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Localisation and the metal–insulator transition in two dimensions

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Abstract

We investigate the insulating and metallic behaviour found in strongly interacting, dilute 2D GaAs hole systems. At the lowest carrier densities insulating behaviour is observed with the resistivity increasing with decreasing temperature. As the carrier density is increased a transition to metallic behaviour occurs. Despite exhibiting all the properties of the metallic behaviour observed in other material systems, localising corrections due *both* to weak localisation and hole–hole interactions are still present in the "metallic" regime. These results suggest that the metallic behaviour may only be a finite temperature effect, and we investigate various semi-classical mechanisms that might be responsible for this metallic-like behaviour. © 2001 Published by Elsevier Science B.V.

Keywords: Metal-insulator transition; Two-dimensional system; GaAs

1. Introduction

It has long been believed that although two-dimensional (2D) systems may appear metallic at high temperatures, there can be no true 2D metallic state at $T = 0$ [1,2]. This view received strong support from early experiments on thin metal films [3] and low mobility silicon MOSFETs [4,5], which revealed a logarithmic increase in the low-temperature resistance as $T \rightarrow 0$. It was further shown, both theoretically [6], and experimentally [7], that weak electron–electron interactions also cause a localising correction to the Drude conductivity in 2D systems. At the other extreme, where interactions are very strong, a dilute 2D system is still an insulator, due to the formation of a Wigner crystal pinned by the residual disorder [8].

In the intermediate regime, where both interactions and disorder are important, experimental studies of a wide variety of 2D semi-conductor systems have however revealed an unexpectedly large decrease in the resistance as the temperature is reduced below $T \sim 1$ K, suggesting the existence of a 2D metal [9–15]. Although numerous theories have been put forward to explain this effect, the origin of this metallic behaviour is still a subject of great debate. Is it a true quantum phase transition, where the resistivity remains finite at $T = 0$, or is it a finite temperature phenomena that

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can be explained by existing semi-classical mechanisms?

In this paper, we summarise recent experimental results from a series of high-quality GaAs 2D hole systems, investigating the behaviour of the temperature-dependent resistivity and examining the evidence for a 2-D metallic phase. We briefly describe the various samples used in Section 2, and then describe the evidence for an apparent transition from a strongly localised state at very low hole densities to a metallic state at slightly higher hole densities. The suppression of this metallic behaviour by an in-plane magnetic field $B_{||}$ is described in Section 3. We then describe a return to insulating behaviour at very high hole densities (at $B = 0$), where the metallic behaviour weakens and logarithmic corrections re-emerge (Section 4). A phase diagram for the observation of the metallic behaviour in the $p_s - B_{||}$ plane is also mapped out. In Section 5, we discuss recent results which show that even in the metallic regime both weak localisation and localising interaction corrections to the Drude conductivity are present, despite the metallic behaviour of $\rho(T)$. Finally, we consider the possibility of a semi-classical origin for the metallic behaviour in Section 6.

2. Experimental details

The GaAs heterostructures used in this study were fabricated by MBE growth on (3 1 1)A substrates, utilising silicon as the acceptor dopant. All samples were patterned into Hall bars aligned along the [$\bar{2}$ 3 3] direction. Sample A consisted of a 200 Å GaAs quantum well, modulation doped on one side with Si as the acceptor. The carrier density p_s was varied with an p^+ back-gate, formed using a combination of in situ ion implantation and MBE regrowth [15,16]. In this sample, which has a peak mobility of 2.5×10^5 cm^2 V^{-1} s^{-1}, the transition between insulating and metallic behaviour occurs at a density of $p_c = 5 \times 10^{10}$ cm^{-2} ($r_s \sim 12$). Sample B [17] is a similarly designed heterostructure to sample A, but with a 30% lower mobility (at the same p_s) than sample A. Sample C was grown in a different MBE system to samples A and B, consisting of an ultra-high mobility modulation-doped GaAs single heterojunction with both an in situ back-gate and a Schottky front-gate [18]. Sample C is of exceptionally high quality, with a peak mobility in excess of 1×10^6 cm^2 V^{-1} s^{-1}, and the transition from insulating to metallic behaviour occurring at a density of $p_c \sim 1 \times 10^{10}$ cm^{-2}.

Measurements were performed using standard low-frequency AC lockin techniques with excitations as low as 0.01 nA or 6 μV to avoid heating effects.

