Transients in the Speech of Organ Flue Pipes – A Theoretical Study

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Summary

A general approach is developed to the calculation of transients in the speech or organ flue pipes through approximate integration of a set of coupled non-linear differential equations for the normal modes interacting through the air jet driving the pipe. Illustrative examples are calculated for the cases of plosive, abrupt and slow rises of pressure in the pipe foot. It is concluded that the form of the pressure transient has a large effect on the pipe speech transient, a plosive attack emphasizing upper partials. In most cases the second pipe mode is found to develop more rapidly than the fundamental and to be dominant in the early part of the speech transient, in agreement with experiment. The frequencies of the upper partials are not initially harmonic, being determined by the pipe resonances and the reactive component of the jet impedance. In the later stages of the transient all modes are locked into harmonic relationship as in the steady tone.

Übergangsvorgänge bei der Ansprache von Orgelpfeifen — Eine theoretische Untersuchung

Zusammenfassung

Es wurde ein allgemeines Verfahren zur Berechnung von Übergangsvorgängen bei der Ansprache von Orgelpfeifen entwickelt, welches in der näherungsweisen Integration eines Systems gekoppelter nichtlinearer Differentialgleichungen für die Normalmoden besteht, die durch den die Pfeife anregenden Luftstrom miteinander in Wechselwirkung stehen. Für die Fälle explosionsartigen, abrupten und langsamen Druckanstieges im Pfeifenfuß werden einige anschauliche Beispiele berechnet. Es wird der Schluß gezogen, daß die Art des Druckverlaufes einen großen Einfluß auf die Übergangsvorgänge bei der Pfeifenansprache besitzt. Ein explosionsartiger Anstieg hebt die hochliegenden Teiltöne an. In den meisten Fällen konnte festgestellt werden, daß die zweite Pfeifenmode sich wesentlich schneller entwickelt als die Grundmode und daß sie im Anfangsteil des Ansprachevorganges in Übereinstimmung mit dem Experiment dominiert. Zu Beginn liegen die hochliegenden Teiltöne unharmonisch zueinander. Ihre Lage wird durch die Pfeifenresonanzen und die Blindkomponente der Luftstromimpedanz bestimmt. In der späteren Phase des Übergangsvorganges werden alle Moden wie beim stationären Ton harmonisch gezogen.

Les transitoires des tuyaux d'orgue: étude théorigue

Sommaire

On met au point une méthode générale pour calculer les transitoires de tuyaux d'orgue, qui passe par l'intégration approchée d'un ensemble d'équations différentielles non linéaires couplées, exprimant les modes normaux qui interagissent par l'intermédiaire du jet d'air qui excite le tuyau. On donne des exemples du calcul pour des montées de pression au pied du tuyau plosives, brusques et lentes; on conclut que la forme du transitoire de pression exerce une influence importante sur le transitoire du son du tuyau, une attaque plosive favorisant les partiels supérieurs. Dans la plupart des cas, on trouve que le second mode du tuyau s'établit plus rapidement que le fondamental, et prédomine dans les débuts du transitoire du son, ce que l'expérience confirme. Les fréquences des partiels supérieurs ne sont pas harmoniques au début, déterminées qu'elles sont par les résonances du tuyau et par la composante réactive de l'impédance du jet. Aux stades ultérieurs du régime transitoire, tous les modes sont fixés en des rapports harmoniques, tout comme ils le sont dans le régime établi.

1. Introduction

It is well known that much of the characteristic quality of many musical sounds derives from the attack transient, although the harmonic structure of the steady tone (if there is a steady tone) is also clearly important. The decay transient for sound sources like organ pipes is generally of less importance, being influenced considerably by the properties of the building.

The attack transient in organ flue pipes has been recognised as an important musical variable for several centuries and organ builders have learnt to control the initial "chiff" of these pipes so as to make it uniform throughout the rank or even to eliminate it through nicking of the languid. It is also well known that the design of the windchest and of the valve admitting air to the pipe foot can have a considerable effect on the character of this initial transient.

Although the sounding mechanism of organ flue pipes has been the subject of detailed investigation for many decades, our understanding of the complex phenomena involved is still little more than rudimentary. The action of the air jet on the air in the pipe is almost completely described by recent work of Elder [1], building on earlier studies by Cremer and Ising [2] and by Coltman [3], but the equally important questions of the action of the acoustic current from the pipe mouth in perturbing the jet [4] and of the propagation of these disturbances along the jet are still largely unexplored.

The study of the harmonic structure of organ pipe sound is well advanced experimentally and the physical adjustments contributing to the timbre are known at least qualitatively [5], but detailed treatment of the underlying physical principles is still in its early stages [6], [7], [8]. The existing theory for flue pipes [8] is, however, sufficiently developed that it is not unreasonable to consider the more difficult problems of transients.

The purpose of the present paper is to lay the foundations for a general theoretical discussion of transients in flue pipes, to identify the significant variables and to calculate several illustrative examples. The theory to be presented is quite general and applies to all sorts of transients including the vibrato produced in organ pipes or flutes by rhythmic variation of blowing pressure [9], [10], but our present interest is largely with the initial transient.

Experimental studies on the attack transient have been carried out by Nolle and Boner [11] and by Keeler [12], among others, but the lack of a theoretical framework for interpretation of the measurements has reduced their usefulness. An extensive investigation has now commenced in this laboratory based upon study of the parameters identified in the theory as being of major significance.

