BOLTZMANN EQUILIBRIUM OF ENDOTHERMIC HEAVY NUCLEAR SYNTHESIS IN THE UNIVERSE AND A QUARK RELATION TO THE MAGIC NUMBERS

HEINRICH HORA[1], GEORGE H. MILEY[2] and FREDERICK OSMAN[3]

[1]*Department of Theoretical Physics, University of New South Wales, Sydney, Australia*
[2]*Fusion Studies Laboratory, University of Illinois, Urbana, USA*
[3]*School of Quantitative Methods and Mathematical Sciences, University of Western Sydney, Penrith, Australia; E-mail: f.osman@uws.edu.au*

(Received 12 April 2004; accepted 3 August 2004)

Abstract. As laser–plasma interactions access ever-increasing ranges of plasma temperatures and densities, it is interesting to consider whether they will some day shed light on questions concerning nuclear synthesis. One such open question is the process of endothermic nuclear synthesis for elements with $A > 60$, thought to have taken place at a point in time during the big bang, or currently in supernovae. We present an explanation based on a Boltzmann equilibrium condition, in combination with the change of the Fermi-statistics from the relativistic branch for hadrons from higher than nuclear densities to the lower density subrelativistic branch. The Debye length confinement of nuclei breaks down at the relativistic change, thus leading to the impossibility of nucleation of the quark-gluon state at higher than nuclear densities. Taking the increment for the proton number Z as $Z' = 10$ of the measured standard abundance distribution (SAD) of the elements for a Boltzmann probability for heavy element synthesis, a sequence 3^n was found with the exponent n for the sequence of the magic numbers. The jump between the magic numbers 20 and 28 does not need then the usual spin-orbit explanation.

Keywords: laser produced plasmas, degenerate electrons, theory of nuclei and nucleation, endothermic element synthesis, quark gluon plasma, magic numbers

1. Introduction

Studies of very high energy density laser–plasma interactions could lead to experiments where violent particle acceleration may produce Hawking-Unruh radiation, similar to that obtained in black holes (Hora et al., 2002). A further consideration of Debye-layer mechanisms of plasma theory may lead to further understanding of a quark-gluon state at higher than nuclear density as will be presented in the this manuscript. The problem we will address is to understand how endothermic syntheses of nuclei heavier than iron can be produced in the universe, while the synthesis up to iron by fusion reactions is exothermic and well understood from the reactions in stars. If a Boltzmann statistics for an unspecified nuclear-chemical process is assumed in the stellar plasmas for creation of the heavy nuclei, we find that the exponential increment fits with the measured standard abundance distribution of

Astrophysics and Space Science **298:** 247–253, 2005.

the elements. This may have the importance of some consistency only if we follow up a relation with the magic numbers of nuclei. Bagge (1948) derived the magic numbers by a purely numerological speculation, where a first connection with the experimental facts was given by a consideration of spin and spin-orbit properties of nuclei (Haxel et al., 1950). We discuss the relation of the magic numbers with the exponent of the mentioned Boltzmann statistics and subsequently find an explanation for the jump between the two Bagge series (Bagge, 1948) without needing the spin and spin-orbit relation. The crucial mechanism for the equilibrium-type generation of all known nuclei in the universe is based on the result that there is a change of the Fermi–Dirac statistics for the nucleons from the relativistic branch at the densities above the nuclear density to the subrelativistic density leading to the nucleation at the well known nuclear densities. This mechanism, due to the surface energy of the nuclei, is combined with the results of the empirically derived Boltzmann increment and then fit together with the magic numbers.

2. Surface Energy and Quark-Gluon Plasma

Surface tension of dielectric materials is explained by the dipole property of molecules. The fact that highly ionized plasmas without any molecules do have a surface tension was derived from the electric fields within the Debye length at a plasma surface due to the gradient of the electrostatic energy density (Hora, 1991a,b, 1992; Hora et al., 1984; Eliezer and Hora, 1989). Extending this to the degenerate electrons in a metal, the surface tension can immediately explain measurements (Hora et al., 1989) where the double layer at the metal surface is produced by a swimming electron layer that is expressed by an exponentially decaying Schrödinger function. In addition, this argument also explains the work function for the emission of electrons. Extending this further to the surface of a nucleus (Hora, 1991a, 1992), the gradient of the energy density leads to a similar Debye length λ_d and a surface energy for compensating the internal energy of the compressed hadrons in a nucleus. This energy is not so much due to the Coulomb repulsion of the protons at the well-known nuclear density n_n but is mostly determined by the Fermi–Dirac energy E_F. In the following argument, we are comparing the surface energy with this internal energy by neglecting the minor contribution by the Coulomb and other effects (Hora, 1991). The surface tension for the nuclei is then