3. Apparent insulator–metal transition in 2D hole systems

Fig. 1(a) shows the temperature dependence of the resistivity for sample A, for a range of different carrier densities. Insulating behaviour is observed for densities below 4.6×10^{10} cm^{-2}, with the resistivity increasing rapidly as the temperature is reduced. At higher densities metallic behaviour is observed, with the resistivity dropping as the temperature is reduced. Remarkably it is possible to 'scale' each of the $\rho(T)$ traces in Fig. 1(a) onto one of two curves, by re-plotting the data against T/T_0 [15]. The scaling parameter T_0 depends only on the density p_s. This is shown in Fig. 1(b), where all the data can be seen to lie either on the 'insulating' branch, or on the 'metallic' one. It has been suggested that this ability to collapse the $\rho(T)$ data onto two distinct branches, first reported for Si MOSFETs, is evidence for a quantum-phase transition between insulating and metallic ground states of the 2D system [9,10].

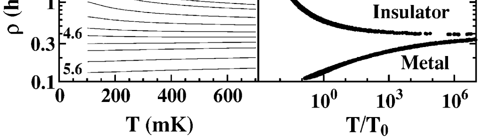

Fig. 1. (a) Temperature dependence of the resistivity for sample A at different carrier densities. (b) The same data plotted as a function of T/T_0.

Despite the low density and strong hole–hole interactions ($r_s > 10$) the insulating behaviour in sample A can be described by conventional variable range hopping (VRH) conduction

$$\rho(T) = \rho_0 \exp(T/T_0)^{-m}. \tag{1}$$

At the very lowest densities $m = \frac{1}{2}$ [15], characteristic of VRH in the presence of a Coulomb gap [19]. At higher densities, close to the transition to metallic behaviour, m crosses over to $\frac{1}{3}$, characteristic of conventional phonon-assisted VRH [17]. The collapse of all the $\rho(T)$ data onto a single trace in the insulating regime is not surprising, as it is a natural consequence of conduction by VRH. If the prefactor ρ_0 is constant then Eq. (1) implies that $\rho(T)$ depends only upon T/T_0. A gradual crossover from $m = \frac{1}{3}$ to $\frac{1}{2}$, with a corresponding gradual change of the prefactor ρ_0, will not affect this collapse as long as the change in ρ_0 is small.

However, the ability to collapse both the insulating and metallic-like $\rho(T)$ data onto two branches is less readily explained, and has attracted considerable attention. Nevertheless, it should be noted that the collapse in the metallic regime is invariable less satisfactory than in the insulating regime – and that not all experimental studies agree with this scaling in the metallic regime.

Attempts to examine the functional form of the individual metallic $\rho(T)$ traces have shown that the metallic behaviour can be well described by the empirical formula [20]

$$\rho(T) = \rho_0 + \rho_1 \exp[-T_1/T]^n. \tag{2}$$

In this four-parameter model, ρ_0, ρ_1, and T_1 are density-dependent fitting parameters, and n is usually (but not always) close to 1. A fit of Eq. (2) to the metallic-like temperature-dependent resistivity is shown in Fig. 2 for three different carrier densities in the metallic regime. Close to the transition, Fig. 2(a), the resistivity drops with decreasing T, and begins to saturate at $T = 100$ mK. As the carrier density is increased and we move further from the transition, the metallic behaviour becomes weaker and the saturation becomes visible at higher temperatures. At the highest density (Fig. 2(c)) the drop in resistance is rather small, and saturates at $T = 350$ mK.

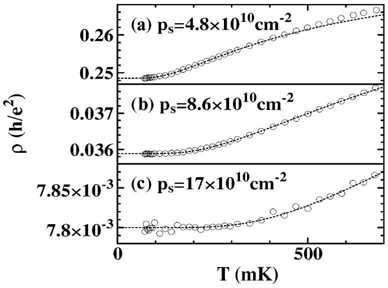

Fig. 2. Metallic-like temperature dependence of the resistance for sample A at different carrier densities. Solid lines are a fit to $\rho(T) = \rho_0 + \rho_1 \exp[-T_a/T]$.

The empirical formula (2) therefore dictates a saturation of $\rho(T)$ as $T \to 0$. Although different from the scaling analysis of Refs. [9,10], and the scaling shown in Fig. 1(b), it is still consistent with the existence of a 2D metallic state because $\rho(T)$ remains finite as $T \to 0$.

4. Influence of an in-plane magnetic field

One of the most intriguing aspects of the metallic behaviour observed in dilute 2D systems is the unusually large effect of a magnetic field $B_{||}$ applied parallel to the 2D plane. In Si MOSFETs, the application of a parallel magnetic field rapidly destroys the metallic behaviour [21,22]. A parallel magnetic field of only a few Tesla produces an extraordinary increase of the resistance, with ρ increasing by up to three orders of magnitude from the $B = 0$ value. A similar suppression of the metallic behaviour was observed in GaAs 2D hole systems [15], as shown in Fig. 3. The parallel magnetic field causes a significant increase in the resistivity, driving the system towards the strongly localised regime. The magnitude of the increase in ρ (a factor of ~ 3) is smaller than in Si MOSFETs, although this is consistent with the observation that the metallic-like increase in resistance with increasing T is smaller in GaAs hole systems than Si MOSFETs.

