigurationen, bei denen die Zunge durch den Blasdruck entweder geöffnet oder geschlossen wird. Die Messungen bestätigen die Ergebnisse von früheren theoretischen Analysen (N. H. Fletcher, [1]). Jede Konfiguration besitzt einen Blasdruck unterhalb dessen die Konduktanz überall positiv ist und keine akustische Generation stattfinden kann. Oberhalb dieses Drucks besitzt eine Zunge, welche sich durch das Anblasen öffnet, negative Konduktanz innerhalb eines schmalen Frequenzbereichs unmittelbar oberhalb der Zungenresonanz und eine Reaktanz, welche überall induktiv ist. Eine Zunge, welche sich schließt, besitzt negative Konduktanz von der Frequenz Null bis zur Resonanzfrequenz und eine Reaktanz, welche überall kapazitiv ist.

L'admittance acoustique des générateurs à anche actionnés par air

Sommaire

On présente des mesures de l'admittance acoustique d'une anche métallique amortie effectuées sur une gamme de fréquences englobant la fréquence de résonance de l'anche et sur une étendue suffisante de pressions de soufflage, de manière à étudier des configurations dans lesquelles l'anche est soit ouverte soit fermée par la pression du souffle. Ces mesures confirment les prévisions d'une analyse théorique antérieure (N. H. Fletcher [1]). Chaque configuration fait apparaître une pression de souffle en dessous de laquelle la conductance est toujours positive et où aucune génération de son ne peut avoir lieu. Au dessus de cette pression, une anche qui est maintenue ouverte par le souffle présente une conductance négative seulement dans un étroit domaine de fréquences juste au-dessus de la résonance de l'anche, tandis que sa réactance est toujours inductive. Par contre, une anche qui est maintenue fermée par le souffle manifeste une conductance négative depuis la fréquence nulle jusqu'au dessus de la résonance et une réactance qui est toujours capacitive.

1. Introduction

In a recent publication [1] we gave a detailed quantitative discussion of excitation mechanisms in woodwind and brass instruments. The basis of the discussion was not new, dating back to the time of Helmholtz [2], and several other workers [3], [4], [5] have also contributed to our understanding. What was new was an explicit calculation of the acoustic admittance of an air-driven reed for the two classes of such generators — those whose aperture is closed by the blowing pressure, as in the oboe or clarinet, and those whose aperture is further opened, as in the trumpet or the human larynx. With the aid of this information it is possible to reach a fairly detailed understanding of the complete musical instrument, considered as a linear resonator coupled to a highly non-linear negative-resistance generator [6].

It is the purpose of the present paper to present the results of measurements on air-driven reed systems which confirm the predictions of the theoretical analysis. We also take the opportunity to correct, in the Appendix, two minor typographical errors in the original publication [1].

2. Theory

As discussed in [1], the acoustic volume flow U through a vibrating-reed valve has the approximate form

$$U = D\,|\xi|^{\alpha}(p_0 - p)^{\beta}, \tag{1}$$

where ξ is the reed opening distance, p_0 the steady blowing pressure and p the acoustic back-pressure in the resonator. α, β and D are constants and $\alpha \approx 1$, $\beta \approx \frac{1}{2}$. When the acoustic pressure p is varying, this causes a variation in the reed opening ξ, the magnitude and phase of which are governed by the mechanical properties of the reed. In the simple case considered, the reed behaves like a simple harmonic oscillator with a single resonance at angular frequency ω_r.

The quantity calculated in the theory is the small-signal acoustic admittance of the reed, defined by

$$Y_r = -\left(\frac{\partial U}{\partial p}\right)_{p_0} \tag{2}$$

the negative sign arising from the direction assigned to the flow U.

At low blowing pressure p_0 the flow U is dominated by the factor $(p_0 - p)^{\beta}$ and Y_r always has a positive real part, so that the reed is simply a dissipative load. When p_0 is large enough, however, the behaviour is dominated by the variation of ξ with p, and the real part of Y_r may become negative, allowing the reed to act as a negative-resistance generator. It is this regime that is principally of interest.

The behaviour of the two configurations of reed generator is quite different. For sufficiently large p_0 the real part of Y_r for a clarinet-like reed is negative from zero frequency up to the reed resonance frequency ω_r. For a trumpet lip-valve generator, on the other hand, the real part of Y_r is negative over only a small frequency range just above ω_r. The exact behaviour depends upon the quality factor Q of the reed. In addition, if p_0 is not strictly constant because of finite reservoir volume, then other effects enter. If the reservoir volume is small, then, as discussed in [1], the effect is to add terms which effectively decrease the Q of a clarinet-type generator and increase the Q of a trumpet-type generator. In fact, the trumpet-type generator is easily made regenerative and thus self-exciting at its resonance frequency without the aid of any external resonator.