2. The organ pipe as a system

Rather than considering the organ pipe as a single entity, it is convenient, and indeed necessary at this stage, to divide it into several sub-systems and to consider the interaction between these. For our present purposes a division into two such sub-systems is sufficient. The first is the air column of the pipe — a very nearly linear resonant system with an infinite number of normal modes. The second is the jet system, including its interaction with the pipe lip — a highly non-linear system which may or may not have natural resonance frequencies but does certainly have regimes for which its incremental impedance is negative. Finally there is the mechanism which couples these two systems together.

Treatment of the pipe as a resonant system is straightforward and is covered in all standard texts on acoustics [13]. For our present purposes it is sufficient to note that the pipe has a series of normal modes with (angular) frequencies n_i which are in approximate but not exact harmonic relationship. These modes are damped by viscosity radiation and thermal conduction [14] so that the displacement x_i of the ith mode obeys an equation of the form

$$\ddot{x}_i + k_i\,\dot{x}_i + n_i^2\,x_i = \lambda_i\,F(t) \qquad (1)$$

when acted on by an external force $F(t)$. If $F(t)$ is held constant in amplitude but is varied in frequency, then the amplitude of the velocity resonance is proportional to λ_i/k_i and its width to k_i. The response curve for the pipe, determined for example by applying a sound wave of frequency ω and constant pressure amplitude to the pipe mouth and measuring the maximum sound pressure inside the pipe as a function of ω (or, to sufficient approximation, by measuring the pressure amplitude of the wave radiated from the other end of the pipe), is a superposition of responses of type (1) from the various normal modes of the pipe.

The jet system is much more difficult to treat, but an approximate description along the lines of that developed previously [3] will suffice at present. The jet, on emerging from the narrow slit defined by the edge of the languid and the lower lip of the pipe mouth, is sensitive to displacement by the fluid motion $v = \sum \dot{x}_i$ through the mouth of the pipe. If the acoustic flow is out of the pipe mouth, then the jet is deflected outside the upper lip and the pressure falls, while if the acoustic flow is inwards the jet is deflected inside the upper lip and raises the pressure in the pipe near the mouth. For a steady velocity v into the pipe mouth, the driving force F produced by the jet has a form like that shown in Fig. 1. The curve is sigmoid in shape and, due to offset of the initial jet direction, is not generally antisymmetric about $v = 0$. The driving force F for this static case can thus be written as a power series expansion

$$F = c_0 + c_1\,v + c_2\,v^2 + c_3\,v^3 + \cdots \qquad (2)$$

where the coefficients c_n are functions of the blowing pressure p. Terms at least up to v^3 are necessary

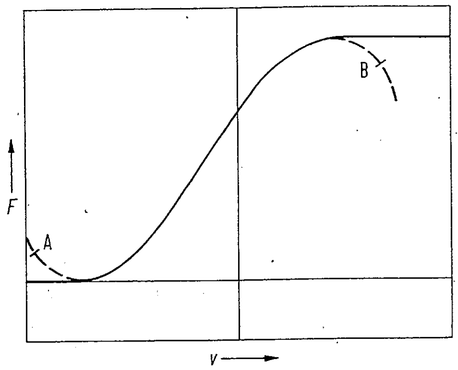


Fig. 1. Static interaction curve showing the force F generated by the jet under influence of an air flow of velocity v into the mouth of the pipe. The broken curve shows a cubic approximation which is adequate between points A and B.

to give a reasonable approximation to the shape of the curve, as shown in Fig. 1.

For the oscillatory case in which we are interested, however, we must recognise that the acoustic velocity v acts on the jet as it leaves the slit [4] while the resulting pressure response is produced at some time δ later, where δ is the time for the displacement to travel across the mouth of the pipe along the jet. To be completely general, we should recognise that the jet may be dispersive, so that δ may depend on frequency, and there may be a further phase shift Δ during the interaction process. If, then, we let ω_i be the actual vibration frequency of the mode associated with the resonance n_i, we can generalize eq. (2) to write

$$F(t) = \sum_{m=0}^{\infty} c_m \left[\sum_{i=1}^{\infty} \dot{x}_i \left(t - \delta_i - \frac{\Delta_i}{\omega_i} \right) \right]^m. \tag{3}$$

The total behaviour of the coupled non-linear pipe/jet system is now specified by the set of equations (1) with the specific interpretation (3) for $F(t)$. It is with the solution of this set of equations that we shall now be concerned.

3. The system equations

The set of equations (1), (3) can be written in the form

$$\ddot{x}_i + n_i^2 x_i = f_i(\dot{x}_j) \tag{4}$$

where

$$f_i(\dot{x}_j) \equiv -k_i \dot{x}_i + \lambda_i F(\dot{x}_1, \dot{x}_2, \ldots) \tag{5}$$

and in this form we recognise eq. (4) as being the equation studied by Van der Pol [15] and subsequently given considerable attention by Bogoliubov and others [16], [17].

An exact solution for this set of coupled non-linear differential equations is, of course, out of the question. Fortunately a solution of sufficient accuracy for our present purpose can be obtained by applying the most elementary methods developed by these workers — the approach known as the method of slowly varying parameters. We expect this approach to be valid when the form of the oscillation does not vary very greatly from one cycle to the next, a situation generally found in organ pipe transients. The formal condition for the validity of the method relates to the smallness of the function f_i relative to the individual terms on the left side of eq. (4). We shall state this condition more precisely later.

