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# Controlling the transients and timbre on single reed instruments

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### Abstract

A beautiful note should have appropriate pitch, loudness and timbre, and be elegantly articulated. On the clarinet and saxophone, the frequency, sound level, spectral envelope and articulation can be controlled using a learned coordination of breath pressure, lip force, tongue motion and vocal tract shape. But how do players achieve this and how do these control parameters affect the sound? Here, the acoustical impedance spectrum of the player's vocal tract, the blowing pressure and its time variation, the tongue-reed contact and their coordination were measured while players played notes of different pitch, loudness, timbre and articulations such as accent, *sforzando*, *staccato* etc. The effects of blowing pressure, lip force and tongue motion on the initial and final transients were studied independently using a clarinet-playing machine. To start a note, players vary the blowing pressure ( $P$ ) over time and adjust the timing of tongue release from the reed to produce different articulations with rates of exponential increase in amplitude (of order  $1000 \text{ dB}\cdot\text{s}^{-1}$ ) in the initial transient. In the sustained part of a note, advanced players can use the vocal tract to vary the spectral envelope whilst keeping the pitch and sound level constant. The harmonics in the radiated sound are enhanced when the magnitude of the player's vocal tract impedance is increased sufficiently to become comparable with that of the instrument bore at nearby frequencies. Notes can be terminated either by decreasing  $P$  below a threshold or by tongue contact with the reed: both produce exponential decreases in sound pressure, often dominated by energy losses in the bore.

**Keywords:** single reed, transients, articulation, spectral envelope, regeneration

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## 1 Introduction

In woodwind playing, performers use their breath, lips, tongues and vocal tract configurations to produce notes with the desired pitch, loudness, timbre and articulation. Developing the skills to control these is an important part of musicianship, taking years to master. This paper reviews and extends studies on the control of transients and timbre in clarinet and saxophone playing.

Initial transients of notes are highly salient and important in identifying instruments [1-2]. Musicians refer to initial and final transients as articulation. On reed instruments, articulation usually involves 'tonguing' — the use of the tongue to touch the reed and to release it to start a note [3-4], and sometimes to stop a note by touching the reed.

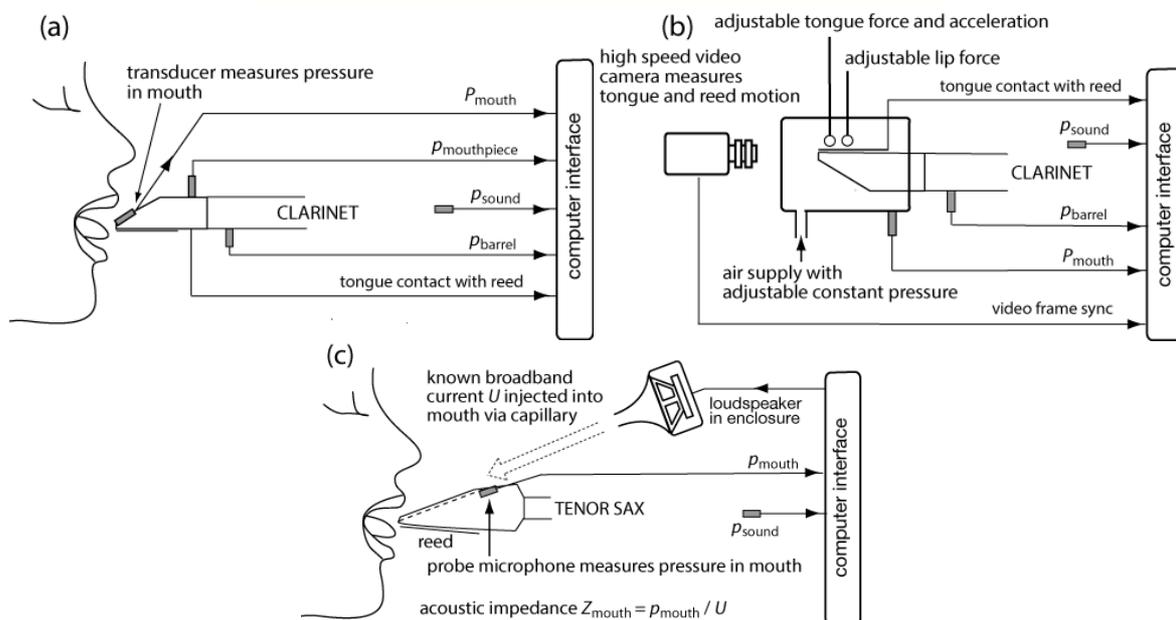
Players also vary the timbre of their sound for musical expression, especially on the saxophone. Many players report that they achieve this by adjusting the shape of the vocal tract. Several studies on clarinetists and saxophonists have previously examined the effects of the vocal tract on advanced playing techniques, such as pitch bending, *glissandi* and multiphonics [5-6]. Here, the contribution of the player's vocal tract in producing timbre variation is reported.