$$\sigma_e = 0.27\, E_F^2 \big/ (8\pi\varepsilon^2\lambda_d) \tag{1}$$

The Fermi energy can be expressed generally (Eliezer et al., 2002)

$$E_F = [(3/\pi)^{2/3}/4][h^2 n^{2/3}/(2m)](\lambda_C/2)^{-1}[n + 1/(\lambda_C/2)^3]^{-1/3} \tag{2}$$

where n is the nucleon density, and m is the nucleon mass. This splits into the branches

$$E_F = \{[(3/\pi)^{2/3}/4]h^2 n^{2/3}/(2m)\} \quad \text{(subrelativistic)} \tag{2a}$$
$$E_F = \{[(3/\pi)^{2/3}/4]hcn^{1/3}\} \quad \text{(relativistic)} \tag{2b}$$

where λ_C is the Compton wave length $h/(mc)$. The surface energy of the nucleus (Hora, 1991a, 1992) is then

$$E_{surf} = 0.27[3A(4\pi)^{1/2}]^{2/3}3^{1/3}E_F^{3/2}/(\pi^{1/2}2^{5/2}n^{1/6}e) \tag{3}$$

For comparison between the surface energy and the internal energy we have

$$E_{surf}/(AE_F) = \{0.27(3^{3/2}/2^{10/3})hn^{1/6}/(em^{1/2}A^{1/3})\} \quad \text{(subrelativistic)} \tag{4a}$$
$$E_{surf}/(AE_F) = \{0.27[3^{8/3}/(2^{7/3}\alpha^{1/2}A^{1/3})]\} \quad \text{(relativistic)} \tag{4b}$$

using the fine structure constant $\alpha = 2\pi e^2/hc$. From (4a) we see that the nucleus cannot be confined for too low densities. The nucleus is stable only when the density reaches a value of the density n_n where the ratio (4a) is equal to one. This is the case for the well-known value of the nuclear density as checked e.g. for bismuth (Hora, 1991a, 1992). The surface "Debye"-layer has a thickness of about 2–3 fm, which is just at the measured Hofstadter decay of the surface charge of heavy nuclei. At relativistic densities just above that of the subrelativistic case reproducing the well-known density of nuclei, we see that the value

$$E_{surf}/(AE_F) = 6.28/A^{1/3} \tag{5}$$

no longer depends on the nucleon mass or density. We then have no nucleation by the surface energy and the result is a soup of matter. The fact that there is no dependence of this ratio on the mass implies that it holds equally well for either hadrons (as assumed in neutron stars) or quark-gluon plasmas. Only when this dense matter is expanding (as at the big bang or from a neutron star in a supernova, when reaching the nuclear density) will the surface energy produce nucleation. The numerical factor in Eq. (5) may mean that values higher than 247 for A are not possible; otherwise a nucleation or surface effect would appear. This just may explain the fact that the nucleation, by expansion of the matter from the relativistic branch to the lower nuclear density, can produce elements only up to uranium (or up to curium at most) within such equilibrium processes.