Consider eq. (4). Provided f_i is small compared with terms on the left side, we expect that the solution can be written

$$x_i = a_i \sin(\omega_i t + \beta_i) \tag{6}$$

where $\omega_i \simeq n_i$ and both a_i and β_i vary slowly with time. There is a certain indeterminancy about this specification which we can remove by requiring that $\dot{x}_i$ have the form

$$\dot{x}_i = a_i \omega_i \cos(\omega_i t + \beta_i) \tag{7}$$

which imposes the conditions

$$\dot{a}_i \sin(\omega_i t + \beta_i) + a_i \dot{\beta}_i \cos(\omega_i t + \beta_i) = 0. \tag{8}$$

Substituting eqs. (6) and (7) back into eq. (4) now gives

$$\begin{aligned} &-a_i \omega_i^2 \sin(\omega_i t + \beta_i) + \dot{a}_i \omega_i \cos(\omega_i t + \beta_i) - \\ &- a_i \omega_i \dot{\beta}_i \sin(\omega_i t + \beta_i) + \\ &+ n_i^2 a_i \sin(\omega_i t + \beta_i) = f_i(\dot{x}_j) \end{aligned} \tag{9}$$

which, using eq. (8) gives

$$\begin{aligned} \dot{\beta}_i = {} & \frac{1}{a_i \omega_i} f_i(\dot{x}_j) \sin(\omega_i t + \beta_i) + \\ & + \frac{n_i^2 - \omega_i^2}{\omega_i} \sin^2(\omega_i t + \beta_i) \end{aligned} \tag{10}$$

$$\begin{aligned} \dot{a}_i = {} & \frac{1}{\omega_i} f_i(\dot{x}_j) \cos(\omega_i t + \beta_i) - \\ & - \frac{a_i (n_i^2 - \omega_i^2)}{\omega_i} \sin(\omega_i t + \beta_i) \cos(\omega_i t + \beta_i). \end{aligned} \tag{11}$$

This is all, so far, quite rigorous. We now, however, follow Van der Pol and replace these exact time derivatives by their average values, found by integrating eqs. (10) and (11) over a period of the vibration. If the vibration were stationary then this too could be done rigorously by extending the integration to infinity and would select Fourier components of frequency ω_i out of $f_i(\dot{x}_j)$. As it is, the integration serves only to make an approximate separation of these components. Denoting the average by $\langle\ \rangle$, we can write

$$\langle\dot{\beta}_i\rangle = -\frac{1}{a_i\,\omega_i}\langle(f_i(\dot{x}_j)\sin(\omega_i t+\beta_i)\rangle + \\ +\frac{n_i^2-\omega_i^2}{2\,\omega_i} \qquad (12)$$

$$\langle\dot{a}_i\rangle = \frac{1}{\omega_i}\,\langle f_i(\dot{x}_j)\cos(\omega_i t+\beta_i)\rangle\,. \qquad (13)$$

In keeping with our aim of describing the oscillations in the pipe by equations like (6) with a_i and β_i varying slowly with time, the averages in eqs. (12) and (13) must be interpreted so as to neglect rapidly oscillating components arising from the short integration interval and retain only the major secular components which have the same trend over many cycles of the oscillation. The solution is satisfactory if $\langle\dot{\beta}_i\rangle$ and $\langle\dot{a}_i\rangle$ are small compared with ω_i and a_i respectively.

In principle the averages in eqs. (12) and (13) could be evaluated using the full form of the functions $f_i(\dot{x}_j)$, but the fact that the mode frequencies ω_i are in approximate harmonic relationship makes the series expansion explicit in eqs. (3) and (4) convenient. This expansion is also necessary if we are to make further progress with the formalism. To make this clear, we note that the nth power term in eq. (3) will contain, when the forms (7) are inserted for $\dot{x}_i$, terms varying with frequency $\omega=\omega_j\pm\omega_k\pm\omega_l\,\ldots$ where there are n terms in this sum. The averaging procedure in eqs. (12) or (13) first converts ω to $\omega\pm\omega_i$ and then retains only the slowly varying terms for which $\omega\pm\omega_i\simeq 0$.

With this prescription an arbitrary number of pipe modes can be considered and the jet characteristic expanded to an arbitrary number of terms. Only the algebra becomes complicated. To illustrate the forms of expressions obtained we consider the first three modes of an open pipe, for which $n_i\simeq i\,n_1$, with the jet characteristic expanded up to cubic terms. We find, from eq. (12),

$$\begin{aligned}\langle\dot{\beta}_1\rangle = &-(\lambda_1/2a_1\omega_1)\{c_1a_1\omega_1\sin(\omega_1\delta_1+\varDelta_1)+c_2a_1a_2\omega_1\omega_2\sin[(2\omega_1-\omega_2)t+\omega_2\delta_2-\omega_1\delta_1+\\ &+2\beta_1-\beta_2+\varDelta_2-\varDelta_1]+c_2a_2a_3\omega_2\omega_3\sin[(\omega_1+\omega_2-\omega_3)t+\omega_3\delta_3-\omega_2\delta_2+\beta_1+\\ &+\beta_2-\beta_3-\varDelta_2+\varDelta_3]+\tfrac{3}{4}c_3a_1^2a_3\omega_1^2\omega_3\sin[(3\omega_1-\omega_3)t+\omega_3\delta_3-2\omega_1\delta_1+3\beta_1-\beta_3-\\ &-2\varDelta_1+\varDelta_3]+\tfrac{3}{4}c_3a_2^2a_3\omega_2^2\omega_3\sin[(\omega_1+\omega_3-2\omega_2)t-\omega_3\delta_3+2\omega_2\delta_2+\beta_1+\beta_3-\\ &-2\beta_2-\varDelta_3+2\varDelta_2]+\tfrac{3}{4}c_3(a_1^3\omega_1^3+2a_1a_2^2\omega_1\omega_2^2+2a_1a_3^2\omega_1\omega_3^2)\sin(\omega_1\delta_1+\varDelta_1)\}+\\ &+(n_1^2-\omega_1^2)/2\omega_1 \end{aligned} \qquad (14)$$

$$\langle\dot{a}_1\rangle = -\tfrac{1}{2}k_1a_1+(\lambda_1/2\omega_1)\{\ldots\cos\ldots\} \qquad (15)$$

where the expression $\{\ldots\cos\ldots\}$ in eq. (15) is the same as $\{\,\}$ in eq. (14) with sin replaced by cos throughout. The expressions for $\langle\dot{\beta}_2\rangle$, $\langle\dot{\beta}_3\rangle$, $\langle\dot{a}_2\rangle$ and $\langle\dot{a}_3\rangle$ have similar forms to these and are given in the appendix.