The clarinet is one of the most popular wind instruments and has become a standard instrument for scientific studies, particularly its acoustical behaviour in the steady state [7-20]. More recently, researchers have also started investigations on the transient behaviour [21-24]. However, the effects of blowing pressure, lip force and tonguing on transients have not been studied in detail: that is one aim of the current paper. Another is to explain the measured properties of transients in terms of known acoustical properties and simple models of the instrument and its reed.

This paper summarises and extends several studies: measurements under playing conditions show how human players control transients and the acoustical properties of their vocal tracts. A complementary study uses an automated clarinet-playing machine to investigate the effects of blowing pressure and lip force on transients. Regeneration and transient behaviour are described by modelling the energy budget of standing waves in the bore.

## 2 Materials and methods

Three experimental setups are shown in Fig. 1. The initial and final transients of notes played by experts are studied using a clarinet fitted with a pressure transducer on the mouthpiece to measure the blowing pressure  $P$  and the acoustic pressure  $p_{\text{mouth}}$  in the mouth — see Fig. 1(a). Other microphones measure pressures  $p_{\text{mouthpiece}}$  in the mouthpiece,  $p_{\text{barrel}}$  in the barrel (10 cm from the reed), and  $p_{\text{sound}}$  outside the bell. A high impedance electrical circuit using a fine wire on the reed detects tongue contact. More details are in [25].



**Figure 1: Schematic diagrams (not to scale) show the experimental setups for the three studies.**

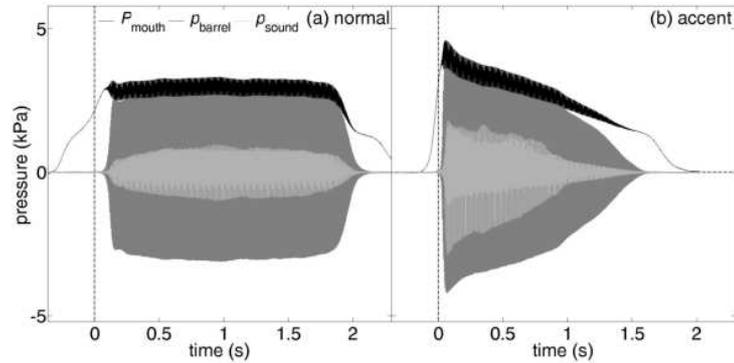
The effects of blowing pressure  $P$ , lip force  $F$  and tongue force and acceleration are studied using a clarinet-playing machine, in which these variables are controlled and varied independently [Fig. 1(b)]. Microphones measure mouth, barrel and bell pressures, and a high-speed video camera measures the tongue and reed motion. More details are in [26].

In the third, the impedance spectra of the vocal tract of expert players were measured while they played using a modified saxophone mouthpiece [Fig. 1(c)]. A synthesised broadband acoustic current is injected into the mouth near the reed. The pressures in the mouth and outside the bell are measured while players vary the timbre by changing vocal tract shape. More details are in [27].