For an infinitely thin 2D system an in-plane magnetic field should couple only to the electron (or hole) spin, suggesting that the suppression of the

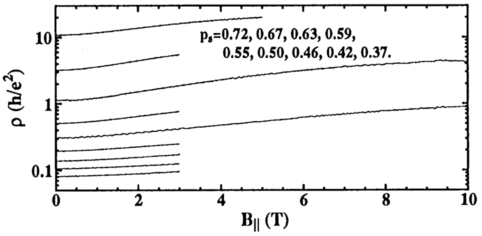

Fig. 3. Resistivity of the 2D hole system in sample A as a function of applied parallel magnetic field $B_{||}$ at $T = 0.27$ K for the hole densities indicated on the graph (in units of 10^{11} cm^{-2}).

metallic behaviour is related to a lifting of the spin degeneracy. For example a spin-polarised electron (or hole) system is less able to screen impurities than a spin degenerate system, so an increase in the applied magnetic field will increase the resistance due to ionised impurity scattering [23].

However, all real 2D systems have a finite thickness, which allows coupling of the in-plane magnetic field to the orbital momentum of the 2D carriers. This coupling becomes increasingly important at low carrier densities, because the 2D system becomes 'thicker' at low densities: $t \propto p_s^{-1/3}$. Recent calculations by Das Sarma and Hwang have shown that this coupling can produce more than an order of magnitude increase in the resistance of 2D hole systems as a parallel magnetic field is applied, in qualitative agreement with experiments [24]. Thus the suppression of the 2D metallic behaviour by an in-plane magnetic field is not necessarily proof of a spin-related origin for the metallic behaviour.

5. Re-entrant insulating–metallic–insulating behaviour

The metallic behaviour described in Section 3 is in contrast to earlier studies of more disordered, weakly interacting, 2D systems which demonstrated that both phase-coherent weak localisation and weak electron–electron interactions caused a logarithmic decrease in the conductivity as $T \to 0$. The natural question that this comparison raises is: why are these logarithmic corrections not observed in the more recent studies of low disorder, strongly interacting 2D systems?

Clearly, if it is possible to simply turn the interactions off, the earlier results from non-interacting systems should be recovered – that there is no 2D metal because all states in 2D are localised. The effective strength of the interactions, r_s decreases with increasing carrier density

$$r_s = m^* e^2/(4\pi\varepsilon\hbar^2\sqrt{\pi p_s}). \tag{3}$$

For GaAs hole systems m^* is not well defined, with values ranging from $0.2m_e$ to $0.4m_e$ in the literature. We take $m^* = 0.3m_e$, although using a larger m^* will simply increase the values of r_s calculated from the measured p_s.

It might therefore be expected from Eq. (3) that a second transition from metallic behaviour to insulating behaviour might occur at *high* carrier densities where interactions are weak, in contrast to the transition shown in Fig. 1(a) which occurs at low densities as p_s is reduced.

Alternatively, it may be that the logarithmic corrections are still present, but are masked by the large exponential drop in $\rho(T)$ shown in Fig. 2. In this case, reducing the magnitude of the metallic-like drop in resistance described by Eq. (2) should make it possible to once again resolve the logarithmic corrections. As shown in Fig. 2, the metallic behaviour becomes weaker as the density p_s is increased, suggesting that if they are still present the logarithmic corrections should re-appear at high carrier densities.

We now examine in detail the behaviour of the resistance at low, intermediate and high carrier densities [17]. In Fig. 4, we plot the temperature dependence of the resistance for sample B. We deliberately use a lower mobility (more disordered) sample for this study, to weaken the metallic behaviour and make it possible to resolve any logarithmic corrections to $\rho(T)$. At the lowest carrier densities, where $\sigma < e^2/h$, the system is strongly insulating and there is a large increase in resistivity with decreasing T (Fig. 4(a)). In this regime, close to the transition, ρ follows the form for Mott variable range hopping, Eq. (1), with $m = \frac{1}{3}$.