With the aid of these formal expressions, explicit values for the jet coefficients c_m (which may be time-dependent), and a set of initial conditions, the problem can now be solved by simple numerical integration. We shall pursue this in the next section. Meanwhile, several important observations can be made.

First consider the steady state. The "instantaneous frequency" (to use an inexact term) for the mode i is $\omega_i+\dot{\beta}_i$. From the form of the expression (14) for $\dot{\beta}_1$ and the similar expressions for the other $\dot{\beta}_i$, the condition that $\dot{\beta}_i$ be independent of time is that

$$\omega_i+\dot{\beta}_i = i(\omega_1+\dot{\beta}_1) \qquad (16)$$

so that the frequencies of the pipe modes are locked into harmonic relationship. This is the normal "musical" mode for a flue pipe. The actual fundamental frequency and the amplitudes of the harmonics are determined by the combined solutions of the mode equations, which are identical with those used in the earlier analysis of the steady tone [8].

The condition for a steady, non-zero amplitude is that $\langle\dot{a}_i\rangle=0$ for some non-zero a_i while $\langle\dot{a}_i\rangle>0$ as $a_i\to 0$. For the ith mode, from the analogue of eq. (15), this requires as a first approximation

$$\lambda_i c_1\cos(\omega_i\delta_i+\varDelta_i)>k_i \qquad (17)$$

which states that the appropriately phase-retarded regeneration coefficient should exceed the damping coefficient. The equilibrium amplitude for which $\langle\dot{a}_i\rangle=0$ depends upon the non-linear terms in eq. (15). Even if eq. (17) is not satisfied, there will generally be some component with frequency near n_i generated by interaction between other modes whose frequencies differ by approximately n_i.

Other "non-musical" modes of pipe behaviour which approximate a steady state also exist over certain ranges of the coefficients and these can be observed experimentally with many organ pipes, usually in the pressure range just before over-blowing occurs. An example frequently found in practice occurs when eq. (17) is strongly satisfied for modes 2 and 3 but is either not satisfied or else only weakly satisfied for the fundamental mode 1. In this case amplitudes a_2 and a_3 are large but a_1 is small. Now an examination of eq. (14) and the similar equations for $\langle\dot{\beta}_2\rangle$ and $\langle\dot{\beta}_3\rangle$ shows that modes 2 and 3 cannot couple except through small high-order terms like $c_4 a_2^2 a_3^2 \sin[(3\omega_2 - 2\omega_3)t]$ or through terms like $c_2 a_1 a_3 \sin[(\omega_3 - \omega_2 - \omega_1)t]$ which are small because a_1 is small. Thus modes 2 and 3 remain nearly uncoupled and can adjust themselves so that $\omega_2 \simeq n_2$, $\omega_3 \simeq n_3$ and are then not harmonically related. The cross terms then produce a component ω_1 near the fundamental resonance frequency n_1 which is modulated by terms varying like $\sin[(\omega_3 - \omega_2 - \omega_1)t]$ which appear as unpleasant beats. This is exactly the observed behaviour [18].

4. Jet and pipe parameters

It is not our purpose here to give any detailed attention to the behaviour of the air jet. Elder [1] has already done this for an axial jet with modulated flow but the discussion of physically realistic flue pipe jets is in a much less well developed state. The discussion given by Coltman [3] as developed in our previous paper [8] will serve for our present purpose. When a better description of the jet becomes available it can easily be inserted in our discussion at this point.

If we adopt the convention of expressing the blowing pressure p in millibars (or, almost equivalently, in centimetres of water head) and accept that the phase velocity for transverse disturbances on the jet 1 to 2 mm thick is a fraction $\alpha \simeq 0.4$ of the jet stream velocity [4], then the propagation delay δ for a jet length (pipe cut-up) l cm is

$$\delta = 8 \times 10^{-4}\, l\, p^{-1/2} \alpha^{-1}\ \mathrm{s}. \tag{18}$$

We neglect any frequency dispersion. We also assume, as discussed previously [8] that

$$\Delta \simeq \pi \tag{19}$$

again neglecting any frequency dependence. Let us also, without further discussion, adopt the jet and pipe dimensions in our earlier paper [8] — jet cross-section $3\ \mathrm{cm} \times 0.1\ \mathrm{cm}$, jet offset 0.01 cm, pipe cross-section 10 cm², mouth cut-up $l = 1$ cm — and accept the derived jet parameters

$$\begin{aligned} c_0 &= 30\, p\,, \\ c_1 &= 0.4\, \gamma\, p^{1/2}\,, \\ c_2 &= -4 \times 10^{-4}\, \gamma^2\,, \\ c_3 &= -7 \times 10^{-7}\, \gamma^3\, p^{-1/2}\,, \end{aligned} \tag{20}$$

where γ (called β in the earlier paper) is a parameter measuring the sensitivity of the jet transverse velocity to the acoustic particle velocity in the pipe mouth. This treatment neglects many things, such as the widening and slowing of the jet as it issues from the slit, but we shall not enter into a discussion of these here. Our only objective is to obtain a qualitatively reasonable set of expansion coefficients which vary in approximately the correct way with blowing pressure.