### 3 Players' control of transients on the clarinet

For the written C4 note, Fig. 2 compares a normal note (presented to the players in normal music notation without articulation instructions) with an accented note (written with ">" over the note). The vertical line at  $t = 0$  indicates the moment when the tongue ceased contact with the reed. In both cases, the player increases the blowing pressure before  $t = 0$ , but with a greater rate and greater maximum value for the accented note.

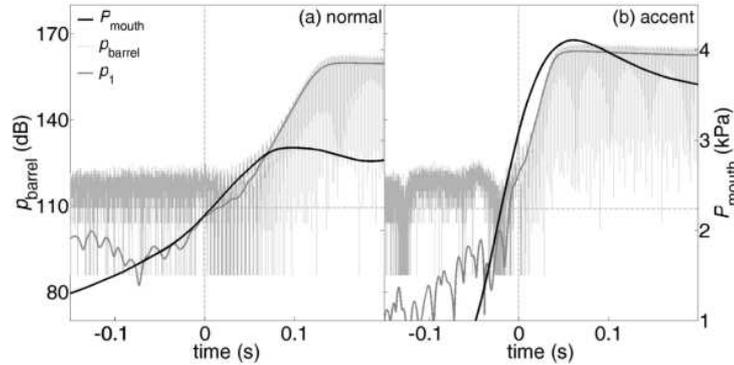
The acoustic flow into the instrument equals that out of the mouth, but the oscillatory component of the mouth pressure is smaller than that in the bore. This is because the instrument operates at a resonance of the bore, so that the acoustic impedance (ratio of acoustic pressure to flow) is larger in the mouthpiece than in the mouth. The envelope of the sound recorded just outside the bell is not simply proportional to that in the mouthpiece, because the bell radiates the high frequencies much better than the low, so the spectra of the two are significantly different.



**Figure 2:** Measured mouth pressure  $P_{\text{mouth}}$  (black, including DC and AC components), mouthpiece pressure  $p_{\text{barrel}}$  (dark grey) and radiated sound  $p_{\text{sound}}$  (light grey, not calibrated) as functions of time for typical examples of the written C4 note with (a) normal and (b) accent, played by one of the expert players.  $t = 0$  is defined as the instant when the tongue ceased contact with the reed.

### 3.1 Initial transients

The initial transients in Fig. 2 are shown in greater detail in Fig. 3, using a log scale. This scale shows that the initial transients both have a phase of exponential increase, maintained over a few tens of dB. For the accent, the tongue is released at a higher value of mouth pressure, which produces a larger rate  $r$  of exponential increase of the fundamental in the barrel pressure:  $1080 \text{ dB}\cdot\text{s}^{-1}$  compared with  $560 \text{ dB}\cdot\text{s}^{-1}$  for the normal note.



**Figure 3:** The slowly varying (DC) component of the mouth pressure  $\bar{P}_{\text{mouth}}$  (black), the amplitude (RMS) of the fundamental  $p_1$  (dark grey) of the barrel pressure  $p_{\text{barrel}}$  (light grey) for the written C4 note: (a) normal and (b) accent, played by an expert player. In each example, the horizontal dashed line indicates the noise level in the barrel in dB calculated over the 0.1 seconds before tongue release.  $t = 0$  is defined as the instant when the tongue ceased contact with the reed.

