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Physica B 249–251 (1998) 819–823

**PHYSICA B**

## Many-body interactions, the quantum Hall effect, and insulating phases in bilayer two-dimensional hole-gas systems

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### Abstract

We have studied the quantum Hall effect in a high mobility double-layer 2D hole gas in which we are able to control the carrier density in the two layers independently. We find that both the  $\nu = 1$  and the  $\nu = \frac{3}{2}$  bilayer quantum Hall (QH) states are destroyed with increasing carrier density. The collapse of  $\nu = 1$  is extremely abrupt, occurring at a carrier density that corresponds to an inter-layer separation of  $1.8l_B$ . However, although the  $\nu = \frac{3}{2}$  QH state disappears without trace, the zero in  $\rho_{xx}$  at  $\nu = 1$  is replaced by a broad minimum similar to that observed at  $\nu = \frac{1}{2}$  when only one layer is occupied. This suggests that we are observing a transition from a correlated incompressible QH state with total filling factor  $\nu = 1$  to a compressible state consisting of two (possibly uncorrelated) layers of 'composite fermions'. At very low densities and filling factors below  $\nu = 1$ , an insulating phase is observed, which is not present when only one layer is occupied. We demonstrate that this state is entirely due to inter-layer interactions and is strong evidence for the existence of a bilayer Wigner crystal. © 1998 Elsevier Science B.V. All rights reserved.

**Keywords:** Double-layer hole systems; Quantum Hall effect; Metal-insulator transition; Wigner solid

In a bilayer system, new integer and fractional quantum Hall states can be observed which have no counterpart in single-layer systems. For example, in a clean single-layer system, the filling factor  $\nu = \frac{1}{2}$  corresponds to a compressible Fermi liquid [1], whereas in a bilayer system with total filling factor  $\frac{1}{2}$ , a well-defined fractional QH state can be observed [2]. Similarly at  $\nu = 1$  (i.e.  $\nu = \frac{1}{2}$  in each layer) an interlayer correlated QH state can be

formed [2–4] although in electron systems it can be difficult to distinguish this state from a single-particle QH state due to inter-layer tunnelling [5]. In addition to these new QH states, there is also the possibility of bilayer Wigner crystallisation, which is expected to occur at larger carrier densities and filling factors than in single-layer systems [6].

The ideal system for studying correlated bilayer states is one in which two clean 2D layers are brought sufficiently close together that the interactions between particles in different layers (characterised by the Coulomb energy scale  $e^2/4\pi\epsilon d$ , where

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$d$  is the layer separation) becomes comparable to that between particles in the same layer (characterised by  $e^2/4\pi\epsilon l_B$ , where  $l_B$  is the magnetic length). In practice, it is difficult to realise a system with strong interlayer correlations (small  $d/l_B$ ) because tunnelling between the layers increases rapidly as  $d$  decreases, weakening the correlations. With bilayer hole gases fabricated with GaAs heterostructures, not only is the disorder extremely low, but the large effective mass ( $m^* \simeq 0.3 m_c$ ) allows the layers to be brought into close proximity without significant inter-layer tunnelling occurring.

The double 2D hole gases used in this work were grown by MBE on (3 1 1)A oriented GaAs substrates, and consist of two modulation doped quantum wells of thickness  $d_w = 150 \text{ \AA}$ , separated by an  $\text{Al}_{0.33}\text{Ga}_{0.67}\text{As}$  barrier of thickness  $d_b = 20 \text{ \AA}$ . Samples were processed into Hall bars, with a Schottky front-gate and an in situ back-gate [7] providing independent control over the carrier density in the upper and lower quantum wells. The as grown carrier densities were  $p_s \simeq 1 \times 10^{11} \text{ cm}^{-2}$  in each well, with an average mobility of  $\simeq 1.1 \times 10^6 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ . Measurements were performed with the sample mounted either directly in the mixing chamber, or via a cold finger, of dilution refrigerators with base temperature less than 30 mK. Temperature dependence measurements were performed using a calibrated resistor mounted in a flux cancelled region of the mixing chamber. Standard low-frequency AC lock-in techniques were used with measurement currents less than 2 nA.