Increasing the carrier density causes a transition to metallic behaviour (regime II), in which the

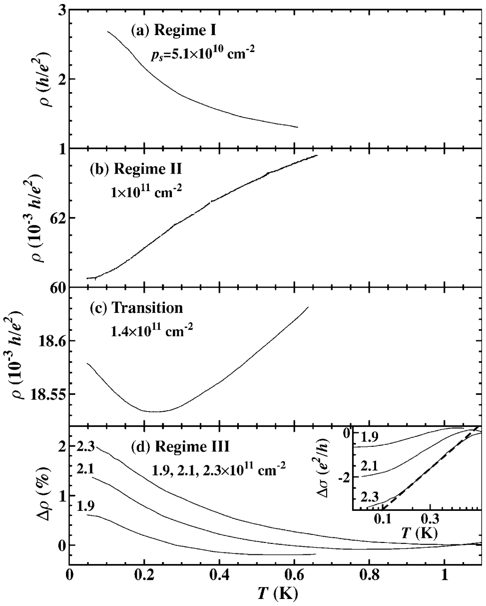

Fig. 4. Temperature dependence of the resistivity at different carrier densities for sample B, showing three distinct regimes. (a) Strongly localised regime: ρ shows hopping conduction. (b) Metallic regime: ρ decreases exponentially as T is reduced. (c) Transition between regimes II and III: The metallic behaviour is weaker at this higher density, with insulating behaviour visible at $T < 0.2$ K. (d) Regime III: Fractional change in resistivity for three different carrier densities. The inset shows the equivalent change in conductivity on a semi-log axis. The dashed line shows that $\Delta\rho \propto \log(T)$ over a limited temperature range.

resistivity decreases as the temperature is lowered (Fig. 4(b)). This metallic behaviour is considerably weaker than observed in the higher quality samples A and C, with only a 6% decrease in ρ at T is reduced.

As the carrier density is further increased the metallic behaviour becomes less pronounced, and a gradual transition to an insulating state occurs. Fig. 4(c) shows the behaviour of the resistivity at the second transition, with $p_s = 1.4 \times 10^{11}$ cm^{-2}. At high temperatures metallic behaviour is observed ($\partial\rho/\partial T > 0$), but for $T < 200$ mK there is a change in sign of $\partial\rho/\partial T$ as a weak insulating state emerges. The presence of both metallic and insulating behaviour at the same carrier density means that this is not a sharp transition from metallic to insulating behaviour, but a gradual cross-over. This insulating state persists to higher temperatures as the carrier density is increased. The strengthening of the insulator with increasing density is shown in Fig. 4(d), where the percentage change in resistivity from the $T = 1$ K value is plotted for three different densities. At a density of $1.9 \times 10^{11}\,\mathrm{cm}^{-2}$ the insulating behaviour is visible to 500 mK, and by 2.3×10^{11} cm^{-2} it is observed up to $T = 1$ K.

To determine whether this high-density insulating behaviour is due to logarithmic corrections [1,6], we plot the change in conductance, $\Delta\sigma = \sigma(T) - \sigma(T = 1\ \mathrm{K})$, against the temperature on a log scale in the inset of Fig. 4(d). At the largest density, furthest from the II → III transition, log(T) behaviour can be observed over a limited temperature range (150–600 mK). This data is very similar to that observed in previous studies of weakly interacting electron gases in lower mobility silicon inversion layers [7]. At high temperatures there is a deviation from log(T) behaviour, as phonon scattering [25] and temperature-dependent screening of impurity scattering become important. There is also an apparent saturation of $\Delta\sigma$ at low temperatures (which is not due to unintentional heating effects), such that it is not possible to unambiguously identify log(T) behaviour as the change in conductance is small ($\Delta\sigma < 2\%$) and the temperature range limited.

Having demonstrated the existence of a re-entrant transition from insulating–metallic–insulating behaviour, a phase diagram for the stability of the metallic behaviour was mapped out, as shown in Fig. 5 [17]. To speed-up the measurement process, a DC electric field E was used to heat the 2D hole system, and locate the transition between insulating and metallic behaviour. Both at low and high densities the sample is insulating, so that $\rho(E) < \rho(0)$. In the intermediate density range metallic behaviour is observed, with $\rho(E) > \rho(0)$. The transition between the two types of behaviour is identified by the point at which $\rho(E) = \rho(0)$. Application of an in-plane magnetic field 'squeezes' the metal from both sides, until at $B_{||} \gtrsim 0.7$ T insulating behaviour is observed for all carrier densities.