3. Cosmic Heavy Nuclear Generation

It is well known from nuclear astrophysics (Audouze and Vauclair, 1980; Rauscher et al., 1994) that there is a standard abundance distribution (SAD) of the elements in the Universe (Hora and Miley, 2000). One interesting feature of the SAD is that for nucleon numbers $A > 60$ (above about iron), a nearly exponential decay of the structure of maxima which are close to the magic numbers can be seen. Element synthesis for heavier nuclides with $A > 60$ cannot be due to fusion because these reactions would be endothermic. There are well known reaction chains in which high density background neutrons may produce reactions for greater than $A = 60$ element synthesis. Some examples are the reactions in supernovae and in white dwarfs known as the r-, s- or p- (rapid, slow or by-pass) processes. Also, for these a rather similar abundance for the elements is gained (Audouze and Vauclair, 1980). Indeed, there is a discussion as to whether these heavy elements can be produced only in the later development of the universe and not before the early development of galaxies (Rauscher et al., 1994; Hora and Miley, 2000; Sneden et al., 1994; McWilliam, 1994; Lefebvre et al., 1997). Nevertheless, there are compelling theories put forth that the heavy elements may have been produced in the state of the big bang when the cosmos had a density close to the nuclear density (Rauscher et al., 1994) and where inhomogeneities provided the conditions for the heavy element generation. This is all related to the conditions by which surface energy of nuclei due to inhomogeneity fields results in stable nuclei of the well known nuclear density, whereas at six times higher density, the Fermi energy of the nucleons changes in the relativistic branch forbidding any nuclear structure and permitting only uniform nucleon or quark-gluon plasmas (Hora, 1991a, 1992). The fact that there is a universally equal distribution of the heavy elements – due to a big-bang or later processes – suggests that without the well known detailed single reactions being taken into account, there seems to be a global reaction equilibrium defined by a Boltzmann-like exponential distribution into which all the heavy nuclei within the background of neutrons may emerge. This is a Boltzmann-like equilibrium process that changes any distribution of nuclides into the well observed standard abundance having the exponentially decaying probability for higher A or proton number Z of nuclei. A distribution of this abundance $N(Z)$ depending on the proton number Z of the form

$$N(Z) = N' \exp(Z/Z') \tag{6}$$

can be written down for the maxima of the SAD (Hora and Miley, 2000) for heavy nuclides. This is therefore rather trivial. Statistically, there is an up and down in nuclei until the exponential distribution has been achieved. One may assume that if this occurs at an early stage of the big bang when all nuclei are some femtometers (Fermi) in distance, then the reaction times may be between femtoseconds and attoseconds or even less. For lower densities, such as those found in supernovae or in white dwarfs, the endothermic element synthesis by the s-, the r- or the p-processes results in a similar Boltzmann equilibrium. This can be seen in Eq. (6), as well as in Fig. VI.1 of Audouze and Vauclair (1980), or Fig. 10a in Rauscher et al. (1994). (Note however, the reaction times in these other cases are up to 10^4 seconds due to the larger distances of the reacting nuclei.) Similar conditions may exist in astrophysical ensembles of nuclei at similar distances and time scales if there is a proton background (Rauscher et al., 1994) where the Coulomb repulsion is compensated thermally and/or there are sufficiently high densities. Following the Boltzmann equilibrium idea further, we evaluate the ratios of the creation probabilities $N(n)$ depending on the numbers n of the sequence of the magic numbers with the only fitting $Z' = 10$:

$$\text{Magic numbers:} M_1 \in 2,\ 8,\ 20,\ 28,\ 50,\ 82,\ 126 \tag{7}$$

These are for protons Z in nuclides as well as for neutrons $N = A - Z$ with the measured well-known maxima of binding energies (see Fig. 2 of Wilets, 1987). We now calculate the ratios $R(n)$ for the astrophysical (Audouze and Vauclair, 1980) SAD-Boltzmann probabilities from Eq. (6) by taking into account the jump between 20 and 28 of the Bagge sequences (Bagge, 1948).

$$R(n) = [N(Z_{n+1})/N(Z_n)] - 1 = \exp[(Z_{n+1} - Z_n)/Z'] \tag{8}$$

where the magic numbers Z_n of the protons are taken with the following indices n $(0, 1, 2, 3, \ldots)$

$$Z_0 = 2,\ Z_1 = 8,\ Z_2 = 20, \quad \text{for relation up to the magic number 20} \tag{9}$$

$$Z_2 = 28,\ Z_3 = 50,\ Z_4 = 82,\ Z_5 = 126 \quad \text{for the magic numbers above 20} \tag{10}$$

As seen in Table I, for $Z' = 10$ in Eq. (6), the ratios R given by Eq. (8) result in values very close to

$$R(n) = 3^n \tag{11}$$

showing the best fit for $Z' = 10$ (Hora and Miley, 2000).