Previously, we have shown how increasing the strength of the inter-layer interactions (by reducing  $d$ ) while keeping the intra-layer interaction strength fixed (keeping  $p_s$  fixed and hence  $l_B$  constant) gives rise to bilayer correlated QH states at  $\nu = 1$  and  $\nu = \frac{3}{2}$  [3]. In this paper we study the evolution of these and other correlated states as a function of the ratio of the intra- to inter-layer interactions ( $d/l_B$ ) by maintaining a constant inter-layer separation and varying the carrier density.

The evolution of the diagonal resistivity  $\rho_{xx}$  as a function of filling factor is shown in Fig. 1 for several different matched carrier densities [8]. At the lowest densities ( $p_s = 0.83 \times 10^{11} \text{ cm}^{-2}$ ) clear  $\rho_{xx}$  minima are seen at  $\nu = 4, 3, 2$  and 1, along with

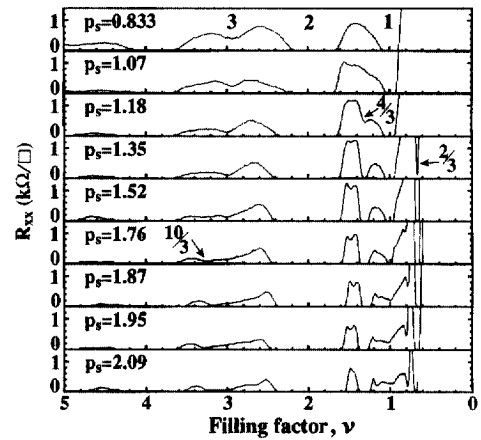


Fig. 1. Resistivity  $\rho_{xx}$  as a function of filling factor for several different total carrier densities  $p_s$  (in units of  $10^{11} \text{ cm}^{-2}$ ), with equal densities in the two wells, at  $T = 30 \text{ mK}$ .

corresponding plateaux in  $\rho_{xy}$  (not shown). The states at even  $\nu$  can simply be explained as two separate non-interacting  $\nu/2$  QH states. Those at odd  $\nu$ , however, can be either due to the single-particle tunnelling gap  $\Delta_{\text{SAS}}$  [9], or due to the formation of many-body correlated states. For filling factors below  $\nu = 1$  there is a strong insulating phase which is discussed later.

As  $p_s$  is increased, the mobility increases and fractional QH states become visible, first at  $\nu = 4/3$  and subsequently at  $\nu = \frac{3}{2}$ . At a density of  $p_s = 1.5 \times 10^{11} \text{ cm}^{-2}$ , the integer QH states at  $\nu = 1$  and particularly  $\nu = 3$  start to weaken, and new fractional states appear at  $\nu = \frac{10}{3}$  and  $\frac{3}{2}$ . Whilst the former corresponds to  $2 \times (\frac{5}{3})$ , the  $\nu = \frac{3}{2}$  state does not have a counterpart in single-layer systems, and is believed to be the electron-hole conjugate of the  $\Psi_{331} \nu = \frac{1}{2}$  bilayer state [3]. Further increasing the carrier density causes the sudden disappearance of the  $\nu = 1$  state in the density interval  $p_s = 1.76 - 1.87 \times 10^{11} \text{ cm}^{-2}$ . At the highest carrier densities there is no remaining trace of the  $\nu = 1$  QH state, with just a broad minima in  $\rho_{xx}$  surrounded on either side by several fractional QH states, reminiscent of the broad minima seen in single-layer systems at  $\nu = \frac{1}{2}$  [1].

To quantify the destruction of the  $\nu = 1$  state with increasing  $p_s$ , we have performed temperature-dependence measurements and extracted the

activation energy from  $\rho_{xx} \propto \exp(\Delta/2T)$  (shown in the inset to Fig. 2). Initially  $\Delta$  decreases slowly with increasing  $p_s$  (in contrast to the QH effect in single-layer systems where  $\Delta$  increases with  $p_s$ ), but drops dramatically as the total density approaches  $1.8 \times 10^{11} \text{ cm}^{-2}$ . This behaviour demonstrates the bilayer origin of the  $\nu = 1$  state, as it is entirely inconsistent with a state stabilised by the single-particle inter-layer tunnelling gap  $\Delta_{\text{SAS}}$  which is only weakly dependent on the density [9].