### 3.2 Players' control of transients

For a given threshold pressure, a player may control the initial transient with two parameters: the mouth pressure  $P_s$  when the tongue is released and the rate  $R$  of the rise in mouth pressure after tongue release. The average mouth pressure  $P_{\text{av}}$  during the pressure rise can be viewed as one control parameter which depends directly on both  $P_s$  and  $R$ . Table 1 shows the percentage of measurements in which the tongue is released below the oscillation threshold,

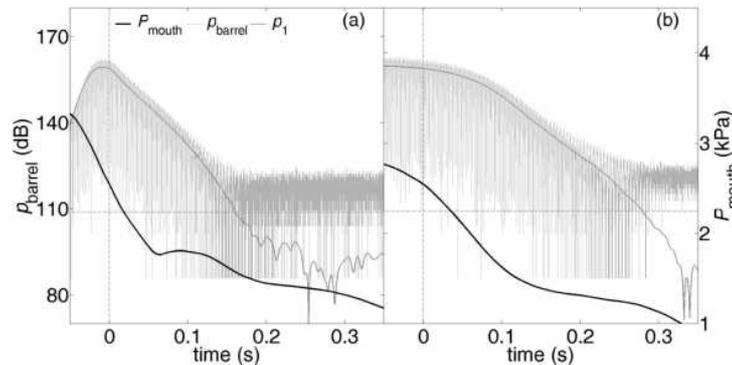
the average mouth pressure  $P_{av}$  during the initial transient and rate  $r$  of exponential increase for the C4 note with four different articulations played by three expert players. For accented and *sforzando* notes, players always released the tongue above the threshold and had a higher average mouth pressure to produce a larger exponential rate in the initial transients.

**Table 1: Tongue release, average mouth pressure and rate of exponential increase for different articulations**

C4	percentage of measurements in which the tongue is released below the threshold	$P_{av}$ during the initial transient (kPa)	rate $r$ ( $\text{dB}\cdot\text{s}^{-1}$ )
normal	17%	$2.70\pm 0.06$	$600\pm 120$
<i>staccato</i>	11%	$3.07\pm 0.12$	$800\pm 100$
accent	0%	$3.23\pm 0.10$	$960\pm 190$
<i>sfz</i>	0%	$3.39\pm 0.10$	$1240\pm 250$

### 3.3 Final transients

For the written C4 note, Fig. 4 compares the final transient of a *staccato* note (written with "." over the note) with that of a normal note. In (a), the vertical line at  $t = 0$  indicates the moment when the tongue touched the reed, producing an exponential decay (with rate  $r = -260 \text{ dB}\cdot\text{s}^{-1}$ ). The normal note [Fig. 4(b)] was stopped by decreasing mouth pressure without using the tongue. For comparison, the vertical line in (b) indicates the moment when the mouth pressure was the same as that at tongue touch in (a). The decay time was  $\sim 0.3 \text{ s}$ , almost twice that of a tongue-stopped note.



**Figure 4: The final transients of a written C4 note played a) *staccato* and (b) normal, by an expert.**

**The slowly varying (DC) component of the mouth pressure  $\bar{P}_{\text{mouth}}$  (black), the amplitude (RMS) of the fundamental  $p_1$  (dark grey) component of the barrel pressure  $p_{\text{barrel}}$  (light grey). In (a),  $t = 0$  is defined as the instant when the tongue touched the reed.**

Once the tongue touches the reed, the auto-oscillation is assumed to stop and the energy in the standing waves in the bore is eventually lost due to viscothermal losses in the walls and, to a lesser extent, radiation from the bell and tone holes [28]. For a resonance, the quality factor  $Q$ , by definition, is  $2\pi$  times the ratio of the energy stored in the oscillating resonator ( $E_S$ ) to the energy dissipated per cycle ( $E_L$ ) by damping processes. Defining  $E$  as the time average of energy stored,  $f$  as the oscillating frequency and  $p$  as the pressure in the barrel:

$$Q = 2\pi \frac{E_s}{E_L} = -2\pi f \frac{E}{\frac{dE}{dt}} = -\frac{2\pi f}{2 \frac{d(\ln p)}{dt}} \quad (1)$$

The rate  $r$  of exponential increase (here negative) can therefore be expressed as

$$r = \frac{d(20 \log_{10} p)}{dt} = -20\pi \frac{f}{Q} \log_{10} e \quad (2)$$

$Q$  is also equal to the frequency-to-bandwidth ratio of the resonator, which is known from acoustic impedance spectra measurements of the clarinet [29]. For the written C4 note, the  $r$  value calculated from equation (2) is  $-250 \pm 40 \text{ dB} \cdot \text{s}^{-1}$ , consistent with that shown in Fig. 4(a).