The pipe is characterized for our present purposes by the frequencies, heights and widths of its first three resonances and we must also assign a value to the interaction parameter γ. The values adopted for our illustrative calculation are as shown in Table I. The quality factors (Q-values) n_i/k_i for the three resonances are all close to 20, so that the damping is not large. The resonances are purposely shifted away from strict harmonic relationship by amounts that are comparable with shifts found in real organ pipes of moderately wide scale. The λ_i and γ act essentially as scaling factors for the interaction between the pipe and air jet and their absolute values, though important if a comparison with specific experiment is sought, need not concern us here.

Table I.
Assumed pipe parameters.

Resonance frequencies	$n_1 = 1000$	$n_2 = 2050$
	$n_3 = 3100$	rad s⁻¹
Damping coefficients	$k_1 = 50$	$k_2 = 100$
	$k_3 = 150$	s^{-1}
Coupling coefficients	$\lambda_1 = \lambda_2 = \lambda_3 = 2,\ \gamma = 100$	
Jet velocity parameter	$\alpha = 0.2$	

5. Pressure transient and initial conditions

The form of the attack transient depends upon the characteristics of the pipe and the geometry of the jet but, once these have been fixed, it depends primarily upon the time variation of the air pressure producing the jet. This effect is generally known to players of organs with mechanical action and, of course, forms the basis of articulation (tonguing) for flute players but, rather surprisingly, it has not been taken into account by those studying the onset transient in pipe speech [11], [12].

We have therefore performed a simple experiment to measure the wind pressure in the foot of a flue pipe seated on a simulated wind chest with a mechanically activated pallet valve. When the valve is opened abruptly the pressure in the pipe foot rises rapidly, in a time of order 10^{-2} s, to a peak value and then decays, with a time constant of order 10^{-1} s, towards a steady-state pressure. The peak pressure is typically several times greater than the steady pressure which itself is considerably less than the pressure in the wind chest. Slower opening of the pallet reduces the magnitude of the pressure peak, while a very slow opening simply allows the pressure in the pipe foot to build slowly to its steady value. A detailed study of these effects is now being commenced but for our present purposes it is sufficient to note that, with abrupt pallet action, the pipe produced a typical "chiff" while this was absent for the case of slow pressure increase.

These forms of pressure transient can all be described to sufficient accuracy for our present purposes by an expression of the form

$$p(t) = p_0 + (p_1 - p_0) \exp(-t/\tau) \qquad (21)$$

where p_1 specifies the pressure peak, p_0 the steady pressure and τ the decay time from the peak. We can then distinguish three possibilities. When $p_1 \gg p_0$ the pressure peak is pronounced and we may refer to the pressure transient as plosive, by analogy with the phonetic term. When $p_1 = p_0$ the steady state is immediately achieved and we call the pressure transient abrupt, while for $p_1 \ll p_0$ the pressure onset is slow.

The form (21) for $p(t)$ can be substituted directly into eq. (20) and thence into eqs. (14) and (15) and the equations of the Appendix. The only other data required are the initial values of the amplitudes a_i and phases β_i. For the simple form of eq. (21), these can be readily evaluated as follows.

The pressure transient begins with a sudden rise of pressure in the pipe foot to a value p_1. We make the simplifying assumption that the jet forms immediately and, after a small delay, interacts with the pipe lip at a time which we shall take as $t = 0$. It is during this part of the initiation of pipe speech that the effects due, for example, to nicking of the languid are most likely to produce an effect, but we have agreed to ignore these here.

Now from eq. (20) we see that a pressure step p_1 (millibars) in the pipe foot produces, for the particular jet and pipe we have in mind, a pressure step $30 p_1$ (dyne cm^{-2}) in the pipe. The large decrease, considering the change of units, is caused by the small ratio of jet cross section to pipe cross section. Initially the pipe presents the specific acoustic impedance $\varrho c = 42$ c.g.s. units characteristic of an infinite pipe [13] so that the resultant velocity step wave has amplitude $0.7\, p_1$. The various Fourier components comprising this step are reflected from the open end of the pipe and again from the jet end, with a slight phase change due to the impedance of the jet. The initial velocity disturbance thus looks like a square wave with frequency close to n_1 and we can make sufficient allowance for anharmonicity of the resonances by assigning frequencies n_i rather than $i n_1$ to the upper partials. Thus

$$v_i \simeq (0.5\, p_1/i) \sin n_i t \qquad (22)$$

which gives for the initial amplitudes, phases and frequencies

$$\begin{aligned} a_i^0 &= 0.5\, p_1/i\, n_i \\ \beta_i^0 &= -\pi/2 \\ \omega_i &= n_i \,. \end{aligned} \qquad (23)$$

Any minor error made in the frequency ω_i is corrected very rapidly by eq. (14) so that the exact choice of ω_i is not important. The main part of this correction is due to the phase change produced by interaction with the jet and, as we shall see from the numerical examples, the initial frequencies $\omega_i + \dot{\beta}_i$ may differ considerably from the pipe resonance frequencies n_i.

In passing we should also note that, for a slow pressure increase, it is still necessary to commence with a finite pressure step p_1 in order to achieve a unique solution. This happens, both in the theory and in practice, since the static situation $\dot{x}_i = 0$ for all i is a valid solution of the original set of eqs. (1). This solution will persist until upset by some random transient, thus giving non-reproducible behaviour. It is therefore necessary to provide a reproducible transient to initiate pipe speech.