For zero applied magnetic field ($B_{||} = 0$) the low-density transition between insulating behaviour (due to strong localisation) and metallic behaviour is sharp and well defined (as can also be seen in Fig. 1). However, the high-density transition

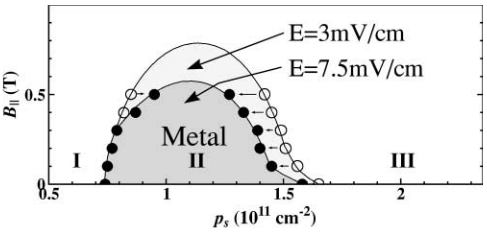

Fig. 5. Phase diagram for the observation of the 2D metallic behaviour as a function of carrier density and in-plane magnetic field for sample B. The transitions are identified from the carrier densities at which $\rho(E) - \rho(0) = 0$, using $E = 3$ and 7.5 mV cm^{-1} for the closed and open symbols, respectively.

is not as sharp, with the cross-over from metallic to insulating behaviour moving to lower densities as E is reduced (Fig. 4). If we compare the transitions identified at higher hole temperatures ($E =$ 7.5 mV cm^{-1}, open symbols in Fig. 5) with those identified at lower temperatures ($E = 3$ mV cm^{-1}, closed symbols) we see that the effect of reducing the hole temperature is to shrink the range of densities over which the metallic behaviour is observed. The first transition is sharp down to the lowest hole temperatures, but Fig. 5 provides a graphic illustration of how the metallic behaviour is extinguished from the high-density side as E (or T) tends to zero, and it raises the question as to the ultimate low-temperature fate of the metallic behaviour. What happens to the quantum corrections at lower densities – will localising corrections always take over at $T = 0$, or is there a density range over which a true 2D metallic phase exists?

6. Weak localisation and interaction corrections

It is extremely difficult to determine the presence (or absence) of quantum corrections solely from the $B = 0$ $\rho(T)$ data, as these logarithmic corrections can easily be masked by other semi-classical mechanisms such as temperature-dependent screening. However, detailed studies of the low-field magnetoresistance in low-density GaAs hole systems [26] have shown that weak localisation effects are still present in the metallic regime, close to the transition to strongly insulating behaviour (i.e. in the vicinity of the transition between regimes I and II). Furthermore similar evidence for the co-existence of weak localisation and metallic behaviour of $\rho(T)$ has also been found in SiGe electron systems [27]. These results prove that phase coherent effects are not responsible for the 2D metallic behaviour, as they provide a localising correction which grows stronger as $T \to 0$.

In addition to phase coherent weak localisation, hole–hole (or electron–electron) interactions can produce an additional correction to the Drude conductivity. It is possible to identify and separate these two quantum corrections through measurements of the low-field Hall effect [7]. Careful measurements of GaAs hole systems [26] have confirmed the presence of this logarithmic correction to the conductivity, which was found to be localising, acting to reduce the conductivity to zero as $T \to 0$.

Both the weak localisation and interaction effects described above were found to produce localising corrections to the conductivity, independent of whether $\rho(T)$ was showing insulating or metallic behaviour. This strongly suggest that neither phase coherent effects nor electron–electron interactions are responsible for the metallic behaviour observed in high-mobility 2D systems. Instead these results indicate that the metallic behaviour may be a finite temperature effect, and not a signature of a true quantum-phase transition.

7. Semi-classical origins of the 2D metallic behaviour

If there is no true 2D metal, one final question remains – what then is the origin of the widely observed metallic-like drop in resistance? A number of semi-classical mechanisms have been proposed, including temperature-dependent carrier–carrier scattering in a two-band system [20,28], and temperature-dependent screening [29,30]. We now consider the relevance of these two semi-classical models, at low carrier densities, close to the transition from metallic-like to strongly localising behaviour.

We first examine the relevance of the two-band model to the metallic behaviour observed in dilute

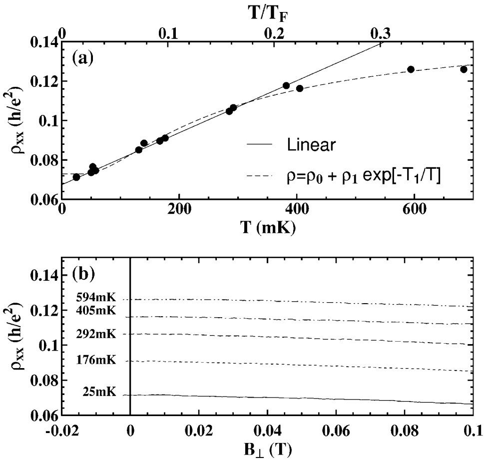

Fig. 6. (a) Temperature dependence of the zero-field ($B = 0$) resistivity in the metallic regime for sample C. The carrier density is 2×10^{10} cm^{-2}, and the back-gate bias is 1.7 V. (b) Low-field resistivity ρ_{xx} as a function of perpendicular magnetic field over the same temperature interval.

2D hole systems. Fig. 6(a) shows the characteristic drop in the zero magnetic field ($B = 0$) resistivity ρ of sample C with decreasing temperature. At this low hole density of 2×10^{10} cm^{-2} (close to, but on the metallic side of, the "metal"–insulator transition) the resistivity decreases by 50% as the temperature is reduced from 700 to 30 mK, with no signs of saturation as $T \to 0$.