3. Experiment

The determination of acoustic admittance is in principle a simple matter using measurement of standing waves in a cylindrical pipe terminated with the unknown admittance. The theory of the method is set out in standard texts [7]. Our present problem is somewhat non-standard, because of the possibility of a negative real part to the admittance over a part of the frequency range, but in practice there is no difficulty in resolving the resulting ambiguity in solution of the equations derived from the measured standing-wave ratio. This is accomplished by observing the sign of the shift in phase with position in the vicinity of a pressure minimum in the standing-wave tube. This sign determines the direction of propagation of the wave of larger amplitude and hence whether the reed is acting as an absorber or a generator.

Reeds for the experiment were made from copper shim stock thickly coated on one side with silicone-based mastic compound to reduce the Q value and broaden the resonance peak for ease of measurement. The reed was clamped to a metal plate with an aperture in it a little smaller than the reed tongue, which was curved to give an appropriate unblown opening ξ_0' (see [1]). Experiment showed the exact shape and curvature of the reed to be critical in determining its performance, a conclusion that was not surprising in view of the skill and experience required of pipe organ reed voicers.

The reed was mounted in measuring equipment as shown in Fig. 1, the air supply being drawn from a throttled compressor line and stabilized in a reservoir of capacity about 20 litres. The reservoir was fitted with a pressure-stabilizing valve consisting of an outward-opening copper reed, this being necessary in the case of measurement of the clarinet-like reed which has a negative static impedance at high pressures and thus tends to shut uncontrollably. The pressure-stabilizing reed was arranged so that it did not vibrate, and the Helmholtz frequency of the reservoir-reed combination was below 50 Hz.

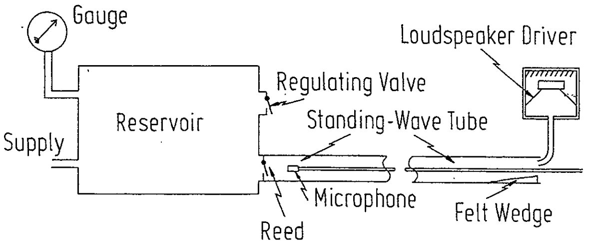

Fig. 1. Schematic diagram of the experimental arrangement. The reed generator is supplied from a 20 litre reservoir with a pressure-stabilizing valve. The 1.5 metre standing-wave tube is one wavelength long at the resonance frequency of the reed and is damped with a felt wedge. The acoustic excitation is fed from a loudspeaker driver through a flexible pipe, and probe measurements are made with a small electret microphone.

The measuring tube was 40 mm in diameter and 1.5 m in length, this being chosen to be almost one wavelength at the resonant frequency of the reed so as to minimize the possibility of oscillation. The pipe was further damped with a felt wedge inserted into its open end, this being adjusted if necessary to stifle any reed-generated sound. Sound waves were introduced into the open end of the pipe through a tube connected to a loudspeaker, and a small electret microphone on the end of a measuring rod was used to explore the standing wave field in the tube. Appropriate filters were used to eliminate higher harmonics and for each measurement the speaker was adjusted to give the same standing wave pressure at the reed in order to minimize the effects of nonlinearity.

The same reed was used for measurements in both configurations by simply reversing it in its holder. The relevant physical parameters are shown in Table I. A range of values is given for the reed resonance since this shifts slightly with static blowing pressure, the reed becoming effectively shorter as the opening closes. The parameters α, β and D were determined from steady flow measurements in both clarinet and trumpet configurations and agree well with those found by Backus [3] for similar reed geometry.

4. Results and discussion

Measured values of the real and imaginary parts of the acoustic admittance for the reed in trumpet-like configuration are shown in Fig. 2 and for the same reed in clarinet-like configuration in Fig. 3. In each case measurements are given for three blowing pressures, one of which is below the critical value so that the real part of the admittance is everywhere positive. In Fig. 4 these curves are combined to show the behaviour of admittance in the complex plane, the parameter along each curve being frequency.

Table I.
Physical parameters of the reed.