In the main part of Fig. 2 we show the phase diagram for the destruction of the QH effect for filling factors  $\nu = 3, \frac{3}{2}$  and 1. In this diagram, the  $x$ -axis denotes the ratio of the inter-layer tunnelling  $\Delta_{\text{SAS}}$  to the Coulomb energy  $e^2/4\pi\epsilon_0 l_B$  and the  $y$ -axis denotes the ratio of the intra- to inter-layer Coulomb interaction strength (with  $d = d_w + d_b$ ). Increasing  $p_s$  at fixed  $\nu$  corresponds to reducing  $l_B$ , essentially moving up the  $y$ -axis. The presence (absence) of a given QH state is marked by solid (open) symbols, showing the destruction of the  $\nu = 3, \frac{3}{2}$  and 1 states at  $d/l_B = 0.95, 1.6$  and  $1.8$ , respectively. In order to compare these values with theoretical predictions for the destruction of the QH effect, we take into account the softening of the Coulomb interaction caused by the finite width of our 2D hole gas by replacing  $l_B$  with  $\sqrt{l_B^2 + (d_w/2)^2}$ . This reduces the above values to 0.8, 1.3 and 1.45, respectively. Recent numerical calculations [10] pre-

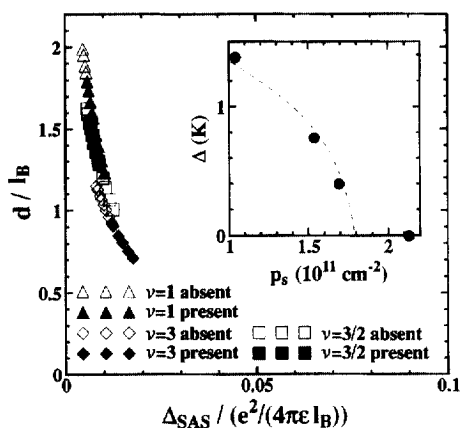


Fig. 2. Measured phase diagram for the existence of the various QH states, with solid and open symbols denoting observed and missing states at filling factor  $\nu$ . The inset shows the activation energy  $\Delta$  measured at  $\nu = 1$  as a function of  $p_s$ .

dict that the charge excitation gap disappears at  $d/l_B \lesssim 1.5$  for the  $\nu = 1$  bilayer QH state, in excellent agreement with our data.

We now examine the nature of the state at  $\nu = 1$ , once the QH effect has been destroyed ( $d/l_B = 1.88$ ). First, we show the evolution of the single-layer QH effect when only the lower quantum well is occupied for carrier densities from  $0.6 - 1.2 \times 10^{11} \text{ cm}^{-2}$  (Fig. 3a). Clear integer and fractional QH states are observed ( $\rho_{xx} \rightarrow 0$ ) which become stronger with increasing  $p_s$ . At the higher densities, a broad  $\rho_{xx}$  minimum develops at  $\nu = \frac{1}{2}$ , characteristic of ‘composite fermions’ in zero-effective magnetic field. Fig. 3b shows the temperature dependence of the QH effect for one of these traces ( $p_s = 1 \times 10^{11} \text{ cm}^{-2}$  in the lower well). In contrast to the strongly  $T$  dependent QH states at  $\nu = \frac{1}{3}, \frac{2}{3} \dots$  the Hall resistance at  $\nu = \frac{3}{4}$  and  $\nu = \frac{1}{2}$  takes the temperature-independent classical value of  $B/p_s e$  as expected for a single-layer system.