A lack of inversion symmetry in a 2D system can lift the "spin" degeneracy of the electrons (or holes), creating two distinct bands. If the carriers in the two bands have different transport properties this can give rise to a metallic $\rho(T)$ over a limited temperature range. Indeed studies of the metallic phase in high density hole systems, far from the metal–insulator transition, show a strongly temperature-dependent positive magnetoresistance in a perpendicular magnetic field that is characteristic of conduction in a two-band system [28,31,32]. Further evidence for carrier–carrier scattering causing the metallic behaviour comes from a comparison of the magnitudes of the increase of the sample resistivity with temperature, and the increase with magnetic field. In these studies it was shown that, ignoring Landau quantisation, $\rho(T = \infty) - \rho(T = 0) = \rho(B = \infty) - \rho(B = 0)$. All of these observations are consistent with the metallic behaviour resulting from scattering in a two-band system. However these experiments have been conducted at high carrier densities, (2–4) $\times 10^{11}$ cm^{-2}, far from the transition to strong localisation, and it is not clear that the results can be related to the behaviour observed at much lower densities where the metallic behaviour is strongest.

Here, we concentrate on the metallic-like behaviour immediately in the vicinity of the transition, with carrier densities an order of magnitude smaller than used in Refs. [28,31,32]. This low density has the additional advantage that complications due to the anisotropy and non-parabolicity of the valence band structure which occur at higher energies are avoided. Fig. 6(b) shows the low-field magnetoresistance for five different temperatures between 25 and 600 mK at $p_s = 2 \times 10^{10}$ cm^{-2}. In these ultra-high-quality samples the mean free path l is sufficiently long, weak localisation effects are almost unobservable even at the lowest measurement temperatures. However, the positive magnetoresistance due to two-band conduction is a semi-classical effect that persists to much higher temperatures. Yet even though there is a factor of two drop in the zero-field resistivity as T decreases from 700 to 30 mK, the absence of a positive magnetoresistance shows that the $B = 0$ "spin-splitting" due to inversion asymmetry is extremely small. These results suggest that the two-band model of temperature-dependent scattering is not appropriate at low densities, and cannot be solely responsible for describing the 2D metallic behaviour.

Further support for this argument comes from direct tests of the effect the asymmetry of the confining potential has on the metallic behaviour close to the transition. A combination of front and back-gates is used to vary the asymmetry of the potential confining the 2D holes while keeping the carrier density almost constant. The influence of the back-gate bias on the potential profile and hole wave function is calculated self consistently [33], and plotted in Fig. 7(a–c) for the three back-gate biases of 0, 1.25 and 1.7 V, corresponding to electric fields of 0–14 kV cm^{-1}.

A measure of the 'strength' of the metallic behaviour is obtained by examining the magnitude of the increase in $\rho(T)$ with increasing T. To allow a direct

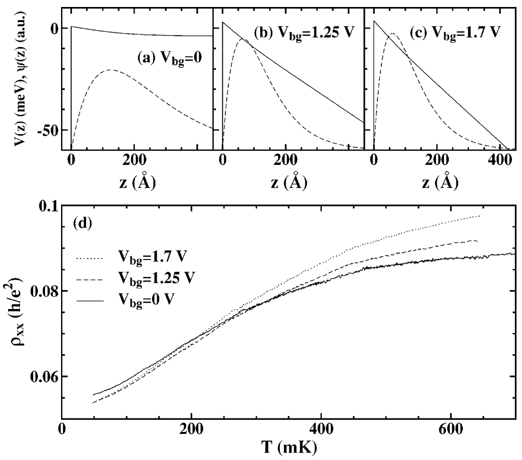

Fig. 7. Properties of sample C as a function of the asymmetry of the confining potential, with the $T = 30$ mK resistivity fixed at $0.055h/e^2$ ($k_F l = 18$). (a–c) Potential profiles $V(z)$ (solid lines) and hole wave functions $\psi(z)$ (dashed lines). (d) Corresponding temperature dependence of the resistivity.

comparison between $\rho(T)$ traces under different symmetry conditions, it is necessary to adjust the carrier density with the front-gate bias so that each $\rho(T)$ trace starts from the same point. Thus, there is a slight increase in p_s from 1.8 to 2.3×10^{10} cm^{-2} as the back-gate bias is increased from 0 to 1.7 V. Although the carrier density is not constant in these measurements, it only varies by 30%, whereas the total electric field across the 2D hole system increases fivefold from 3 to 17 kV cm^{-1}. This can be seen in the plots of the potential profiles and hole wavefunctions in Fig. 7(a–c). The corresponding temperature-dependent resistivity traces for the three back-gate biases are shown in Fig. 7(d). Despite the large change in the symmetry of the confining potential, the traces are almost indistinguishable. This confirms that the strength of the metallic behaviour at low densities is not solely determined by the asymmetry of the confining potential, and is therefore not simply due to the degree of band splitting in a two-band model [34].