6. The attack transient

To compute the attack transient we have now only to substitute the initial conditions (23) and the pressure variation (21) into eqs. (14), (15) and their analogues in the Appendix, using the expressions (20) for the c_n and the numerical coefficients given in Table I. The resulting equations are then easily integrated numerically.

Illustrative calculations have been performed for a wide variety of cases from which we have selected the examples shown in Fig. 2 as being representative of typical flue pipe behaviour. Computation time was only 20 s, on a medium-sized computer, for each set of curves. The only modification made to our previous discussion was to take α, the disturbance velocity parameter on the jet of eq. (18), to have a value 0.2 rather than 0.4. This revised value is in accord with indications from a study of flute

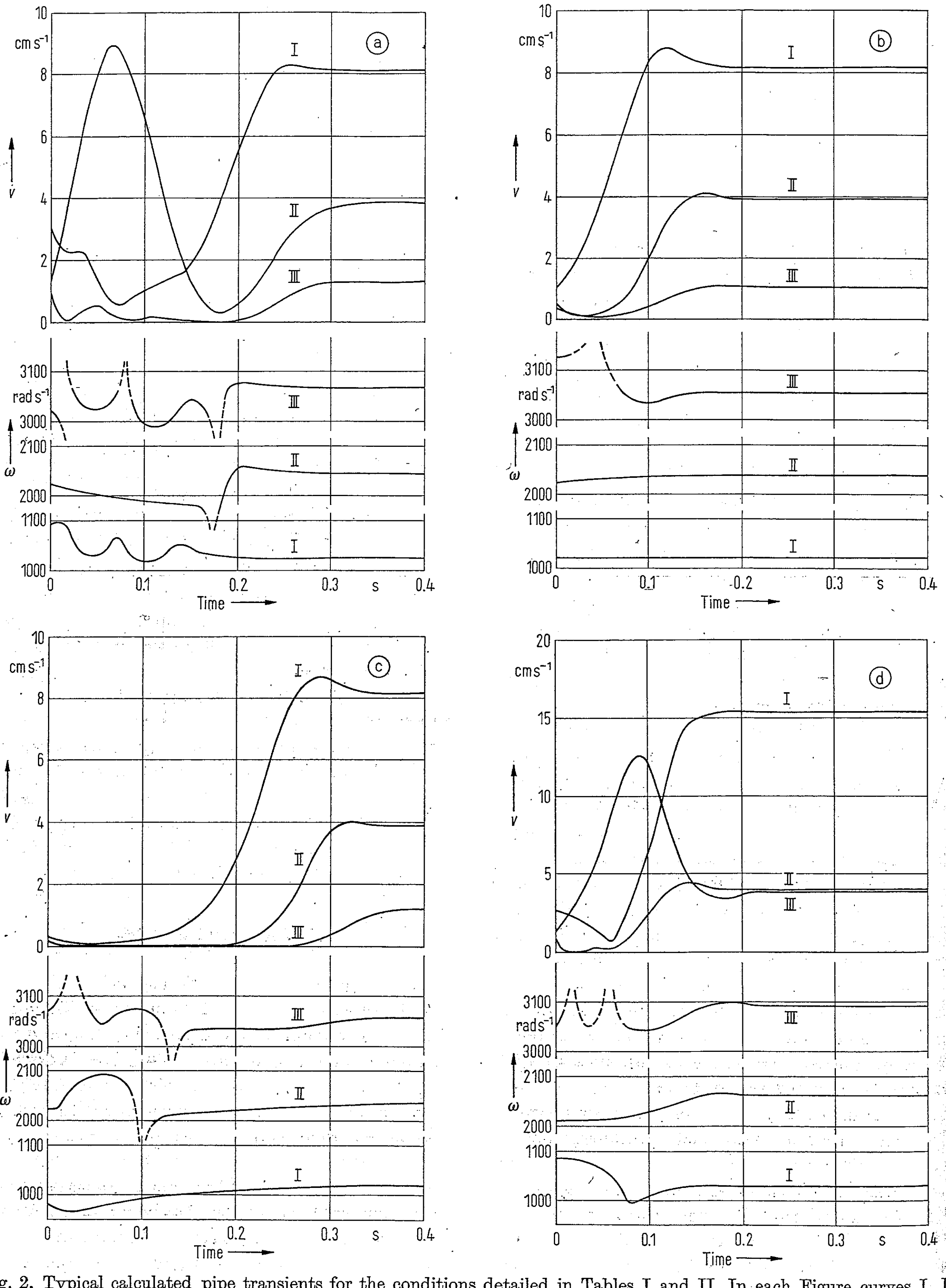
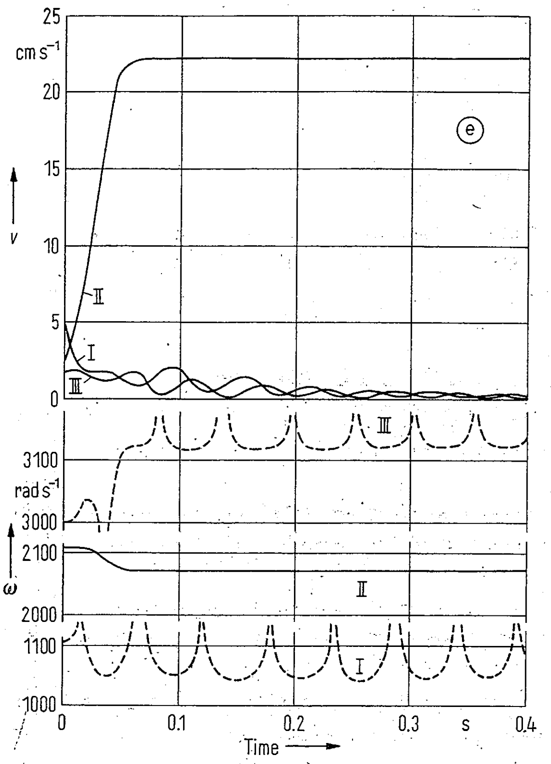
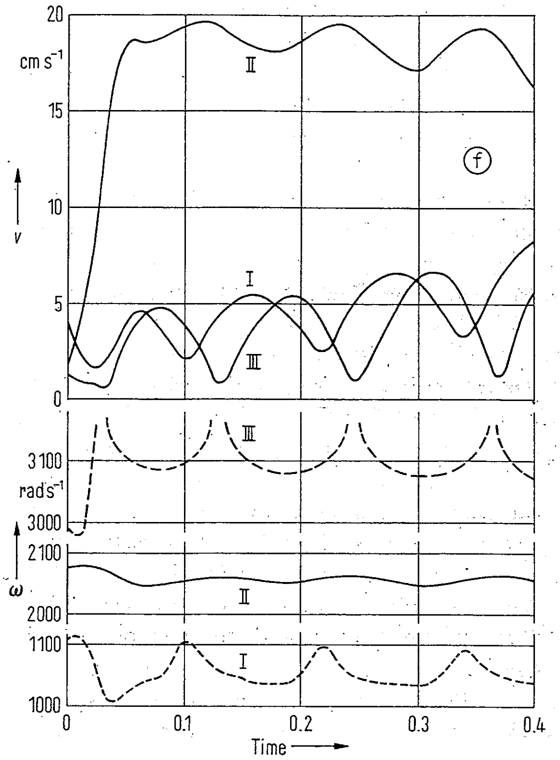