## 4 Controlled transients and regeneration

An exponential rise and decay were also observed in the initial and final transients produced by the automated clarinet-playing machine when the blowing pressure  $P$ , the lip force  $F$  and other parameters were controlled. The reed loses its mechanical energy within  $\sim$ ms after tongue release. However, a sudden change in the reed aperture leads to a sudden change in the volume flow  $U$ , which reflects and interacts with the regenerative reed to give rise to the exponential increase in the standing waves. In initial transients, the rate  $r$  of exponential increase of the fundamental in the barrel pressure depends strongly on  $P$  and  $F$ . Further, when the tongue was released with  $P$  below the oscillation threshold, an exponential decay was observed (details in [25]).

Figure 5 shows a sketch of the clarinet mouthpiece, reed and embouchure, and sketches the volume flow  $U$  versus blowing pressure  $P$  for large and small lip forces without reed oscillation [18, 30]. For a constant lip force, at low  $P$ , the 'Bernoulli term' gives  $U$  proportional to  $P^{1/2}$ . The (AC) acoustic conductance of the reed,  $\partial U / \partial P$ , is positive, so energy associated with the oscillation is dissipated. At sufficiently high  $P$ ,  $F$  or combination of these, the reed is closed completely against the mouthpiece lay, making  $U = 0$ . So on the right hand side  $\partial U / \partial P$  is negative. Consider small oscillations at constant lip force, and neglect the inertia of the reed and losses associated with its motion. Auto-oscillation can occur with a resonant load when  $\partial U / \partial P$  is sufficiently negative to overcome the finite losses in the bore, the reed and the lip.

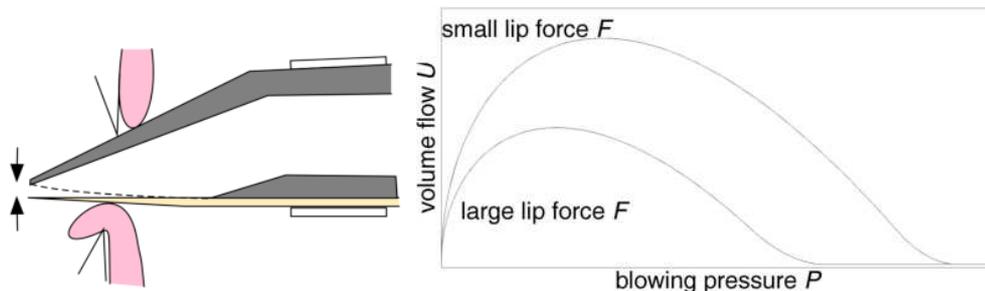


Figure 5: A cross-section of the clarinet mouthpiece (grey), reed (pale), lips and teeth. The arrows indicate the height of the aperture for air flow  $U$  produced by the blowing pressure  $P$  in the mouth. The graphs sketch  $U(P)$ . At low  $P$ , the 'Bernoulli term' gives the flow approximately proportional to  $P^{1/2}$ . High  $P$  closes the reed aperture, so  $\partial U / \partial P$  is negative at high  $P$ .

#### 4.1 Simple models for regeneration

A simple model proposed for the auto-oscillation of the reed [31] can explain the dependence of  $r$  on  $P$  and  $F$ . Treat the reed as a linear spring, which both  $P$  and  $F$  act to close against the mouthpiece. For small oscillations, the  $U(P)$  relation can be written as

$$U = w(H_f - \frac{P}{k})\sqrt{\frac{2P}{\rho}} \quad (3)$$

where  $w$  is the width of the reed opening;  $H_f$  is the height of the reed opening with lip force applied;  $\rho$  is the air density, and  $k$  is a constant with unit of  $\text{N}\cdot\text{m}^{-3}$ .

$$\frac{\partial U}{\partial P} = w(H_f - \frac{3P}{k})\sqrt{\frac{1}{2\rho P}} = \frac{1}{R_{reed}} \quad (4)$$

where  $R_{reed}$  is the reed resistance.