We now move on to consider the effects of disorder and screening, looking in particular at the functional form of $\rho(T)$. Although the exponential formula (2) provides a reasonable fit to both our and other data [14,20,26], as shown in Figs. 2 and 6(a), we note that this is a four parameter fit, and it could be argued that with four free parameters it is possible to fit most smoothly varying, monotonic curves using a number of different functions. Furthermore, it is not easy to distinguish between the exponential formula and other kinds of polynomial behaviour. For example, temperature dependent screening produces (to first order) a linear increase in ρ with increasing T [29], which saturates at low temperatures (as $T \to 0$), and is overtaken by a $1/T$ decrease in ρ when the 2D system becomes non-degenerate ($T > T_F$) [35]. Interestingly, the experimental data plotted in Fig. 6(a) and in Ref. [34] show that the resistivity increases approximately linearly with increasing temperature for $T/T_F < 0.2$. A similar linear increase of ρ with increasing T has also been reported in silicon MOSFETs [36].

The metallic behaviour in these 2D systems, therefore, has similarities to that expected from temperature-dependent screening. Indeed, previous experiments have shown that in the metallic-like regime the fractional decrease in conductivity with increasing T depends only upon T/T_F for temperatures down to 0.3 K [15]. We now re-examine this result, using a variety of 2D systems and extending the measurements to lower temperatures. Fig. 8(a) shows the temperature dependence of ρ at different carrier densities in the range 5.1–14×10^{10} cm^{-2} for the quantum well sample A. This data is re-plotted in Fig. 8(b) to show the fractional change in conductivity $\Delta\sigma/\sigma$ against T/T_F. Here $\Delta\sigma$ is the change (decrease) in conductivity with increasing T, and T_F is the Fermi temperature. Remarkably, despite a three-fold variation in carrier density, all the data collapse onto a common curve which is linear at low T/T_F, and saturates at higher T/T_F as the 2D system becomes non-degenerate.

The metallic behaviour observed in the higher quality single heterojunction sample C is shown in Fig. 8(c), where we plot the temperature dependence of the resistivity for carrier densities in the range 1.6–3.5×10^{10} cm^{-2} [34]. The resistivity shows strongly metallic behaviour, with up to a twofold decrease in ρ as T is reduced. Despite the higher quality and lower densities achievable in this sample, the corresponding $\Delta\sigma/\sigma$ data again collapse onto a common trace when plotted against T/T_F [Fig. 8(d)]. As with sample A, $\Delta\sigma/\sigma$ is linear in

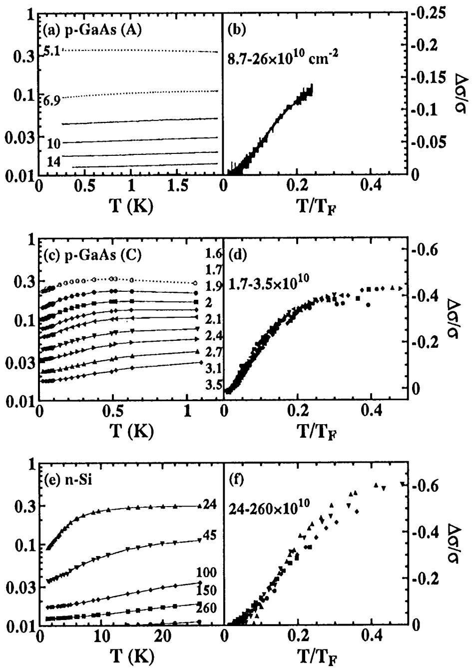

Fig. 8. Metallic behaviour in different p-GaAs 2D hole systems and n-Si MOSFETs. (a) Temperature dependence of the zero-field resistivity at different carrier densities in sample A. (b) corresponding $\Delta\sigma/\sigma$ as a function of the dimensionless temperature T/T_F, for a wider range of carrier densities. (c) Metallic behaviour of $\rho(T)$ in sample C, and (d) corresponding $\Delta\sigma/\sigma$ for 14 equally spaced densities in the range $1.7\text{–}3.5\times10^{10}$ cm^{-2}. (e, f) Equivalent data for the 2D electron system in a silicon MOSFET.

T/T_F at low temperatures ($T/T_F < 0.2$), and flattens off at higher temperatures.