Fig. 2. Typical calculated pipe transients for the conditions detailed in Tables I and II. In each Figure curves I, II and III refer to the first, second and third pipe modes respectively. Velocity amplitude v for each mode is given in centimetres per second and frequency f in radians per second. Broken lines indicate regions of unstable or uncertain frequency.

(a), (b) and (c) show plosive, abrupt and slow attacks, respectively, to a steady pressure of 2 mb,
(d) is an abrupt attack to 5 mb,



(e) an abrupt attack to an overblown condition at 10 mb and



(f) shows an unstable situation after abrupt attack to 8 mb.

jets [10] but the only effect in the present study is to double the effective cut-up distance, which is somewhat arbitrary in any case. The same results could have been achieved with $\alpha = 0.4$ and $l = 2$ cm.

The quantities calculated and presented here are the velocity amplitudes of the various modes inside the pipe. The sound radiated from the open pipe end emphasizes the upper partials in proportion to their frequency [13], which should be borne in mind when comparing our calculations with the measurements of Keeler and others [11], [12]. The actual sound pressure level measured at a distance of 1 m from our pipe is calculated [13] to be 60 to 70 dB, which is reasonable. This is, however, not significant since the absolute level is determined by the ratio λ/γ and the values of these parameters have been simply assigned rather than derived.

Details of the pressure transients considered in the calculations are set out in Table II. Fig. 2a, b and c show respectively plosive, abrupt and slow attacks to a steady pressure of 2 mb. The plosive attack gives a speech transient in which the second pipe mode is dominant for the first 0.15 s or so, the three modes not being harmonically related during this time. After about 0.2 s the three modes

Table II.
Pressure transients.

Transient	p_1 mbar	p_0 mbar	τ s
(a) plosive	6.0	2.0	0.1
(b) abrupt	2.0	2.0	—
(c) slow	0.5	2.0	0.1
(d) abrupt	5.0	5.0	—
(e) abrupt	10.0	10.0	—
(f) abrupt	8.0	8.0	—

become locked into harmonic relationship and build steadily to their final amplitudes after about 0.3 s. For an abrupt pressure attack the fundamental is dominant throughout, harmonic relation between the modes is achieved after 0.15 s and the steady state is reached in 0.2 s. For a slow attack the speech is delayed, with the fundamental always dominant, and the steady state does not occur until about 0.35 s after the pressure is applied.

Fig. 2d illustrates an abrupt attack to a final pressure of 5 mb, showing that with such a higher blowing pressure a speech transient with dominant second mode can be produced without the necessity for plosive attack. Such an attack does, however, emphasize the second mode peak, while a slow attack eliminates it. At this higher blowing pressure the steady state is also achieved in about 0.2 s.

When the steady blowing pressure is increased much above 6 mb, the pipe in our calculations overblows to its second mode to sound the octave. Fig. 2e shows an abrupt attack to an overblown state at a steady blowing pressure of 10 mb. The second mode rises to its steady state after rather less than 0.1 s, its frequency shifting a small amount during this time. The first and third modes decay rapidly with an oscillatory behaviour which can be attributed to the term in $(\omega_3 - \omega_2 - \omega_1)t$ in eq. (15).

Finally, Fig. 2f shows an abrupt attack to a marginally overblown condition at a steady pressure of 8 mb. At this pressure the linear term in the jet impedance has a negative real component for the second and third pipe modes but a positive component for the first mode. The interaction terms in $(\omega_3 - \omega_2 - \omega_1)t$ are sufficiently large to cause all modes eventually to build to comparable amplitudes and to exhibit the unpleasant beating behaviour remarked by Lough [18]. A slow attack is actually able to eliminate the first and third modes in this case but pipe speech would, in practice, be regarded as unreliable.