Tongue length		17 mm
Tongue width		12 mm
Copper thickness		0.13 mm
Silicone thickness		$\approx$1 mm
Unblown opening		1.7 mm
Pressure response		0.3 mm kPa^{-1}
Resonance frequency		210 ··· 240 Hz
Q-value		17
Flow parameters	α	1.3
	β	0.7
	D	2.1×10^{-2} m$^{3-\alpha}$ Pa$^{-\beta}$ s^{-1}

It is immediately clear that the measured curves are closely similar to those calculated from the theory [1]. To examine the quantitative agreement it is necessary to repeat the computations, since the present reed has rather different physical parameters from that considered in [1]. We show in Fig. 5 the calculated complex admittdance for the pressures used in the experiment and, from comparison between Figs. 4 and 5, it is immediately clear that quantitative agreement is excellent, the residual discrepancies being accounted for by uncertainty in the specification of reed opening geometry.

Specifically we note that both reed configurations show a critical pressure below which the acoustic conductance (the real part of the admittance) is always positive and no sound generation or amplification can take place. Above this critical pressure the trumpet-like reed configuration shows a conductance which is negative over only a small frequency range just above the resonance and an imaginary part that is always inductive with a peak

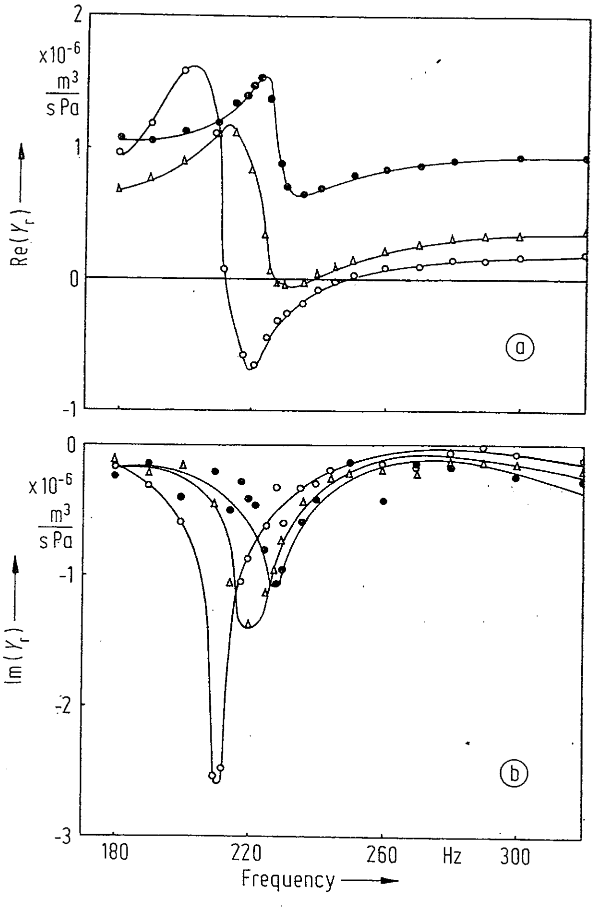

Fig. 2. Real (a) and imaginary (b) parts of the measured acoustic admittance Y_r of a reed in the trumpet configuration for three blowing pressures: ●●● 0.3 kPa, △△△ 0.6 kPa and ○○○ 1.5 kPa.

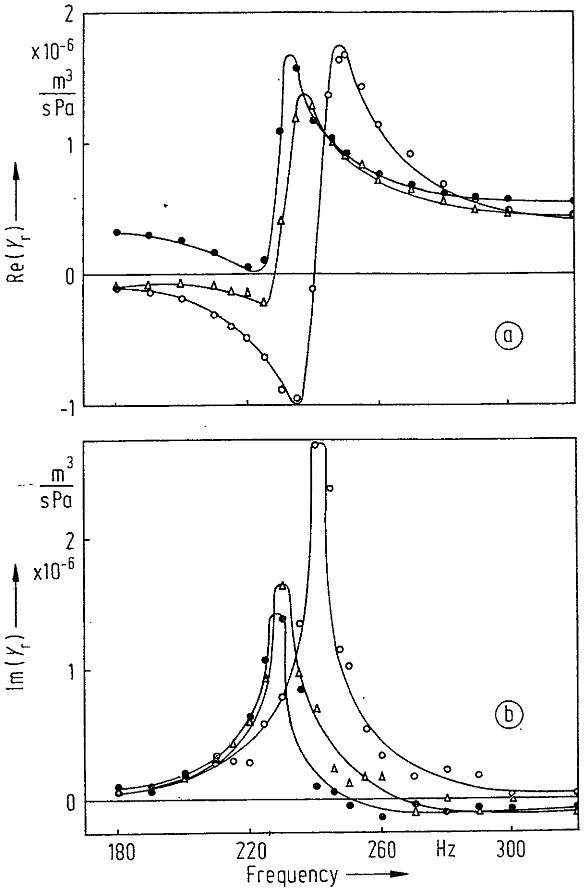

Fig. 3. (a) Real and (b) imaginary parts of the measured acoustic admittance Y_r of the same reed in the clarinet configuration for three blowing pressures: ● ● ● 0.3 kPa, △ △ △ 0.6 kPa and ○ ○ ○ 1.6 kPa.