In Fig. 3c, temperature-dependent  $\rho_{xx}$  and  $\rho_{xy}$  data is plotted for a total density of  $p_s = 2 \times 10^{11} \text{ cm}^{-2}$ , spread equally between the two

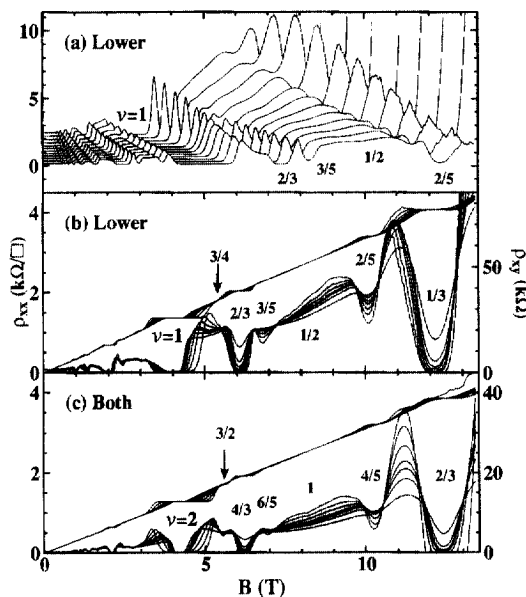


Fig. 3. (a) Evolution of the fractional QH effect and the  $\rho_{xx}$  minimum at  $\nu = \frac{1}{2}$  as  $p_s$  is increased (traces offset for clarity). (b) Temperature dependence of  $\rho_{xx}$  and  $\rho_{xy}$  for the lower layer only;  $T = 80, 102, 142, 205, 275, 400, 580 \text{ mK}$ . (c) As (b), but with both wells equally occupied.

wells. The data is very similar to that when only one layer is occupied, except at  $\nu = \frac{3}{2}$  where a bilayer QH state forms, with a corresponding plateau in  $\rho_{xy}$ . Activation energies for the  $\nu = \frac{2}{3}$  and  $\frac{4}{3}$  states when both wells are occupied (1.54 and 1.07 K) are very similar to the energies measured when only one well is occupied (1.28 and 1.08 K), indicating that these states are not bilayer in origin, unlike the  $\nu = \frac{3}{2}$  state. Most importantly, the data at  $\nu = 1$  in the bilayer case (Fig. 3c) is almost identical to that in the  $\nu = \frac{1}{2}$  single-layer case (Fig. 3b). This suggests that at large  $d/l_B$  when the bilayer  $\nu = 1$  QH state has been destroyed, the properties of the bilayer system are the same as that of two isolated single-layer systems with  $\nu = \frac{1}{2}$  in each layer, namely two layers of uncorrelated ‘composite fermions’.

Finally, having shown how bilayer correlated states are destroyed with increasing  $p_s$ , we turn to the opposite limit of low densities when inter-layer interactions are at their strongest. In particular, we examine the possibility of correlated bilayer insulating phases, and discuss how these can be distinguished from insulating states that do not involve inter-layer correlations. We note that Hyndman et al. have recently observed a re-entrant insulating phase in lower mobility bilayer hole gases between QH states at  $\nu = 1$  and  $\nu = \frac{2}{3}$  (with a  $\rho_{xx}$  maxima at  $\nu = 0.74$ ) [4]. However, it is not clear that this is a bilayer insulator, as a similar re-entrant insulating phase has been observed at  $\nu = 0.37$  (between  $\nu = \frac{2}{3}$  and  $\frac{1}{3}$  QH liquids) in single-layer 2D hole gases [11]. This single-layer insulating phase would also occur at  $\nu = 0.74$  in a system of two 2D hole gases without any inter-layer interactions, consistent with the observations of Ref. [4].

Fig. 4 shows the strong insulating phase that develops at low (matched) densities in our samples for  $\nu < 1$ , with the remnants of the  $\nu = \frac{2}{3}$  QH state and a weak  $\nu = \frac{1}{2}$  bilayer QH state also visible. To answer the question as to whether this insulating phase is single or bilayer in origin, we compare the magnetotransport data for the case when both wells are occupied with equal carrier densities (Fig. 5a) with the case where we have removed all the carriers from the upper quantum well (Fig. 5b). When only the lower well is occupied, a strong QH state is observed at  $\nu = \frac{1}{3}$ ; for  $\nu < \frac{1}{3}$  an insulating phase develops, which has been attributed to the