A comparison of the curves of Fig. 2 with the measurements of Keeler [12] shows very good qualitative agreement. Keeler's transients typically extend over 20 to 40 periods of the pipe fundamental which, for our 160 Hz pipe ($n_1 = 1000$ rad s^{-1}) implies 0.1 to 0.3 s in agreement with our calculations. Lack of knowledge of the pressure transient in the pipe foot for Keeler's measurements precludes any closer comparison.

Our calculations also give further insight into the nature of the chiff in pipes with plosive or abrupt attack. The dominance of the second pipe mode is one important feature while another, perhaps equally important, is the lack of harmonic relationship and the rapid shifts of phase and frequency in all the pipe modes during the transient.

We have, perforce, neglected the effects of most voicing adjustments on the initial transient, simply assuming the pipe to have normal speech. There is, however, one observation which can be made. The time interval before the pipe achieves its steady state depends not only on the form of the pressure transient, but also on the initial values of the mode amplitudes, and particularly that of the fundamental. A moderately large initial amplitude hastens steady speech, while a very small initial amplitude delays it. This initial amplitude is, by eq. (23), proportional to the initial pressure p_1, but it is also proportional to the fraction of the undisturbed jet (assumed to be half in our calculation) which enters the pipe mouth. If the jet is adjusted to pass largely outside the pipe lip then we expect the speech to be slow. This is in general accord with experience.

7. Conclusions

The qualitative success of the theory developed above in describing the onset transient of organ flue pipes suggests that the theory is basically correct and that no vital mechanisms have been overlooked in its formulation. There are, of course, subtleties to be considered but these can safely be left to a later stage in the development.

The theory is based upon a greatly simplified model for the air jet and its interaction with the acoustical currents in the pipe and it is evident that this model requires a great deal of refinement before we can attempt a realistic comparison between theory and experiment for a blown pipe. Theoretical and experimental work to this end is now under way in these laboratories.

In advance of this proper understanding of the jet mechanism, however, the theory has made clear the important rôle played by the pressure transient in determining pipe speech. This must be taken carefully into account in any further experimental studies.

Elaboration of the theory to treat a pipe with four or more significant resonances or to include higher terms in the non-linear jet characteristic is simply a matter of tedious algebra which, when completed, will not significantly complicate the calculations or extend the small amount of computer time necessary for the integration. Any such development is left in abeyance until the more significant study of the jet characteristic has been completed.

This work is part of a project designed to investigate the acoustics of traditional musical instruments which is supported by the Australian Research Grants Committee.

Appendix

$$\begin{aligned}\langle \dot{\beta}_2 \rangle = &- (\lambda_2/2 a_2 \omega_2) \{c_1 a_2 \omega_2 \sin(\omega_2 \delta_2 + \Delta_2) + \tfrac{1}{2} c_2 a_1^2 \omega_1^2 \sin[(\omega_2 - 2\omega_1)t + 2\omega_1 \delta_1 + \\ &+ \beta_2 - 2\beta_1 + \Delta_1] + c_2 a_1 a_3 \omega_1 \omega_3 \sin[(\omega_2 + \omega_1 - \omega_3)t - \omega_1 \delta_1 + \omega_3 \delta_3 + \beta_2 + \\ &+ \beta_1 - \beta_3 - \Delta_1 + \Delta_3] + \tfrac{3}{2} c_3 a_1 a_2 a_3 \omega_1 \omega_2 \omega_3 \sin[(2\omega_2 - \omega_1 - \omega_3)t - \omega_2 \delta_2 + \\ &+ \omega_1 \delta_1 + \omega_3 \delta_3 + 2\beta_2 - \beta_1 - \beta_3 - \Delta_2 + \Delta_1 + \Delta_3] + \tfrac{3}{4} c_3 (a_2^3 \omega_2^3 + 2 a_2 a_1^2 \omega_2 \omega_1^2 + \\ &+ 2 a_2 a_3^2 \omega_2 \omega_3^2) \sin(\omega_2 \delta_2 + \Delta_2)\} + (n_2^2 - \omega_2^2)/2\omega_2\end{aligned}$$

$$\langle \dot{a}_2 \rangle = -\tfrac{1}{2} k_2 a_2 + (\lambda_2/2\omega_2)\{\ldots\cos\ldots\},$$

$$\begin{aligned}\langle \dot{\beta}_3 \rangle = &-(\lambda_3/2 a_3 \omega_3)\{c_1 a_3 \omega_3 \sin(\omega_3 \delta_3 + \Delta_3) + c_2 a_1 a_2 \omega_1 \omega_2 \sin[(\omega_3 - \omega_1 - \omega_2)t + \delta_1 \omega_1 + \\ &+ \delta_2 \omega_2 + \beta_3 - \beta_1 - \beta_2 + \Delta_1 + \Delta_2] + \tfrac{3}{4} c_3 a_2^2 a_1 \omega_2^2 \omega_1 \sin[(\omega_3 - 2\omega_2 + \omega_1)t + \\ &+ 2\omega_2 \delta_2 - \omega_1 \delta_1 + \beta_3 - 2\beta_2 + \beta_1 + 2\Delta_2 - \Delta_1] + \tfrac{3}{4} c_3 (a_3^3 \omega_3^3 + 2 a_3 a_1^2 \omega_3 \omega_1^2 + \\ &+ 2 a_3 a_2^2 \omega_3 \omega_2^2) \sin(\omega_3 \delta_3 + \Delta_3)\} + (n_3^2 - \omega_3^2)/2\omega_3,\end{aligned}$$

$$\langle \dot{a}_3 \rangle = -\tfrac{1}{2} k_3 a_3 + (\lambda_3/2\omega_3)\{\ldots\cos\ldots\}.$$

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