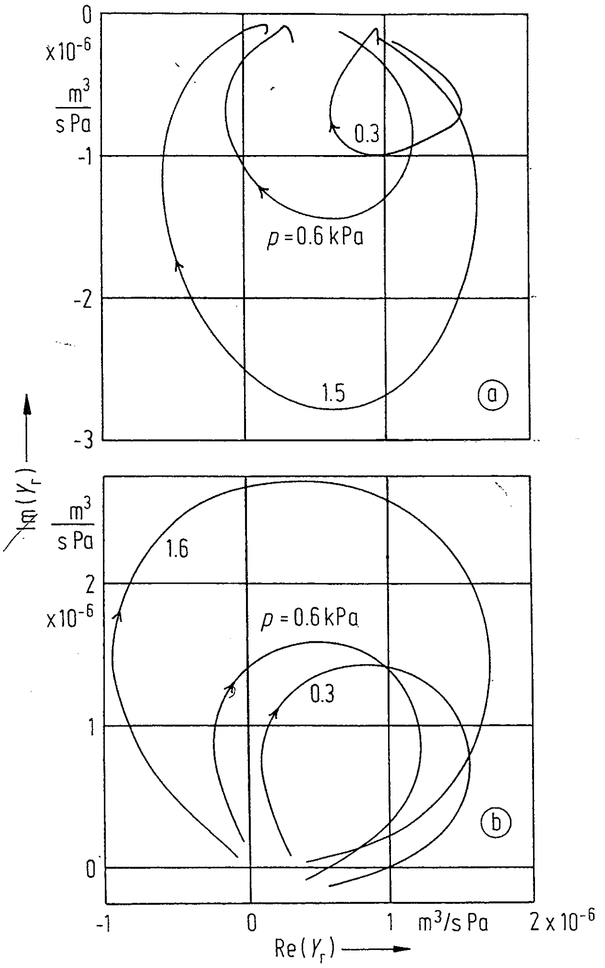

Fig. 4. Complex acoustic admittance plots, with frequency increasing in the direction shown, for the same reed blown (a) in trumpet configuration and (b) in clarinet configuration.

at the resonance frequency. The clarinet-type of configuration on the other hand shows a negative conductance at essentially all frequencies below resonance and an imaginary part that is nearly everywhere capacitive, with a peak again at the resonance frequency. The negative peak in the conductance of this configuration just below resonance may affect the tone of reed-driven woodwind instruments as has been discussed by Thompson [8].

It is interesting to note that the curves of Figs. 2 and 3 bear a close resemblance to the mechanical admittance curves for a simple harmonic oscillator, except that real and imaginary parts have been interchanged. The reason for this is clear. The reed itself vibrates as a simple oscillator under the influence of the acoustic pressure in the pipe and its mechanical admittance (velocity/force) has the expected complex form. The acoustic volume flow through the reed is proportional, however, to reed opening (in the high pressure limit) and this lags 90° behind the reed velocity.

Two other matters remain to be commented upon, the first being the effect of the volume of the reservoir cavity behind the reed, which is equivalent to the mouth cavity in a wind instrument player. We have already remarked that if this cavity is small, say comparable with the human mouth volume in our experiment, then its acoustic impedance acts so as to increase the damping in a clarinet-like configuration and to decrease it in a trumpet-like configuration. In the latter case even a moderate blowing pressure, well below the normal critical pressure for generation, leads to autonomous oscillation at the reed resonance frequency even in the absence of a pipe resonator. In the standing-wave equipment the acoustic impedance of a small cavity is comparable with that of the tube and this leads to large phase shifts in the reed response and consequent confusing results.