TABLE I

Sequence $n = 0, 1, 2, \ldots$ of magic numbers with the values $\exp(Z_n/Z')$ and $R(n) = \exp[(Z_{n+1} - Z_n)/Z']$ of Eq. (3) with $Z' = 10$ from Eq. (6) as measured

n	Magic number	exp(Z/Z′)	$R(n)$	3n
0	2	1.221	1.822	1
1	8	2.2225	3.321	3
2 ($n + 1$ in (8))	20	7.389	–	–
2 (as n in (8))	28	12.1824	9.025	9
3	50	148.413	24.53	27
4	82	3640.95	81.45	81
5	126	296558.5		

4. Quark and Hadron Structure of Nuclei

The combination of two research topics: “From quarks to the cosmos” and “How were the elements from iron to uranium made?” was the focus of a recent panel of astronomers and physicists that were asked to come up with a list of the key questions in astronomy and physics today (Turner, 2001). We hope that the considerations in this paper may provide insight into the answers of some of those questions. The Boltzmann equilibrium process of nucleation that occurs when matter of higher density than that of the nuclei is expanding, but bound to the surface energy mechanisms, may well explain why nuclei not much larger than that of uranium may be possible. For the properties of the generated nuclei, it is interesting to note that both conflicting properties are present; the hadron structure of the nuclei which determined the Fermi–Dirac statistics and its transition into the relativistic branch by the mass of the hadrons, while the relation for the shell structure for the magic numbers, Eq. (11) indicated the quark property by the threefold multiplicity. Hofstadter’s theoretically predicted decay for large nuclear surfaces of 2–3 fm thickness may also indicate the range of the Yukawa potentials of about 2 fm as tangling bonds at the surface, and not mutually saturated as is the case within the nucleus by mutual hadron interaction (Hora, 1991). It should be mentioned that the motivation to study the Boltzmann plots from Eq. (6) from the empirically given maxima of the standard abundance distribution, was initially motivated by similar measurements of element distribution observed in a fully reproducible way by low energy nuclear reactions of high density protons in palladium, nickel and zirconium (Hora and Miley, 2000).

Acknowledgements

Valuable discussions and support for the final formulations are most gratefully acknowledged to Scott C. Wilks, LLNL Livermore, further discussions with Edward Shuryak, SUNY Stony Brook, I. Krypokluk, Budker Inst. Novosibirsk and N. Ghahramany, Univ. Shiraz.

References

Audouze, J. and Vauclair, S.: 1980, *An Introduction to Nuclear Astrophysics*, D. Reidel, Dordrecht.

Bagge, E.: 1948, *Naturwissenschaften* **35**, 376.

Eliezer, S. and Hora, H.: 1989, *Phys. Rep.* **172**, 339.

Eliezer, S., Ghatak, A.J., Hora, H. and Teller, E.: 2002, *Fundamentals of Equations of State*, World Scientific, Singapore.

Haxel, O., Jensen, J.H.D. and Suess, H.E.: 1950, *Z. Physik* **128**, 295.

Hora, H.: 1991a, *Plasma Model for Surface Tension of Nuclei and the Phase Transition to the Quark Plasma*, Report CERN-PS/DL-Note-91/05.

Hora, H.: 1991b, *Plasmas at High Temperature and Density,* Springer, Heidelberg.

Hora, H.: 1992, *Laser Interaction and Related Plasma Phenomena,* Vol. 10, Plenum, New York, p. 19.
Hora, H., Lalousis, P. and Eliezer, S.: 1984, *Phys. Rev. Lett.* **53**, 1650.
Hora, H. and Miley, G.H.: 2000, *Czech. J. Phys.* **50**, 433.
Hora, H., Min Gu, Eliezer, S., Lalousis, P., Pease, R.S. and Szichman, H.: 1989, *IEEE Trans. Plasma Sci.* **PS-17**, 284.
Hora, H., Osman, F., Castillo, R., Collins, M., et al.: 2002, *Laser Particle Beams* **20**, 79.
Lefebvre, A., Vouzoukas, S., Agner, P., Bogaert, G., Coc, A., Denker, A., de Olivera, F., Forier, A., Görres, J., Kiener, J., Maison, J.M., Porquet, M.G., Rosier, L., Tatischeff, V., Thibaud, J.P. and Wiescher, M.: 1997, *Nucl. Phys. A* **621**, 199.
McWilliam, A.: 1994, *Annu. Rev. Astron. Astrophys.* **35**, 503.
Rauscher, T., Applegate, J.H., Cowan, J.J., Thielemann, F.-K. and Wiescher, M.: 1994, *Astrophys. J.* **429**, 499.
Sneden, Ch., Preston, G.W., McWilliam, A. and Searle, L.: 1994, *Astrophys. J.* **431**, L27.
Turner M.: 2001, *Phys. World* **14**(2), 6.
Wilets, L.: 1987, in: R.A. Meyers (ed.), *Encyclopedia of Physical Sciences and Technology*, Vol. 9, Academic Press, New York, p. 300.