It is instructive to compare the metallic behaviour observed in these 2D hole systems with that found in the 2D electron system of a high quality silicon MOSFET, to see if a similar scaling of $\Delta\sigma/\sigma$ against T/T_F occurs. In Fig. 8(e), we plot the temperature dependence of the resistivity of a standard silicon MOSFET with a peak mobility of 15×10^3 cm^2 V^{-1} s^{-1} [37]. Strong metallic-like behaviour is observed, with up to a three-fold decrease in ρ as T is reduced below ~ 10 K. The corresponding fractional change in conductivity over the same range of T/T_F as for the p-GaAs data is plotted in Fig. 8(d), where the data again collapse onto a common trace despite spanning an order of magnitude in carrier density. As with the 2D GaAs hole system, $\Delta\sigma/\sigma$ is linear in T/T_F at low temperatures ($T/T_F < 0.2$), and flattens off at higher temperatures. The similarity to the p-GaAs data in Fig. 8(b) is remarkable. This result is even more surprising given that the peak mobility is almost two orders of magnitude lower than for the p-GaAs, and the carrier densities are 1–2 orders of magnitude larger.

The observation that $\Delta\sigma/\sigma$ scales as T/T_F, for samples from different heterostructures, grown in different MBE systems, and the similarity to behaviour observed in an entirely different material system, is an intriguing result. The data show that although the metallic-like behaviour takes a similar form, in the p-GaAs samples it is clearly stronger for the higher quality devices where lower carrier densities can be reached. Temperature-dependent screening is an appealing mechanism for the metallic behaviour as it does not depend on the details of the semiconductor system, and can cause an approximately linear decrease in conductivity with increasing temperature [29]:

$$\Delta\sigma/\sigma = -C(T/T_F) - D(T/T_F)^{3/2} + O(T/T_F)^2. \quad (4)$$

Furthermore, if the parameters C and D are density independent, the fractional change in conductivity will depend only on T/T_F. This would cause a collapse of the $\Delta\sigma/\sigma$ data on to a common trace, as seen in Fig. 8.

The analytical theory of Ref. [29] shows that C and D are density dependent, becoming larger as the carrier density decreases. This is consistent with the trend we observe between p-GaAs samples, where the magnitude of $\Delta\sigma/\sigma$ is larger for the low-density data (sample C) than the data at higher densities (sample A). However, it cannot explain the observation that within a given sample all the $\Delta\sigma/\sigma$ data in the metallic regime collapse onto a single trace, with a deviation of less than 13% despite a two- or three-fold variation in carrier density. A more complex form of temperature-dependent screening which takes into account self-consistent screening and multiple scattering effects [38,39], may therefore be necessary to fully explain the dependence of $\Delta\sigma/\sigma$ upon T/T_F, as well as any possible relationship between Eq. (2) and T-dependent screening. Nevertheless, whatever the functional form of $\rho(T)$, temperature dependent screening is clearly important in dilute 2D systems, and can even qualitatively account for the experimentally observed suppression of the metallic behaviour by an in-plane magnetic field [23].

8. Conclusions

It is now well established that a metallic-like behaviour of $\rho(T)$ can be observed in low density, low disorder 2D systems, although the physical origins of this phenomena are still a subject of some controversy. The central issue is: is this metallic behaviour a signature of a true metallic state that persists to $T = 0$, or is it only a finite temperature effect? To shed light on this question, we have performed a systematic study of the metallic behaviour observed in high quality, low density 2D hole systems. These systems show all the signatures of a $B = 0$ "metal"–insulator transition, including a significant decrease in ρ below $T = 1$ K, and a suppression of the metallic behaviour by an in-plane magnetic field. However, a number of experiments indicate that neither phase coherent effects nor hole–hole interactions are responsible for the apparent 2D metal–insulator transition. Both of these effects are present in the metallic regime and *both* give rise to localising corrections to the classical Drude conductivity at low temperatures. Despite the strong hole–hole interactions ($r_s > 10$), we find that both mechanisms can be well described by theories developed for weakly interacting systems – a puzzling result that merits further study.

As these results indicate that the metallic behaviour observed in these dilute 2D systems is a finite temperature effect, we finally consider the relevance of semi-classical mechanisms to the metallic behaviour. In particular, we focus on the possibility of (i) temperature dependent hole–hole scattering in a two-band system, and (ii) temperature-dependent screening. Our data suggest that the former model is insufficient to fully describe the metallic behaviour observed in low-density 2D hole gas systems. However, we find that despite large differences in sample quality and 2D carrier densities, the metallic behaviour in *both* GaAs hole systems and Si electron systems takes a similar form: $\Delta\sigma/\sigma$ depends only on T/T_F. Although temperature-dependent screening may account for some aspects of the data, there are many unexplained discrepancies, and further work will be needed to fully explain the cause of the metallic behaviour in dilute 2D systems.

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