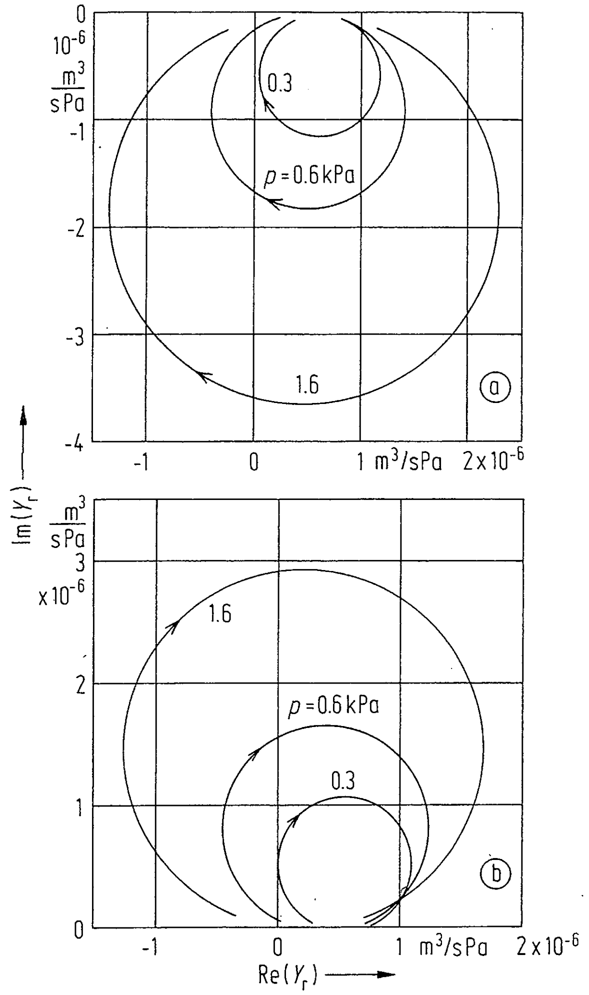

Fig. 5. Calculated acoustic admittance for the experimental reed blown in (a) trumpet and (b) clarinet configurations as in Fig. 4.

In a woodwind or brass instrument the situation is not as complex as this, for the tube of the instrument is operating at one of its resonances and so presents a large acoustic impedance to the reed generator. From a small-signal point of view it is therefore probably reasonable to neglect the finite impedance of the mouth cavity unless it too is near one of its resonances in which case it may have an effect on tone quality. Coltman [9] has examined similar effects in the case of the flute.

Finally we must emphasize that all our considerations here have been of the small-signal behaviour of reed generators. While an understanding of this regime is essential, we must recognise that most reed generators, specially those in musical instruments, operate in the large-signal regime in which non-linearities are an essential feature determining the behaviour [6, 10]. We do not attempt to consider this here.

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Appendix

Two typographical errors occur in the published version of the theory [1]. In the expression (21) of [1] for the phase angle ψ, the sign preceding the last term in the denominator should be $+$ rather than $-$. In the expression (23) for B, a division sign should be inserted before the last two factors $K^2 s_r$ so that the form is similar to that of the expression (22) for A. The correct forms of these two equations were used in the calculation of Fig. 2 and Fig. 3 of [1].

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References

[1] Fletcher, N. H., Excitation mechanisms in woodwind and brass instruments. Acustica **43** [1979], 63.
[2] Helmholtz, H. L. F., On the sensations of tone, 1877. Trans. by A. J. Ellis, reprinted by Dover, New York 1954, pp. 390–394.
[3] Backus, J., Small vibration theory of the clarinet. J. Acoust. Soc. Amer. **35** [1963], 305, and Erratum **61** [1977], 1381.
[4] Worman, W. E., Self-sustained nonlinear oscillations of medium amplitude in clarinet-like systems. Thesis, Case Western Reserve University 1971, Ann Arbor, University Microfilms (ref. 71-22869).
[5] Wilson, T. A. and Beavers, G. S., Operating modes of the clarinet. J. Acoust. Soc. Amer. **56** [1974], 653.
[6] Fletcher, N. H., Sound generation in musical wind instruments. Ann. Rev. Fluid Mech. **11** [1979], 123.
[7] Morse, P. M., Vibration and sound. McGraw-Hill, New York 1948, pp. 133–143, 233–243.
[8] Thompson, S. C., The effect of the reed resonance on woodwind tone production. J. Acoust. Soc. Amer. **66** [1979], 1299.
[9] Coltman, J. W., Mouth resonance effects in the flute. J. Acoust. Soc. Amer. **54** [1973], 417.
[10] Backus, J. and Hundley, T. C., Harmonic generation in the trumpet. J. Acoust. Soc. Amer. **49** [1970], 509.