For the early part of the transient, well before saturation, only the fundamental is considered. The instrument plays at a frequency near a peak of the acoustic impedance of the bore  $Z_{bore}$ . As shown in the inset of Fig. 6, the peak is represented empirically by the parallel resonance of a compliance  $C$ , inertance  $L$  and resistance  $R_{bore}$ , all of which are in parallel with  $R_{reed}$ . At resonance, the bore is almost completely resistive ( $Z_{bore} = R_{bore}$ ), so  $R_{reed}$  is real and can be either positive (dissipative) or negative, in which case it produces auto-oscillation.

Taking  $p$  as the RMS acoustic pressure across all the elements in Fig. 6, the total energy stored in the compliance and inertance together is  $Cp^2$  at any time. Averaged over an integral number of cycles, the rate of increase in the stored energy is therefore  $2Cp \cdot dp/dt$ . Conservation of energy requires that

power produced by reed source = power dissipated in bore + rate of increase in energy stored

$$-\frac{p^2}{R_{reed}} = \frac{p^2}{R_{bore}} + 2Cp \frac{dp}{dt} \quad (5)$$

$$\frac{dp}{dt} = -\frac{p}{2R_{//}C} \quad (6)$$

where the parallel conductance  $\frac{1}{R_{//}} = \frac{1}{R_{reed}} + \frac{1}{R_{bore}}$

Thus, equation (6) has the solution

$$p = p_0 e^{-t/\tau} \quad (7)$$

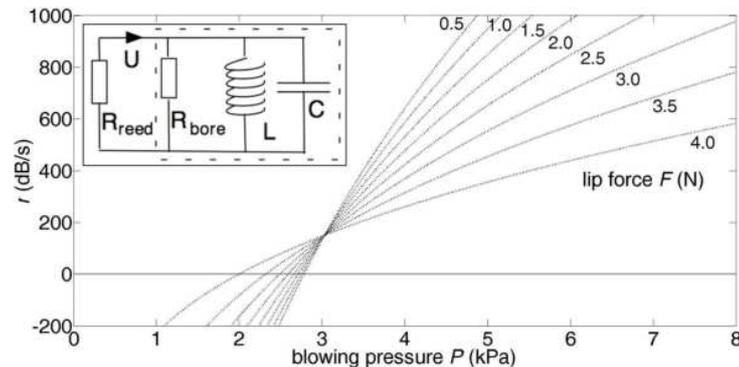
where  $\tau = 2R_{//}C$  is the time constant.

Then the amplitude of the oscillation increases exponentially with rate (in  $\text{dB}\cdot\text{s}^{-1}$ )

$$r = -10 \log_{10} e \cdot \frac{R_{reed} + R_{bore}}{R_{reed} R_{bore} C} = -10 \log_{10} e \cdot \frac{R_{bore} w (H_f - \frac{3P}{k}) \sqrt{\frac{1}{2\rho P} + 1}}{R_{bore} C} \quad (8)$$