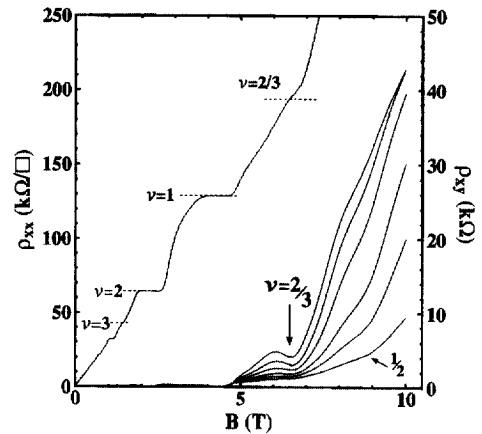


Fig. 4. Temperature dependence of  $\rho_{xx}$  for a total (matched) carrier density of  $1 \times 10^{11} \text{ cm}^{-2}$ , showing insulating behaviour for  $\nu < 1$ .  $T = 50, 100, 142, 200, 250, 330 \text{ mK}$ ;  $T = 50 \text{ mK}$  for the  $\rho_{xy}$  data.

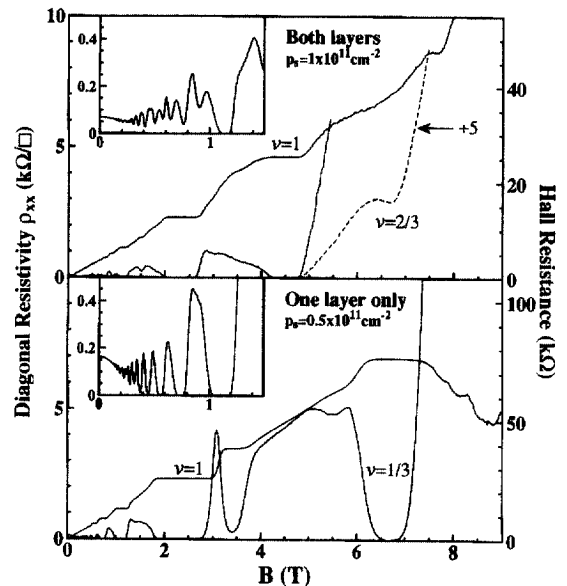


Fig. 5. Comparison of magnetotransport data at  $T < 30 \text{ mK}$ : (a) both wells occupied with equal carrier densities ( $p_s = 1 \times 10^{11} \text{ cm}^{-2}$ ) and (b) only the lower well occupied with  $p_s = 0.5 \times 10^{11} \text{ cm}^{-2}$ . Insets show the low-field behaviour.

formation of a weakly pinned Wigner crystal in single-layer hole gases of similar carrier densities [11]. By adding carriers to the upper well, the average hole mobility increases slightly, yet the insulating phase moves to lower magnetic field and

the QH state at  $\nu = \frac{2}{3}$  (i.e.  $2 \times \frac{1}{3}$ ) is destroyed. This is conclusive proof that the insulating phase we observe is a *correlated* many-body bilayer state, which only occurs at low densities when inter-layer Coulomb interactions are strong ( $d/l_B$  is small). Fig. 3 shows that at higher densities, this insulating phase does not occur, with transport data for the case when both quantum wells are occupied looking almost identical to the case when only one well is occupied. We believe that this data provides compelling evidence for the existence of a pinned bilayer Wigner crystal.

In summary, we have observed the transition from a correlated bilayer QH state at  $\nu = 1$  to a state that strongly resembles two independent layers of composite fermions with  $\nu = \frac{1}{2}$  as the carrier density is increased. At very low densities we observe a *correlated* bilayer insulating phase at  $\nu < 1$ . We demonstrate that this state is entirely due to inter-layer interactions, and is strong evidence for the existence of a bilayer Wigner crystal.

We thank Ana Lopez for many helpful discussions. This work was funded by the EPSRC. ARH and MYS gratefully acknowledge financial support from the British Council in Japan; MYS acknowledges support from the British Association of Crystal Growth; ARH, MYS, TGG and AKS acknowledge support from Exeter University Research Fund.

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