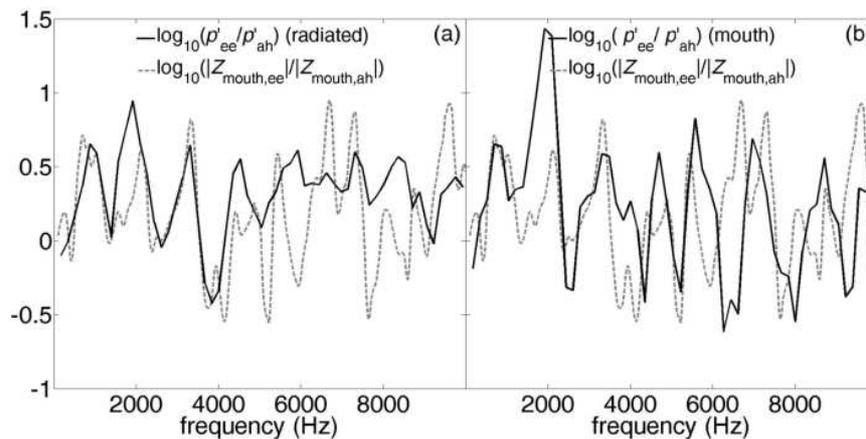
Using the values from [29],  $C$  is  $0.21 \text{ mm}^3 \cdot \text{Pa}^{-1}$ ,  $R_{bore}$  is  $\sim 80 \text{ MPa}\cdot\text{s}\cdot\text{m}^{-3}$  and  $\rho$  is  $1.225 \text{ kg}\cdot\text{m}^{-3}$ . If we simply assume that  $w$  is 11 mm, it is possible to fit the constants  $H_f$  and  $k$  using  $r(P)$  for each set of lip force obtained [25]. Figure 6 shows the  $r(P, F)$  curves thus calculated. In practice, the

measured  $r(P, F)$  curves do not all intersect at the same point because of the non-linear spring and the different effects of  $P$  and  $F$  (data in [25]). Also, in reality, the curvature of the mouthpiece [Fig. 5(a)] means that the length of the vibrating part of the reed decreases with increasing  $F$  and  $P$ , making it stiffer and reducing the slope in  $U(P)$ , as was shown by Dalmont and Frappé [18]. Subject to all these caveats and simplifications, the models can qualitatively describe the auto-oscillation of the reed and the dependence of  $r$  on  $P$  and  $F$  (see also [25]).



**Figure 6:** Calculated rate  $r$  vs blowing pressure for different lip forces. The parallel circuit elements (inset) inside the dashed border ( $L$ ,  $C$  and  $R_{\text{bore}}$ ) are an empirical representation of the acoustic impedance of the bore  $Z_{\text{bore}}$  at frequencies near the peak at which the instrument plays.  $R_{\text{reed}}$  is the small-signal acoustic resistance  $\partial P/\partial U$  of the reed, which may be positive or negative.

## 5 Players' control of the spectral envelope



**Figure 7:** The ratios of the magnitudes of  $Z_{\text{mouth}}$  (dashed line), and the relative amplitudes of harmonics (a) in the radiated sound and (b) in the mouth, as a function of frequency for two vocal tract configurations, described by the player as 'ee' and 'ah'.

Figure 7 compares the different effects of vocal tract (acoustic impedance spectrum) on the sound spectra in the mouth and in the radiated sounds. An expert player played the written G4 note on a tenor saxophone (sounded F3), using two different vocal tract configurations: 'ee' and 'ah'. The altered tract produces substantial changes in the sound spectra. The maxima in the

impedance spectrum of the vocal tract correlate with the increase in the amplitude of harmonics at nearby frequencies: the relative level of several harmonics in the radiated spectra is varied by 5–10 dB and that measured inside the mouth sometimes is varied by more than 15 dB [27].

## 6 Conclusions

The initial and final transients have exponential phases with positive and negative rates respectively. In the initial transients, different accents are produced with different exponential rates  $r$ , which depend on  $P$  and  $F$  with timing depending on tongue release. The tongue action determines the timing of transients but has little effect on  $r$ . Final transients can have an exponential phase if the tongue stops the reed, or a gradual decay as  $P$  decreases. A simple model of energy balance explains the initial and final transient behaviour. In the sustained part of a note, maxima in the impedance spectrum of the vocal tract produce formants in the radiated sound (bands of enhanced radiated power) at nearby frequencies. Thus, players can control the beginning, middle and end of a note to produce a desired result.

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